Manipulability Optimization for Redundant Dual-Arm Robots at the Acceleration Level

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Abstract

Existing manipulability optimization schemes typically solve at the velocity level, which cannot consider joint acceleration limits and are unsuitable for torque control of robotic arms. Therefore, this paper constructs a cost function that considers both joint torque constraints and manipulability optimization of the manipulator, and equivalently transforms it into a convex quadratic function. The proposed scheme addresses the non-convexity issue of manipulability with respect to the robotic arm joint acceleration and the inversion problem of the generalized Jacobian matrix. Simulation results show that the proposed method can maximize the manipulability of redundant dual-arm robots at the acceleration level, verifying the effectiveness of the scheme.

Keywords: Manipulability optimization, Dual-arm robot, Quadratic programming, Redundancy resolution

1. Introduction

In the context of rapid advancements in robotics, dual-arm robots have emerged as a significant research focus in fields such as industrial automation, surgical operations, and service robotics due to their flexibility and capability in handling complex tasks [1], [2], [3]. Dual-arm robots are capable of completing intricate operational tasks through coordinated movements, such as assembly, transportation, and precision handling, which are crucial for enhancing production efficiency and quality [4]. However, the key to achieving these complex tasks lies in improving the robot's manipulability, which refers to its motion capability and responsiveness in the task space [5].

Manipulability was initially proposed and studied to address singularity issues in manipulator motion planning and control. High manipulability allows a manipulator to achieve the desired end-effector motion with smaller joint velocities, making it an essential performance indicator for evaluating manipulator joint configurations and trajectory planning algorithms. Current research predominantly focuses on optimizing manipulability for single-arm systems, mainly relying on velocity-level analytical methods [6], [7], [8]. These methods are typically based on the characteristics of the velocity Jacobian matrix, such as condition numbers and singular values, to enhance the movement capabilities of robotic end effectors. However, velocity-level optimization has significant limitations: firstly, it fails to adequately consider the dynamic characteristics of the robotic system, especially when inertia and torques are involved; secondly, for the collaborative operations of dual-arm systems, velocity-level optimization struggles to address the complex coupling relationships and synchronized coordination between the two arms. These shortcomings restrict the applicability and efficiency of dual-arm robots in complex tasks.

To overcome these limitations, this paper proposes an acceleration-level optimization method for the manipulability of dual-arm robots. By introducing the acceleration Jacobian matrix and a detailed dynamic model, we can more comprehensively analyze the dynamic performance of dual-arm robots, particularly the impact of inertial coupling and torque distribution on system operational capability. This approach not only considers the dynamic coupling characteristics in dual-arm collaboration but also enhances the robot's response speed and stability in dynamic environments by optimizing acceleration-level indicators.

2. System Model

This section will provide a detailed introduction to the mathematical model of the dual-arm robot. The dual-arm system under study is shown in Fig. 1, where both the left and right arms of the robot are redundant manipulators with 7 degrees of freedom.

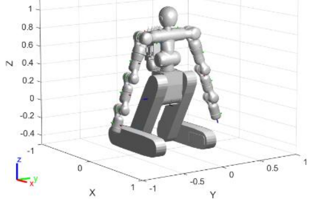

Fig. 1 The dual-arm robot platform

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2.1. *Kinematic model of the manipulator*

The mapping relationship between the position and orientation vector $\chi \in \mathbb{R}^6$ of the end-effector of the manipulator in the task space and the joint space variables $\Theta \in \mathbb{R}^7$ is as follows:

$$\chi = f(\Theta) \tag{1}$$

where $f(\cdot): \mathbb{R}^7 \to \mathbb{R}^6$ represents a nonlinear forward kinematics function. From the above kinematic equations, we can derive the differential-level kinematic equations.

$$\dot{\chi} = J\dot{\Theta} \tag{2}$$

$$\ddot{\chi} = \dot{J}\dot{\Theta} + J\ddot{\Theta} \tag{3}$$

where, $\dot{\chi} \in \mathbb{R}^6$ and $\ddot{\chi} \in \mathbb{R}^6$ are the generalized velocity and acceleration of the end-effector, $\dot{\Theta} \in \mathbb{R}^7$ and $\ddot{\Theta} \in \mathbb{R}^7$ are the joint angular velocity and angular acceleration, J is the corresponding Jacobian matrix, and $\dot{J} = dJ/dt$.

