A Novel Approach to Reducing Ranking Discrepancies in Tennis Based on Tournament Choices

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Abstract

In tennis, discrepancies between rankings and head-to-head results arise because players select different tournaments for ranking. Existing methods using directed graphs cannot address discrepancies caused by varying tournament choices. This study proposes a ranking aggregation method that considers players' selected tournaments to reduce these inconsistencies. The method aggregates all chosen tournaments to form a collective ranking. Experimental results show a reduction in overall discrepancies, although some players saw an increase. This approach offers a partial solution to ranking inconsistencies caused by differing tournament selections in tennis.

Keywords: Tennis Rankings, Ranking Discrepancies, Tournament Selection

1. Introduction

1.1 Research background

In sports, consider two players, A and B, and the creation of a ranking that includes them. Despite A being ranked higher than B, there may be situations where B achieves better results than A. Such a situation is referred to as a "contradiction" in this study. In sports like baseball and basketball, the relative strengths (rankings) of players can be represented using directed graphs, and the contradictions with match results can be counted. Rankings that minimize the total number of contradictions can then be created [1]. However, in tennis, players participate in different tournaments, leading to situations where one context recognizes a contradiction, while another does not. In other words, existing formulations cannot address cases where contradictions are perceived differently between two players.

1.2 Objective of the study

The objective of this research is to develop a generalized ranking system that addresses discrepancies between competition outcomes and rankings, using tennis as a case study. While acknowledging the inherent challenges in perfectly aligning results and rankings, the proposed system aims to provide a more consistent and fair evaluation framework applicable across various sports or competitive domains.

2. Formulation

2.1 ATP Rankings

The ATP ranking is the global ranking system for tennis. Rankings are determined based on the total points from the top 19 tournaments in which a player earned the most points within the past 52 weeks [2]. The ATP determines points based on tournament categories and placements.

2.2 Proposed Formulation

In ATP rankings, players participate in different tournaments. In this study, we propose creating a new ranking by aggregating match results from all tournaments played by the top 10 ranked players. Let $\mathrm{N} = \{\mathbf{1},\mathbf{2},\cdots,n\}$ represent the set of players, and $T = \{t_1, t_2, \cdots, t_m\}$ the set of tournaments. For a player i, the tournaments they select (number of tournaments I) are $T_i = \{t_i^1, t_i^2, \cdots, t_i^I\}$. Match tables are created for each tournament selected by player i. The match table is an $n \times n$ antisymmetric matrix, where the performance of element i against element j is represented by w_{ij} determined as follows:

$$w_{ij} = \begin{cases} \mathbf{1} & win\ for\ i, \\ \mathbf{0} & drow, \\ \mathbf{-1} & loss\ for\ i. \end{cases}$$

Here, a "win" means achieving better results than the opponent in a tournament, even without a direct match (e.g., the tournament champion is considered to have "won" against the players who placed second, third, or fourth based on their relative rankings). Conversely, worse results indicate a "loss," and identical results mean a "draw." To prevent players participating in more tournaments from having an advantage, tournaments not selected during ATP ranking calculation are treated as losses. If two players did not participate in the same tournament, it is treated as a draw.

Under these conditions, match tables are created for all tournaments selected by the top 10 players.

These match tables are then aggregated. The aggregation method proposed in this study is based on the method devised by Yoshitsugu Yamamoto et al. [3].

The similarity constant c_{ij} between rankings when elements i and j exist is defined as follows:

$$c_{ij} = i \text{ higher} - j \text{ higher}$$

(Net tournaments where i ranks higher than j)

Using the matrix C, whose elements are c_{ij}, a comprehensive ranking is created by maximizing the consistency between rankings. The decision variable x_{ij} is defined as follows:

$$x_{ij} = \begin{cases} 1 & i \text{ is ranked higher than } j, \\ 0 & \text{otherwise.} \end{cases}$$

The objective function and constraints are:

$$\begin{aligned} &\text{Maximize} && \sum_{i=0}^{n}\sum_{j=0}^{n} c_{ij}x_{ij} \\ &\text{Subject to} && x_{ij} + x_{ji} = \mathbf{1}, x_{ij} + x_{jk} + x_{ki} \le 2, \\ & && x_{ij} \in \{0,1\}. \end{aligned}$$

Once the X-matrix is determined, the sum of each row in the X-matrix is sorted in descending order to determine the final comprehensive ranking.

