Flocking Control for Multiple Convex Polygonal Agents with Obstacle Avoidance

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Abstract

This paper addresses the distributed flocking control for second-order convex polygonal multiagent systems with obstacle avoidance. Typically, agent shapes are reduced to mere points or circles in existing research. Overlooking the actual geometries of entities such as autonomous ships and vehicles may lead to suboptimal utilization of spatial resources. To rectify this, the paper introduces an approach for calculating the relative distance between two convex polygonal agents. A potential function, which balances attractive and repulsive forces, is designed based on these calculated distances. A flocking trajectory guides collective motion, and a separate obstacle-avoidance trajectory is activated when the agent's proximity to an obstacle falls below a specified safety threshold. The proposed control strategy integrates potential function and reference trajectory is designed to achieve flocking behavior and obstacle avoidance. Stability analysis proves the effectiveness of the algorithm.

Keywords: Flocking control, Multi-agent systems, Convex polygonal, Obstacle avoidance

1. Introduction

Natural phenomena such as the collective behavior of birds, fish, and ants have attracted researchers from various fields [1]. Inspired by these, Minsky introduced the concept of an agent. Advances in artificial intelligence, big data, and networking have led to the development of multi-agent systems (MASs) with autonomous and intelligent capabilities. MASs have been successfully applied in areas such as autonomous driving, drones, robotics, and sensor networks [2]. They excel at distributed cooperative control, tackling complex tasks that single agents cannot. Flocking, a key aspect of collective behavior, is a major research focus within MAS cooperative control [3].

In 1987, Reynolds introduced the Boid model [4], which describes the behavior of groups with rules for cohesion, alignment, and separation. Vicsek et al. refined this in 1995 [5] with a distributed alignment rule. Tanner et al. addressed control of varying topology in MASs [6], Subsequent flocking research addressed real-world challenges such as obstacle avoidance and limited communication. In particular, Olfati-Saber developed innovative control algorithms for flocking in complex environments, and Jafet addressed communication and input constraints [7].

Many studies simplify agents as particles or circles, ignoring practical shapes such as rectangles or polygons, which are more space efficient, especially in confined spaces such as traffic lanes (Fig. 1). This can lead to inaccurate distance estimates and suboptimal collision avoidance. Early studies [8] modeled agents as ellipses to solve these problems, but issues such as discontinuous control signals limited their applicability. Later studies [9] modeled agents as rectangles, which is good for calculating distances and interactions between agents, reducing the risk of collision and improving safety. However, in practical applications, the shape of the agents may not be monolithic. Therefore, considering the shape of the agents only as a rectangle also has limitations.

Fig.1 Advantages of considering the realities of agents for space utilization.

Motivated by the above discussion, the main contributions of this paper can be summarized as follows: (i) Adoption of convex polygonal shapes for agents, addressing spatial inefficiency in confined spaces and improving motion efficiency. (ii) Introduction of a continuous control input method for calculating relative distances between convex polygonal agents, applicable to shapes with varying numbers of vertices for greater flexibility. (iii) Integrate obstacle avoidance into the flocking process, ensuring that agents follow a reference trajectory and employ avoidance strategies when approaching obstacles, improving safety in complex environments.

The rest of this paper is organized as follows. Section 2 gives some preliminaries. Section 3 describes the problem we studied and presents the distributed flocking algorithm. Simulations are carried out in section 4. Finally, the conclusion is given in Section 5.

2. Preliminaries

2.1. *Convex Polygonal Agent Model*

Suppose that agent i is enclosed by a polygon P_i. Let's define agent i by the vector $f_i(t)=[p_i^T(t),\theta_i(t)]^T$, in which $p_i(t)=[p_{ix}(t),p_{iy}(t)]^T$ indicates the coordinates of the center O_i, and $\theta_i(t)$ is the orientation angle of the global coordinate system OXY. The coordinates of the n-th vertex of agent i are given by $\left\{p_i^k \mid 1,2,\cdots,\Lambda_i\right\}$ in the global frame OXY, and by $\left\{\tilde{p}_i^k \mid 1,2,\cdots,\Lambda_i\right\}$ in the local frame $O_iX_iY_i$, as illustrated in Fig 2.