2.2. *Task-Oriented Coordination Operation equations*

During the coordinated operation of dual-arm robots, it is essential to maintain certain kinematic constraints between the robotic arms. Task-oriented coordination operation equations define the workspace and interaction dynamics in a manner that optimizes the performance of the cooperative task. First, a set of geometrically clear manipulation variables are defined to describe the coordinated operation tasks of the dual arms, namely the absolute motion variable χ_a, which describes the motion state of the manipulated object, and the relative motion variable χ_r, which describes the motion states of the two robotic arms.

By differentiating the aforementioned variables, one can obtain the absolute and relative motion variables in terms of velocity and acceleration, specifically described as:

$$\begin{cases} \dot{\chi}_a = (\dot{\chi}_R + \dot{\chi}_L)/2 \\ \dot{\chi}_r = \dot{\chi}_R - \dot{\chi}_L \end{cases} \tag{4}$$

$$\begin{cases} \ddot{\chi}_a = (\ddot{\chi}_R + \ddot{\chi}_L)/2 \\ \ddot{\chi}_r = \ddot{\chi}_R - \ddot{\chi}_L \end{cases} \tag{5}$$

Define $\dot{\chi}_G = [\dot{\chi}_a^T, \dot{\chi}_r^T]^T \in \mathbb{R}^{12}$ as the generalized variable describing the state of the manipulated object. In conjunction with Eq. (2), the task-oriented coordination operation equations for the velocity level can be expressed as follows:

$$\dot{\chi}_G = \begin{bmatrix} J_a \\ J_r \end{bmatrix} \begin{bmatrix} \dot{\Theta}_R \\ \dot{\Theta}_L \end{bmatrix} = J_G \dot{\Theta}_G \tag{6}$$

where, $J_G \in \mathbb{R}^{12\times14}$ is the generalized Jacobian matrix of the coordination operation equation, which represents the mapping relationship between the generalized variables $\dot{\chi}_G$ and the joint space variables , and $\dot{\Theta}_G \in [\dot{\Theta}_R^T, \dot{\Theta}_L^T]^T$, $J_a \triangleq [J_R/2 \quad J_L/2] \in R^{6\times14}$, $J_r \triangleq [-J_R \quad J_L] \in R^{6\times14}$.

Differentiating Eq. (6), one can obtain the coordination operation equation at the acceleration level, specifically described as follows:

$$\ddot{\chi}_G = \dot{J}_G \dot{\Theta}_G + J_G \ddot{\Theta}_G \tag{7}$$

where , $\dot{J}_G$ is the is the time derivative of the generalized Jacobian matrix. Eqs. (6) and (7) describe the task-oriented coordination operation equations for dual-arm robots. Since the manipulator has redundant degrees of freedom, meaning the generalized Jacobian matrix is not a square matrix, the solution to this equation is infinite.

For the aforementioned kinematic equations, the solution can be found using the pseudoinverse method, which is specifically described as follows:

$$\dot{\Theta}_G = J_G^\dagger \dot{\chi}_G + (I - J_G^\dagger J_G)\Pi_1 \tag{8}$$

$$\ddot{\Theta}_G = J_G^\dagger (\ddot{\chi}_G - \dot{J}_G \dot{\Theta}_G) + (I - J_G^\dagger J_G)\Pi_2 \tag{9}$$

where $J_G^\dagger \in \mathbb{R}^{14\times12}$ is the generalized inverse of J_G, assuming that J_G is full row rank, $J_G^\dagger = J_G^T (J_G J_G^T)^{-1}$. Π_1 and Π_2 are arbitrary vectors representing the gradients of certain selected performance metrics.

3. Acceleration-level Manipulability Optimization

In this section, a scheme for optimizing manipulability at the acceleration level is developed to address the redundancy resolution of dual-arm robots.

