

# Multi-Frame Track-Before-Detect with Adaptive Number of Frame as Noise Level

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## Abstract

Multi-frame Track-Before-Detect (MF-TBD) is a batch processing method used to enhance detection and tracking performance in low SNR environments. Unlike traditional filtering techniques, MF-TBD does not apply thresholding and instead uses all observed data to reduce the risk of target loss. By integrating observations across multiple frames, it leverages space-time correlations to improve detection robustness. However, as the number of frames increases, the computational cost grows exponentially due to the need to correlate data over a larger dataset, leading to inefficiencies. Especially in high SNR conditions, where fewer frames are sufficient for accurate detection. To address this, we propose an Adaptive MF-TBD framework that dynamically adjusts the number of frames based on SNR levels.

*Keywords:* multi-frame track-before-detect, target tracking, adaptive number of frame, maneuvering target

## 1. Introduction

Target tracking is a critical process that estimates the state of a target based on continuous measurements, playing a key role in both military and commercial applications. Conventional tracking algorithms apply a threshold to each measurement, using filtered measurements for tracking [1]. However, these methods face significant challenges when the target's SNR (Signal-to-Noise Ratio) is low. In such cases, actual target-originating measurements may be filtered out by the threshold, leading to track loss [2]. To address this issue, Track-Before-Detect (TBD) was introduced, enabling tracking of targets with low SNR. However, TBD faces limitations, such as susceptibility to false alarms when clutter density is high, which reduces its tracking reliability. To improve upon these limitations, more advanced approaches like Multi-Frame Track-Before-Detect (MF-TBD) were developed [3].

MF-TBD improves tracking performance by integrating target energy across multiple consecutive frames, leveraging the spatiotemporal correlation of the target. This makes it particularly effective for weak signal

targets. However, the primary drawback of MF-TBD is its computational complexity and high memory requirement. Each frame used for computation contains a large number of measurements, and processing several frames simultaneously significantly increases computational demand. This is because MF generates trajectory candidates for the target using a fixed number of consecutive frames. As the number of frames increases, the number of potential trajectory combinations grows exponentially, leading to a significant rise in computational load [4].

To overcome these limitations, this paper proposes a novel adaptive MF-TBD method that adjusts the number of frames based on signal conditions. Using the Mean Absolute Error (MAE) metric to quantify signal qualities [5], the framework minimizes computational overhead in high-SNR conditions (low MAE) by reducing the number of frames while maintaining performance. Conversely, in low-SNR conditions (high MAE), it retains a larger number of frames to ensure robust tracking performance. This adaptive strategy ensures optimal performance across diverse SNR conditions, achieving significant computational efficiency gains compared to conventional

fixed-frame methods while maintaining tracking accuracy.

The adaptive MF-TBD framework and the use of the MAE metric for dynamic frame adjustment are detailed in Section II. Section III compares the performance of the proposed method with conventional TBD and fixed-frame MF-TBD in terms of accuracy and efficiency. Section IV concludes with a summary and discussion of its applicability to various tracking scenarios.

## 2. MF-TBD with adaptive number of frame

The target model assumes a point target performing constant velocity (CV) motion within a 2D surveillance region. The continuous target state  $x_k = [x_k, \dot{x}_k, y_k, \dot{y}_k]$  evolves according to the following Markov process

$$x_k = Fx_{k-1} + w_k \quad (1)$$

where  $F$  is the state transition matrix defined as

$$F = I_{s/2} \otimes \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \quad (2)$$

$T$  is the sampling interval, and  $w_k$  is the process noise.

The measurement model uses a pixelized approach commonly employed in TBD techniques to describe low-resolution sensors. The surveillance region is assumed to be divided into a grid of  $N_x \times N_y$ , with  $\Delta_x$  and  $\Delta_y$  representing the cell sizes in the  $x$ - and  $y$ -directions, respectively. The measurement at time  $k$  is expressed as

$$z_k(i, j) = \begin{cases} A_k + n_k^{H_1}, & \text{if target is in } (i, j) \\ n_k^{H_0}, & \text{otherwise} \end{cases} \quad (3)$$

where  $A_k$  represents the signal amplitude from the target, and  $n_k^{H_1}$ , and  $n_k^{H_0}$  are samples from independent and identically distributed (i.i.d.) zero-mean Gaussian noise processes with variances  $\sigma_k^{H_1^2}$  and  $\sigma_k^{H_0^2}$ , respectively.