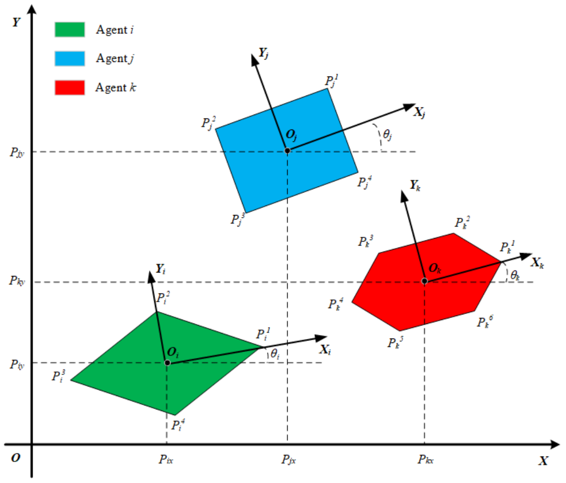

Fig.2 Convex polygonal agents in global-fixed frame and local-fixed frame

The relationship between p_i^k and $\tilde{p}_i^k$ is given as $p_i^k(t)=p_i(t)+R_i(t)\tilde{p}_i^k(t)$, $R_i(t)$ is the rotation matrix $R_i(t)=\begin{bmatrix}\cos(\theta_i(t)) & -\sin(\theta_i(t))\\ \sin(\theta_i(t)) & \cos(\theta_i(t))\end{bmatrix}$.

Agent i is equipped with a circular communication zone, having a radius of r_i centered at O_i. It can transmit its data and receive information from neighboring agents within this range, including position, direction, and speed. The communication radii for agents i and j are as follows:

$$\min\left(r_i,r_j\right)>\max_{1\le k\le\Lambda_i}\left(d\left(O_i,p_i^k\right)\right)+\max_{1\le k\le\Lambda_j}\left(d\left(O_j,p_j^k\right)\right)$$

3. Main results

3.1. *Problem formulation*

Consider a second-order multi-agent system consisting of N convex polygonal agents. We assume that each agent i has the following dynamics:

$$\begin{cases}\dot{f}_i=v_i\\ \dot{v}_i=u_i\end{cases} \tag{1}$$

where $v_i(t)=[v_{ix}(t),v_{iy}(t),\omega_i(t)]^T$ denotes the linear velocity and angular velocity of agent i, respectively. $u_i(t)=[u_{ix}(t),u_{iy}(t),u_{i\theta}(t)]^T$ denotes the control input.

The objective of this paper is to develop a distributed flocking control strategy for the multi-agent system (1) ensuring that all agents flock together with constrained control inputs and obstacles. To meet these goals, we present the following definition.

Definition 1. The multi-agent system (1) is said to achieve flocking if and only if for all agents i, the following conditions are satisfied:

(i) (Separation) The relative distance between any two agents i and j is greater than 0,

(ii) (Cohesion) The relative distance between any two agents i and j is bounded, and the error between the actual and reference trajectories is bounded, $f_r(t)=[p_r(t),\theta_r(t)]^T$, is the motion reference trajectory, $p_r(t)=[p_{rx}(t),p_{ry}(t)]$ and $\theta_r(t)$ denote the desired position the desired heading angle, respectively.

(iii) (Alignment) The velocity direction of all agents is the same.

3.2. *Control design*

Let us define a set of all successive vertices of the agent i: $S_i=\{(1,2),(2,3),\ldots,(\Lambda_i,1)\}$.