3.1. *Optimization metrics*

Manipulability is a crucial performance metric for assessing the configuration of robotic arm joints and trajectory planning algorithms. According to [6], the manipulability of a dual-arm system is defined as:

$$M = \sqrt{\det(J_G J_G^T)} \tag{10}$$

The larger of M, the stronger the manipulability of the robotic arm. When $M = 0$, the robotic arm will be in a singular configuration. The derivative of maneuverability M with respect to time is

$$\begin{aligned} \frac{dM}{dt} &= \sqrt{\det(J_G J_G^T)}\, Tr\left((J_G J_G^T)^{-1}(\dot{J}_G J_G^T + J_G \dot{J}_G^T)\right) \\ &= \sqrt{\det(J_G J_G^T)}\, Tr(\dot{J}_G J_G^\dagger) \end{aligned} \tag{11}$$

where, $Tr(\cdot)$ represents the trace of the matrix. The derivative of the generalized Jacobian matrix can be expressed as

$$\dot{J}_G = \sum_{i=1}^{N} \frac{\partial J_G}{\partial \Theta_{Gi}} \dot{\Theta}_{Gi} \tag{12}$$

From this point, we can obtain the optimization vector in terms of velocity .

$$\Pi_1 = -\frac{dM}{dt} \tag{13}$$

The vector representation for optimization at the acceleration level is given by

$$\Pi_2 = \frac{d\Pi_1}{dt} = -\frac{d}{dt}\left(\sqrt{\det(J_G J_G^T)}Tr(\dot{J}_G J_G^\dagger)\right)$$
$$= -\frac{d\sqrt{\det(J_G J_G^T)}}{dt}Tr(\dot{J}_G J_G^\dagger) - \sqrt{\det(J_G J_G^T)}\frac{d}{dt}\left(Tr(\dot{J}_G J_G^\dagger)\right)$$
(14)

with which, $d\left(Tr(\dot{J}_G J_G^\dagger)\right)/dt$ is expressed as

$$\frac{d}{dt}\left(Tr(\dot{J}_G J_G^\dagger)\right) = Tr\left(\frac{d}{dt}\left(\dot{J}_G\right)J_G^\dagger + \dot{J}_G \dot{J}_G^\dagger\right) \quad (15)$$

As to $\dot{J}_G^\dagger$, which can be calculated as follows:

$$\dot{J}_G^\dagger = (I - J_G^\dagger J_G)\dot{J}_G^T(J_G J_G^T)^{-1} - J_G^\dagger \dot{J}_G J_G^\dagger \quad (16)$$

Additionally, to reduce control efforts, we further considered the corresponding optimization criteria, that is Minimum acceleration norm

$$\min \quad (\ddot{\Theta}_G + k\dot{\Theta}_G)^T(\ddot{\Theta}_G + k\dot{\Theta}_G) \quad (17)$$

At this point, the optimization criteria for the dual-arm robot in acceleration level can be expressed as

$$\min_{\dot{\Theta}_G,\ddot{\Theta}_G} \quad \Upsilon_1 = \frac{1}{2}\ddot{\Theta}_G^T\ddot{\Theta}_G - (a_1\dot{\Theta}_G^T + a_2\Pi_2^T)\ddot{\Theta}_G \quad (18)$$

where, the design parameters $a_1 \in (0,1)$ and $a_2 \in (0,1)$ are used to adjust the weights of each criterion, respectively.

3.2. *Joint physical constraints*

In engineering applications, virtually all robotic arms have physical limitations on joint angles and joint angular velocities. Therefore, it is essential to consider the actual range of angles and angular velocities that can be achieved by each joint of the robotic arm. This constraint is specifically described as:

$$\Theta_G^{\min} \le \Theta_G \le \Theta_G^{\max} \quad (19)$$
$$\dot{\Theta}_G^{\min} \le \dot{\Theta}_G \le \dot{\Theta}_G^{\max} \quad (20)$$
$$\ddot{\Theta}_G^{\min} \le \ddot{\Theta}_G \le \ddot{\Theta}_G^{\max} \quad (21)$$

The aforementioned optimization scheme is based on acceleration level solution; therefore, the joint angle and angular velocity constraints need to be converted into descriptions based on joint angular acceleration. According to [8], the physical limits can be expressed as:

$$\Theta_G^- \le \ddot{\Theta}_G \le \Theta_G^+ \quad (22)$$

The boundary constraints Θ_G^- and Θ_G^+ are defined as follows:

$$\begin{cases} \Theta_G^- = \max\left\{\eta_1\eta_2(\Theta_G^{\min} - \Theta_G), \eta_1(\dot{\Theta}_G^{\min} - \dot{\Theta}_G), \ddot{\Theta}_G^{\min}\right\} \\ \Theta_G^+ = \min\left\{\eta_1\eta_2(\Theta_G^{\max} - \Theta_G), \eta_1(\dot{\Theta}_G^{\max} - \dot{\Theta}_G), \ddot{\Theta}_G^{\max}\right\} \end{cases} \quad (23)$$

where, $\eta_1 \ge \eta_2 > 0$ are design parameters used to adjust the feasible domains of joint acceleration and velocity, respectively.