3. Experiments

3.1 Data Used

Table 3.1 lists the top 10 players in the ATP rankings and their points as of December 4, 2023 [2].

Table 3.1 ATP Rankings

Ranking	Player	Points
1	Djokovic	11,245
2	Alcaraz	8,855
3	Medvedev	7,600
4	Sinner	6,490
5	Rublev	4,805
6	Tsitsipas	4,235
7	Zverev	3,985
8	Holger Rune	3,660
9	Hurkacz	3.245
10	Fritz	3,100

3.2 Experimental Method

A match-up table was created for the tournaments used to determine the ATP rankings of the 10 players in Table 3.1. The total match-up data was aggregated to compute the constants c_{ij}. Based on the resulting C-matrix, the decision variable x_{ij} was determined using Gurobi [4]. By summing each row of the X-matrix and sorting the values in descending order, the final rankings were created.

3.3 Experimental Results

The C-matrix obtained by summing the match-up tables

$$C = \begin{pmatrix} 0 & 33 & 5 & 10 & 11 & 27 & 43 & 11 & 42 & 39 \\ -33 & 0 & -12 & -24 & 22 & 42 & 9 & 47 & 42 & 77 \\ -5 & 12 & 0 & -14 & 34 & 48 & 62 & 64 & 60 & 64 \\ -10 & 24 & 14 & 0 & -19 & 35 & 59 & 33 & 45 & 59 \\ -11 & -22 & -34 & 19 & 0 & 6 & 52 & 37 & 78 & 42 \\ -27 & -42 & -48 & -35 & -6 & 0 & 39 & 10 & 32 & 16 \\ -43 & -9 & -62 & -59 & -52 & -39 & 0 & -24 & -1 & 10 \\ -11 & -47 & -64 & -33 & -37 & -10 & 24 & 0 & 17 & 21 \\ -42 & -42 & -60 & -45 & -78 & -32 & 1 & -17 & 0 & 13 \\ -39 & -77 & -64 & -59 & -42 & -16 & -10 & -21 & -13 & 0 \end{pmatrix}$$

is shown below:

The optimization results obtained using Gurobi for the C-matrix are as follows:

$$X = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

By summing each row of the X-matrix and arranging them in descending order, the rankings were determined, as shown in Table 3.2:

Table 3.2 Rankings Based on the Proposed Method

Ranking	Player
1	Djokovic
2	Sinner
3	Alcaraz
4	Medvedev
5	Rublev
6	Holger Rune
7	Tsitsipas
8	Hurkacz
9	Zverev
10	Fritz

The ranking changes between the ATP rankings and those derived from the proposed method are illustrated in Fig. 3.1.

In Figure 3.1, the vertical axis represents the ranks, while the horizontal axis denotes the ranking methodology. The figure shows that, compared to the ATP rankings, Sinner, Holger Rune, and Hurkacz improved their rankings, while Alcaraz and Zverev dropped in rank. Djokovic, Medvedev, Rublev, Tsitsipas, and Fritz retained their positions.

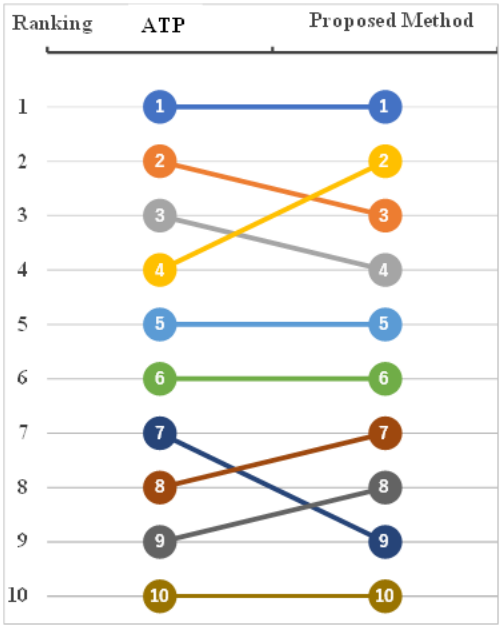

Fig.3.1 Ranking Changes Based on the Methodology

3.4 Evaluation

The ATP rankings and the rankings generated by the proposed method were evaluated based on the "number of ranking discrepancies relative to match results" from the perspective of each player. The match results for each tournament were calculated using w_{ij} as defined in Section 2.2. The ranking discrepancies were classified into the following four patterns:

Pattern A: Discrepancies are counted from both the player's and the opponent's perspectives.
Pattern B: Discrepancies are counted from the player's perspective but not from the opponent's perspective.
Pattern C: Discrepancies are not counted from the player's perspective but are counted from the opponent's perspective.
Pattern D: No discrepancies are counted from either perspective.

The number of discrepancies between match results and ranking positions in tournaments selected by each player is shown in Tables 3.3 and 3.4.

Table 3.3 Ranking Discrepancies in ATP Rankings (from each player's perspective)

Player	Pattern A	Pattern B	Pattern C	Pattern D	Total Discrepancies
Djokovic	13	0	0	95	13
Alcaraz	23	9	0	121	32
Medvedev	38	17	0	125	55
Sinner	37	23	0	111	60
Rublev	29	28	0	114	57
Tsitsipas	29	31	0	102	60
Zverev	24	38	0	118	62
Rune	32	52	0	96	84
Hurkacz	27	47	0	97	74
Fritz	26	67	0	69	93
Total	278	312	0	1048	590

Table 3.4 Ranking Discrepancies in Proposed Method (from each player's perspective)

Player	Pattern A	Pattern B	Pattern C	Pattern D	Total Discrepancies
Djokovic	13	0	0	95	13
Alcaraz	24	19	0	110	43
Medvedev	37	14	0	129	51
Sinner	33	12	0	126	45
Rublev	29	28	0	114	57
Tsitsipas	29	31	0	102	60
Zverev	22	54	0	104	76
Rune	30	44	0	106	74
Hurkacz	27	42	0	102	69
Fritz	26	67	0	69	93
Total	270	311	0	1057	581

From Tables 3.3 and 3.4, the number of discrepancies increased for Alcaraz and Zverev, whose rankings dropped. In contrast, the discrepancies decreased for Medvedev, Sinner, Rune, and Hurkacz, whose rankings improved. The increase in discrepancies for Alcaraz and Zverev is likely due to a tendency for lower-ranked players to exhibit more discrepancies. Furthermore, both the total of Pattern A and Pattern B discrepancies and the overall number of discrepancies decreased.

3.5 Discussion

Comparing the ATP rankings and the rankings generated by the proposed method, positional changes are observed between ranks 2-4 and ranks 7-9. Sinner's rise in ranking can be attributed to his head-to-head winning record against Alcaraz and Medvedev. Additionally, Alcaraz participated in fewer tournaments than Medvedev and Sinner, leading to a drop in his ranking.

For ranks 7-9, the aggregated C matrix of head-to-head match results indicates that Zverev has negative scores against both Rune and Hurkacz. This means Zverev lost more matches than he won against these players, contributing to his lower ranking.

Moreover, while the original rankings lacked a clear definition for the matchup matrix, the formulation proposed in this study treats tournaments not selected as "unattended." This definition eliminates situations where both players simultaneously win or lose, thus removing discrepancies counted only from one perspective (Pattern C). Consequently, the number of discrepancies due to

differing perspectives was reduced, addressing a key issue in this study.

4. Conclusion

4.1 Summary

This study proposed a ranking method to reduce "discrepancies between rankings and match results" in sports where the tournaments considered in the rankings differ for each player. As a result, the integration of all tournaments selected by the 10 players partially resolved the issue of tournament selection differences. Evaluation showed that while some players experienced an increase in discrepancies, the overall number of discrepancies decreased.

4.2 Future Work

The proposed formulation resulted in lower rankings for players with fewer selected tournaments (i.e., tournaments used to generate ATP rankings). Future work should develop a formulation that prevents players with fewer selected matches from being disadvantaged.

Acknowledgements

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Authors Introduction

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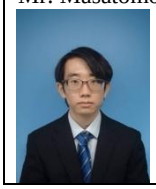

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