The distance between agent i and agent j is defined as:

$$\delta_{ij}=\left(\prod_{a=1}^{\Lambda_i}\prod_{(b,c)\in S_j}\delta_{ij}^{abc}\right)\left(\prod_{a=1}^{\Lambda_i}\prod_{(b,c)\in S_i}\delta_{ji}^{abc}\right)$$

$$\delta_{ij}^{abc}=\sigma\left(d_{ij}^{abc}\right) \tag{2}$$

$$d_{ij}^{abc}=\left\|p_i^a-p_j^b\right\|_2+\left\|p_i^a-p_j^c\right\|_2-\left\|p_j^b-p_j^c\right\|_2$$

Remark 1. From equation (2), it follows that δ_{ij} equals 0 if any δ_{ij}^{abc} is 0, indicating a vertex of agent i lies on the edge of agent j. Consequently, if δ_{ij} remains greater than 0 for all $t\ge 0$, it implies no collisions between agents.

To maintain separation and cohesion in flocks, we use a potential function. This function has a dual role: it generates a repulsive force to prevent collisions when agents are close together, and an attractive force to pull agents closer together when they are too far apart.

The potential function between two agents can be written as

$$\phi_{ij} = \varepsilon_1 Rp(\delta_{ij} \mid \delta_{ijD}, \lambda) + \varepsilon_2 Ar(\delta_{ij} \mid \delta_{ijD}, \delta_{ijM}) \tag{3}$$

where ε_1, ε_2 are positive real numbers, δ_{ijD} is a desirable relative distance, and δ_{ijM} is a further relative distance. $Rp(\delta_{ij} \mid \delta_{ijD}, \lambda)$ and $Ar(\delta_{ij} \mid \delta_{ijD}, \delta_{ijM})$ represents the repulsion and attraction, respectively. Similarly the potential function between the agent and the obstacle can be set as

$$\phi_{ik} = \varepsilon_3 Rp(\delta_{ik} \mid \delta_{ikD}, \lambda) + \varepsilon_4 Ar(\delta_{ik} \mid \delta_{ikD}, \delta_{ikM}) \tag{4}$$

We choose the following repulsive function and attraction function.

$$Rp(\delta_{ij} \mid \delta_{ijD}, \lambda) = \begin{cases} \dfrac{\lambda(\delta_{ijD} - \delta_{ij})^k}{\lambda\delta_{ij} + \delta_{ijD}{}^k}, & if \ \ \delta_{ij} \in [0, \delta_{ijD}] \\ 0, & otherwise \end{cases}$$

$$Ar(\delta_{ij} \mid \delta_{ijD}, \delta_{ijM}) = \begin{cases} 0, & if \ \ \delta_{ij} \in [0, \delta_{ijD}] \\ \ln \dfrac{\delta_{ijM}{}^2}{\delta_{ijM}{}^2 - \delta_{ij}{}^2}, & if \ \ \delta_{ij} \in [\delta_{ijD}, \delta_{ijM}] \\ 1, & if \ \ \delta_{ij} \in [\delta_{ijM}, \infty) \end{cases}$$

To obtain the control law, we have first to derive the partial derivatives of the potential function.

$$\begin{aligned} \dot{\phi}_{ij} &= E_{ij}(\dot{\theta}_i - \dot{\theta}_j) + F_{ij}(\dot{p}_i - \dot{p}_r - \tilde{R}(p_i - p_r)\dot{\theta}_i) + \\ &\quad F_{ji}(\dot{p}_j - \dot{p}_r - \tilde{R}(p_j - p_r)\dot{\theta}_j) \\ &= E_{ij}(\dot{\theta}_i - \dot{\theta}_r) + F_{ij}(\dot{p}_i - \dot{p}_r - \tilde{R}(p_i - p_r)\dot{\theta}_i) \\ &\quad - E_{ij}(\dot{\theta}_j - \dot{\theta}_r) + F_{ji}(\dot{p}_j - \dot{p}_r - \tilde{R}(p_j - p_r)\dot{\theta}_j) \end{aligned} \tag{5}$$

$$\begin{aligned} \dot{\phi}_{ik} = \big[& E_{ik}(\dot{\theta}_i - \dot{\theta}_r) + F_{ik}(\dot{p}_i - \dot{p}_r - \tilde{R}(p_i - p_r)\dot{\theta}_i) \\ & - E_{ik}(\dot{\theta}_k - \dot{\theta}_r) + F_{ki}(\dot{p}_k - \dot{p}_r - \tilde{R}(p_k - p_r)\dot{\theta}_k) \big] \end{aligned} \tag{6}$$