3.3. *Acceleration-level optimization scheme*

By incorporating the end-effector trajectory tracking constraints Eq. (7) and joint acceleration constraints Eq. (22) into the optimization criterion Eq. (18), the redundancy resolution problem of a dual-arm robot can be formulated as an optimization scheme in the following form.

$$\begin{aligned} \min_{\dot{\Theta}_G,\ddot{\Theta}_G} \quad & \Upsilon_1 = \frac{1}{2}\left\|\ddot{\Theta}_G\right\|_2^2 - (a_1\dot{\Theta}_G^T + a_2\Pi_2^T)\ddot{\Theta}_G \\ s.t. \quad & \ddot{\chi}_G - J_G\ddot{\Theta}_G - \dot{J}_G\dot{\Theta}_G = 0 \\ & \Theta_G^- \le \ddot{\Theta}_G \le \Theta_G^+ \end{aligned} \quad (24)$$

4. Simulation Results and Discussion

In order to evaluate the performance of the proposed manipulability optimization scheme, we applied the scheme to the dual-arm robot as shown in Fig. 1.
Simulation experiments were conducted in the MATLAB software environment. To demonstrate the effectiveness of the scheme, we designed the following working scenario: Both arms collaboratively move a circular component, rotating it 180 degrees in the world coordinate system. The simulation results are presented in Figs. 2-4.

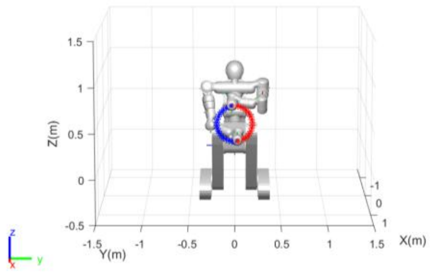

Fig. 2. The simulation process of the operational task.

First, Fig. 2 displays the motion trajectory of the robotic arm's end-effector from a front view.

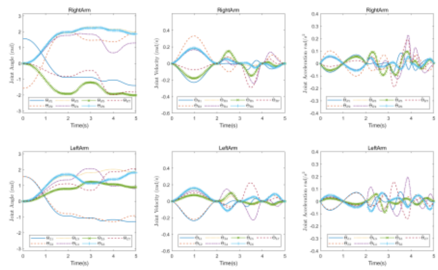

Fig. 3 Joint angles, velocities, and accelerations

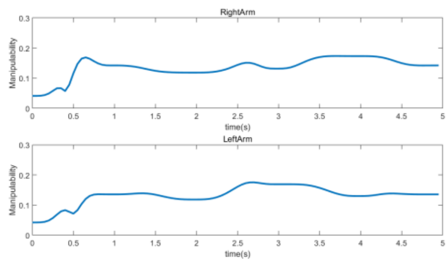

Fig. 4. Manipulability measures of the dual-arm robot

Fig. 3 shows that the joint angles, angular velocities, and angular acceleration of the robotic arm consistently remain within the constraint limits, demonstrating the effectiveness of boundary constraints. It can be observed that the coordinated movement of the dual arms allows the posture tracking task to be successfully accomplished.

Moreover, the joint trajectory planning results show smooth and continuous changes over time, meeting the needs of practical applications. Fig. 4 presents the variability in manipulability of each operational arm during the process.

From the above simulation results, it can be clearly seen that the manipulability optimization scheme proposed in this paper effectively addresses the problem of redundancy resolution at the acceleration level for the dual-arm robot.

5. Conclusion

The proposed scheme effectively addresses the limitations of previous methods that optimize at the velocity level without considering joint acceleration limits. It does so by constructing a cost function that simultaneously considers joint torque constraints and manipulability optimization. This method successfully tackles the issue of non-convexity between manipulator joint angle acceleration and manipulability by converting the non-convex problem into an equivalent convex quadratic function. This method enables manipulability optimization at the acceleration level, thereby enhancing the algorithm's applicability and robustness. Simulation results demonstrate that the proposed method can maximize the manipulability of redundant dual-arm robots at the acceleration level, verifying its effectiveness. This offers a novel solution for robotic control tasks demanding high precision and efficiency in practical applications.

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Authors Introduction

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