Following structure of conventional MF-TBD framework is based on [6]. Merit function  $I_k(x_k|Z_{1:K})$  evaluates the likelihood of the current state  $x_k$  being target-related while incorporating the transition probabilities from the previous state  $x_{k-1}$ . This function plays a critical role in identifying the most probable trajectory of the target over  $K$  consecutive frames. Here,  $K$  represents the number of consecutive frames used to estimate the trajectory, with larger  $K$  values enabling the algorithm to better leverage spatiotemporal correlations, even though at the cost of increased computational complexity. The merit function is computed as follows

$$I_k(x_k|Z_{1:K}) = \max_{\tau(x_k)} I_{k-1}(x_{k-1}|Z_{1:K-1}) + \lambda_k(x_k) \quad (4)$$

$$\tau(x_k) = \{x_{k-1} | y_{k-1}\}^T \in \mathbb{R}^2, \\ |x_k - x_{k-1}| \leq B_1, |y_k - y_{k-1}| \leq B_2 \quad (5)$$

The set of possible previous states,  $\tau(x_k)$ , represents all  $x_{k-1}$  states that can transition to  $x_k$  while satisfying the boundary constraints. Specifically, it includes states where the displacement in the  $x$ - and  $y$ -directions lies within the respective boundaries  $B_1$  and  $B_2$ . These

boundaries defined using the Kinematics Boundary Constraint (KBC) method [7], represent the maximum allowable transitions in each direction, ensuring that only realistic and feasible state transitions are considered based on the target's dynamics.  $\lambda_k(x_k)$  denotes the likelihood of the current state  $x_k$ .

The function  $\psi(x_k)$  identifies the most likely previous state  $x_{k-1}$  that maximizes the merit function  $I_{k-1}$  while meeting the constraints defined by  $\tau(x_k)$ . It is given by

$$\psi(x_k) = \operatorname{argmax}_{\tau(x_k)} I_{k-1}(x_{k-1}|Z_{1:K-1}) \quad (6)$$

The state sequence history  $\chi_i$  represents the recovered trajectory estimate of the target from initial state up to time  $k$ .

$$\chi_{1:k} = \begin{cases} \psi_i(\chi_{i+1}(x_k)), & i = 1, \dots, k-1 \\ x_i, & i = k \end{cases} \quad (7)$$

As the number of frames  $K$  increases, the merit function for the true target tends to improve due to the accumulation of spatiotemporal information across frames. However, this comes with a significant drawback: the set of candidate states  $\tau(x_k)$ , representing all possible transitions, grows exponentially with  $K$ , leading to a rapid increase in computational complexity. Furthermore, in high Signal-to-Noise Ratio (SNR) conditions, maintaining a high frame count is unnecessary, as the target signal is already strong and distinguishable.

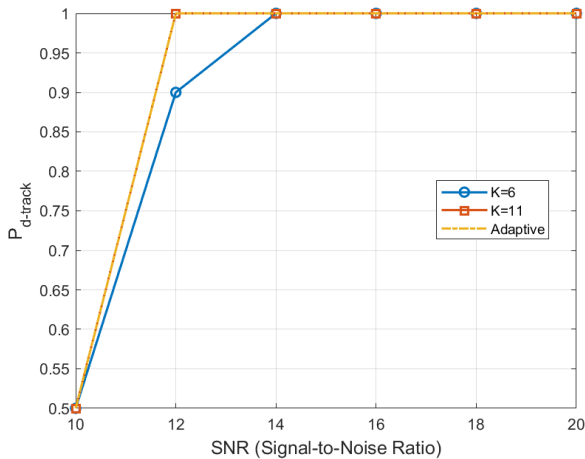
So, the proposed method adjusts the number of frames  $K$  based on the SNR. Specifically, when the SNR is high, a smaller value of  $K$  is selected to minimize computational complexity, whereas a larger  $K$  is chosen under low SNR conditions to ensure robust tracking performance. To quantify the SNR level and make this adjustment, the Mean Absolute Error (MAE) is employed as a metric, calculated as:

$$\text{MAE} = \frac{\sum |x - \hat{x}|}{n} \quad (8)$$

where  $x$  represents the observed values,  $\hat{x}$  the estimated values, and  $n$  the total number of measurements. This SNR-dependent frame adjustment ensures both computational efficiency and tracking accuracy across varying signal environments.

## 3. Simulations

In this simulations, two metrics were used to evaluate performance:  $P_{d-track}$ , which measures the probability of the algorithm returning a valid track, and computation time.  $P_{d-track}$  is defined as the case where the cell difference between the true target and the estimated target is within designed threshold  $\epsilon$ . The experiments were conducted under the following conditions. The target maneuver constant velocity (CV) with 100m/s. The observation area was structured as a  $100 \times 100$  grid, with each cell having a size of  $\Delta_x = \Delta_y = 15$  m. The tolerance to  $\epsilon = 4$ , and the maximum number of frames  $K_{max} =$

Fig. 1.  $P_{d-track}$  per SNR.

11. The adaptive frame number  $K$  was adjusted within the range of 6 to 11.

Figure 1 shows that the  $P_{d-track}$  values for each method ( $K=6$ ,  $K=11$ , adaptive) as SNR level. The results show that the  $K=11$  method outperforms the  $K=6$  method in terms of tracking performance. Furthermore, the adaptive method achieves identical performance to the  $K=11$  method, demonstrating its ability to maintain high tracking accuracy.

Figure 2 presents the computation time for each method as SNR level. When  $K=11$ , the large number of  $\tau$  values that need to be processed results in higher computation time. Conversely, the computation time is significantly lower for  $K=6$ , as fewer  $\tau$  values are required. The adaptive method exhibits selects a higher  $K$  value at low SNR, leading to increased computation time. However, as the SNR improves and the signal quality becomes better, the adaptive method reduces the  $K$  value, which progressively decreases the computation time. Proposed method demonstrated similar  $P_{d-track}$  to the  $K=11$  method, while significantly reducing computation time. Although it required more computation time than the  $K=5$  method, it achieved better tracking performance. Overall, the results show that the proposed method strikes a favorable balance between maintaining high tracking performance and reducing computation time.

#### 4. Conclusion

The proposed algorithm evaluates the signal quality using the MAE metric to appropriately adjust the number of frames, thereby reducing unnecessary computational costs. Compared to conventional MF-TBD method, it maintains similar tracking performance while significantly reducing computation time. This proposed algorithm will contribute to efficiently detecting and tracking maneuvering targets using the MF-TBD approach under various SNR conditions.

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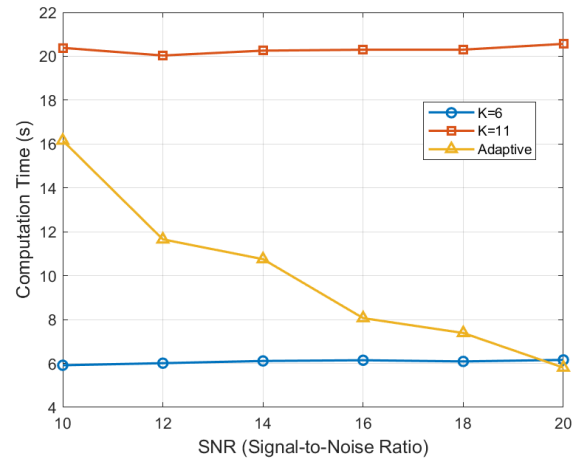


Fig. 2. Computation time per SNR.

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