where $E_{ij} = \dfrac{\partial \phi_{ij}}{\partial \delta_{ij}}\left[\left(A_{ij}\tilde{R}(p_i - P_r) + B_{ij}\right) - \left(A_{ji}\tilde{R}(p_j - p_r) + B_{ji}\right)\right]$

$$F_{ij} = \frac{\partial \phi_{ij}}{\partial \delta_{ij}}(A_{ij} - A_{ji})$$

$$A_{ij} = \sum_{a=1}^{\Lambda_i} \sum_{(b,c)\in S_j} \frac{\partial \delta_{ij}}{\partial \delta_{ij}^{abc}} \frac{\partial \delta_{ij}^{abc}}{\partial d_{ij}^{abc}} M_{ij}^{abc}$$

$$B_{ij} = \sum_{a=1}^{\Lambda_i} \sum_{(b,c)\in S_j} \frac{\partial \delta_{ij}}{\partial \delta_{ij}^{abc}} \frac{\partial \delta_{ij}^{abc}}{\partial d_{ij}^{abc}} N_{ij}^{abc}$$

we take the following Lyapunov function

$$V = \Phi_1 + \Phi_2 + V_1(t) + V_2(t) \tag{7}$$

where

$$\Phi_1 = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \phi_{ij}$$

$$\Phi_2 = \sum_{i=1}^{N} \sum_{k=1}^{M} \phi_{ik}$$

$$V_1(t) = \sum_{i=1}^{N} \left(\varepsilon_{i\theta}\left(\sqrt{(\theta_i - \theta_r)^2 + \eta_\theta{}^2} - \eta_\theta\right) + \varepsilon_{ip}\left(\sqrt{\|p_i - p_r\|_2^2 + \eta_p^2} - \eta_p\right) \right)$$

$$V_2(t) = \frac{1}{2}\sum_{i=1}^{N} \left((\omega_i - \dot{\theta}_r)^2 + (v_{ix} - \tilde{v}_{ix})^2 + (v_{iy} - \tilde{v}_{iy})^2 \right)$$

Then the control law of the system (1) is designed as

$$\begin{cases} u_{i\theta} = -\Delta_{i\theta} - \mu_{i\theta}\sigma(\omega_i - \dot{\theta}_r) \\ u_{ix} = -\Delta_{ix} - \mu_{ix}\sigma(v_{ix} - \tilde{v}_{ix}) \\ u_{iy} = -\Delta_{iy} - \mu_{iy}\sigma(v_{iy} - \tilde{v}_{iy}) \end{cases} \tag{8}$$

where

$$\Delta_{i\theta} = D_i + C_{ik} - \ddot{\theta}_r + \frac{\varepsilon_{i\theta}(\theta_i - \theta_r)}{\sqrt{(\theta_i - \theta_r)^2 + \eta_\theta^2}}$$

$$\Delta_{ix} = C_{ix} + D_{ikx} - \ddot{p}_{rx} + \frac{\varepsilon_{ip}(p_{ix} - p_{rx})}{\sqrt{\|p_i - p_r\|_2^2 + \eta_p^2}} + (v_{iy} - \dot{p}_{ry})\omega_i + (p_{iy} - p_{ry})u_{i\theta}$$

$$\Delta_{iy} = C_{iy} + C_{iky} - \ddot{p}_{ry} + \frac{\varepsilon_{ip}(p_{iy} - p_{ry})}{\sqrt{\|p_i - p_r\|_2^2 + \eta_p^2}} - (v_{ix} - \dot{p}_{rx})\omega_i - (p_{ix} - p_{rx})u_{i\theta}$$

$$\tilde{v}_{ix} = \dot{p}_{rx} - (p_{iy} - p_{ry})\omega_i$$

$$\tilde{v}_{iy} = \dot{p}_{ry} + (p_{ix} - p_{rx})\omega_i$$

Theorem 1. Consider a multi-agent system described by the dynamics (1) and governed by the control law (8). If $\lambda > V(0)$, the following results hold:

(i) The system can achieve flocking,

(ii) The agents can avoid obstacles,

(iii) The inputs $u_{i\theta}$, u_{ix}, u_{iy} are continuous and bounded.

Proof.

Consider the Lyapunov function and take the derivative,

When $\delta_{ik} \geq \delta_{ikM}$, $D_{ik} = 0$, $C_{ik} = [0, 0]$. When $\delta_{ik} < \delta_{ikM}$, we design an obstacle-avoidance reference trajectory to make $\dot{\theta}_r = \dot{p}_{rx} = \dot{p}_{ry} = 0$.

$$\begin{aligned} \dot{V} = -\sum_{i=1}^{N} \big[& \mu_{i\theta}\sigma(\omega_i - \dot{\theta}_r)(\omega_i - \dot{\theta}_r) + \mu_{ix}\sigma(v_{ix} - \tilde{v}_{ix})(v_{ix} - \tilde{v}_{ix}) \\ & + \mu_{iy}\sigma(v_{iy} - \tilde{v}_{iy})(v_{iy} - \tilde{v}_{iy}) \big] \end{aligned} \tag{9}$$

we get $V \leqslant 0$, so $V(0) \geqslant V(t)$, $\forall t \geqslant 0$. Since $\lambda > V(0) \geqslant V(t)$, it means $\lambda = \phi_{ij}(0) > \phi_{ij}(\delta(t))$, $\forall t \geqslant 0$. The condition ”separation” is satisfied.

Since equation (53), $V(t)$ is bounded, so $V_1(t)$ is bounded $\forall t \geqslant 0$, $\|f_i - f_2\|_2$ is bounded. The ”cohesion” of flocking is satisfied.

Take the derivative of equation (9)

$$\begin{aligned} \ddot{V} = -\sum_{i=1}^{N} \big[& \mu_{i\theta}\sigma'(\omega_i - \theta_r')(\omega_i - \dot{\theta}_r) + \mu_{i\theta}\sigma(\omega_i - \dot{\theta}_r)(\mu_{i\theta} - \ddot{\theta}_r) \\ & + \mu_{ix}\sigma'(v_{ix} - \tilde{v}_{ix})(v_{ix} - \tilde{v}_{ix}) + \mu_{ix}\sigma(v_{ix} - \tilde{v}_{ix})(\mu_{ix} - \dot{\tilde{v}}_{ix}) \\ & + \mu_{iy}\sigma'(v_{iy} - \tilde{v}_{iy})(v_{iy} - \tilde{v}_{iy}) + \mu_{iy}\sigma(v_{iy} - \tilde{v}_{iy})(\mu_{iy} - \dot{\tilde{v}}_{iy}) \big] \end{aligned} \tag{10}$$

since $\|f_i - f_2\|_2$ and $\|v_i - \hat{f}_r\|_2$ and control inputs (we'll prove it next) are bounded, we can get $\ddot{V}(t)$ is bounded $\forall t \geqslant 0$, hence $\dot{V}(t)$ is uniformly continuous, according to "Lyapunov-Like Lemma", $\dot{V}(t)$ approaches 0 as t approaches ∞ . The condition "alignment" of flocking is satisfied. (i) of the theorem is proved. The proof (ii) of the theorem is similar to the proof (i).

Using trigonometric inequalities, we can show that (iii) holds true.

4. Conclusion

This paper presents a distributed flocking control algorithm for convex polygonal multi-agent systems in an obstacle environment. The optimized relative distance improves space utilization and flocking efficiency. The proposed potential function promotes cohesion and collision avoidance, while an obstacle avoidance strategy ensures safe navigation. The algorithm enables stable flocking in complex scenarios, as confirmed by simulations. In the future, we will consider higher-order and complex systems and the application of these methods to more complex obstacle environments.

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