Trajectory Analysis for a Mobile Robot Adapted Three Omni Rollers in Constant Roller's Speed

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Abstract

The mobile robot is being developed for use in the logistics industry. A mechanism with multiple omni rollers as an omni-directional moving mechanism has been developed and its kinematics analyzed. In this study, the kinematics were verified in a simulation environment as a preliminary step in order to reduce the cost and duration in the verification experiments with good prospects. The trajectory of the robot was derived and analyzed when a constant roller speed was given in the robot kinematics.

Keywords: Trajectory analysis, Mobile robot, Angular velocity

1. Introduction

Recently, efficient mobility has become a requirement for mobile robots in areas like logistics. Thus, omnidirectional movement with either non-holonomic or holonomic characteristics can produce a total of 3 degrees of freedom motion (sum of 2 degrees of freedom translational motion and 1 degree of freedom rotational motion). Robotic vehicle development is getting attention.

Among them, holonomic mobile mechanisms are easy to control and have excellent omni-directional mobility, as the wheels are driven independently. For this reason, mobile robots arranged in equilateral triangles have been developed [1].

In the RoboCup MSL, a mechanism with three omni-rollers is used: the RV-infinity [2], the Musashi150 [3] and the NuBot [4] have three omni-rollers arranged in an equilateral triangle. The kinematics of a sphere with two roller velocities as inputs has been proposed for the RoboCup MSL as a kinematics study of the moving mechanism [5] and validated on a real machine [6][7]. There are studies on the region of existence of the velocity of the mobile robot with respect to the roller velocity [8][9][10] and on the rotational efficiency of the sphere [11][12]. As for the analysis of robot trajectories, [13] has successfully analyzed the trajectory of a sphere robot driven by rollers using a simulator.

To verify whether the derived equations adequately represent the intended conceptual model, as in this study, it is necessary to measure the actual values on the actual machine and check the errors with the theoretical values. However, verification using actual equipment is costly and time-consuming to develop, and feedback is also time-consuming, making it difficult to shorten the speed of development. There is an example of verification of the trajectory of a moving mechanism with three spherical rollers using the simulation function of 3DCAD. This allows simulations to be carried out on a PC at the pre-experimental stage to reduce development costs and shorten the time required.

In this study, a simulation on the trajectory of a mechanism with three constant values as input for a three-wheeled wheeled mobile mechanism is carried out using the mechanism analysis function in the 3DCAD software. The results are then compared with theoretical values using a mathematical model to verify the consistency of the mathematical model. The rest of this study is as follows: Chapter 2 discusses the kinematics and trajectory for mobile robots. Chapter 3 shows simulation resalt. Finally, we present the summary and future tasks.

2. Kinematics and Trajectory for Omnidirectional mobile robot

This section introduces kinematics which the roller placement position can be changed arbitrarily and analysis robot trajectory in constant rollers speed.

2.1. *Kinematics*

Figure 1 shows a top view of the mobile robot that has a common radius of all omni-wheels adapted the *i-th* rollers ($i = 1,2,3$) contact point $\boldsymbol{P_i}$ (contact position angle α_i) on a circle, which has a radius L (the distance from the robot center to the contact points between wheels and floor)(See Table 1).

$\{x_w, y_w\}$ is the global coordinate system (origin $\boldsymbol{O}$) and $\{x_m, y_m\}$ is robot coordinate system (origin $\boldsymbol{\acute{O}}$). And. The robot orientation ϕ is referred as the angle between x_w-axis and x_m-axis.

Thus, $[v_1, v_2, v_3]^{\boldsymbol{T}}$ in robot coordinate is represented as follows.

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -\sin\alpha_1 & \cos\alpha_1 & 1 \\ -\sin\alpha_2 & \cos\alpha_2 & 1 \\ -\sin\alpha_3 & \cos\alpha_3 & 1 \end{bmatrix} \begin{bmatrix} V_{x_m} \\ V_{y_m} \\ L\dot{\phi} \end{bmatrix} \quad (1)$$

And. $\boldsymbol{V_w}$ denote robot direction vector in robot coordinate system.

Global coordinate system $\{x_w, y_w\}$ is a rotation of the robot coordinate system $\{x_m, y_m\}$ by ϕ. Therefore, following expression can be valid.

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -\sin(\alpha_1-\phi) & \cos(\alpha_1-\phi) & 1 \\ -\sin(\alpha_2-\phi) & \cos(\alpha_2-\phi) & 1 \\ -\sin(\alpha_3-\phi) & \cos(\alpha_3-\phi) & 1 \end{bmatrix} \begin{bmatrix} V_{x_w} \\ V_{y_w} \\ L\dot{\phi} \end{bmatrix} \quad (2)$$

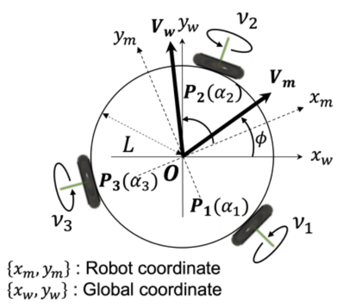


Figure 1 Robot speed vector by robot coordinate system and global coordinate system

2.2 *Kinematics for isosceles triangle three rollers arrangement*

In this section, the trajectory of the center of gravity of the robot is analyzed for a constant roller speed $[v_1, v_2, v_3]^T$ at a constant time t. And as an equilateral triangle roller arrangement we assumed $(\alpha_1, \alpha_2, \alpha_3) = (\pi/6, 5\pi/6, 3\pi/2)$. Here, $v_1 + v_2 + v_3$ is considered by dividing the cases as follows.

(A) Case of $v_1 + v_2 + v_3 \neq 0$
Solving Eq. (1) for, $L\dot{\phi}$ is expressed using the linear expression for v_1, v_2 and v_3 . Therefore, $\dot{\phi}$ is constant. As shown in Figure 2, if the robot moves with an initial speed of $\boldsymbol{V_0}$ and a constant attitude rotation speed of $\dot{\phi}$, the centre $\overrightarrow{OC}$ and radius of the trajectory are expressed as follows

$$\frac{t}{\phi}\left[V_y^o, -V_x^o\right]^T, \quad \frac{\|\boldsymbol{V_0}\| t}{\phi} \quad (3)$$

Thus, the velocity vector $\boldsymbol{V_T}$ after t seconds is the initial velocity $\boldsymbol{V_0}$ rotated by ϕ.

$$\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \left\{ -\frac{t}{\phi} \begin{bmatrix} V_y^o \\ -V_x^o \end{bmatrix} \right\} + \frac{t}{\phi} \begin{bmatrix} V_y^o \\ -V_x^o \end{bmatrix} \quad (4)$$

$|\boldsymbol{V_w}|^2$ is a constant independent of ϕ and $\phi = \dot{\phi}t$ ($0 \leq t \leq T$) is constant. Therefore, position $[x_T, y_T]^{\boldsymbol{T}}$ is represented as following linear trajectory.

$$\begin{bmatrix} x_T \\ y_T \end{bmatrix} = \frac{t}{\phi} \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} V_y^o \\ -V_x^o \end{bmatrix} - \frac{t}{\phi} \begin{bmatrix} V_y^o \\ -V_x^o \end{bmatrix} \quad (5)$$

$$= \frac{1}{\dot{\phi}} \begin{bmatrix} -1+\cos\dot{\phi}t & -\sin\dot{\phi}t \\ \sin\dot{\phi}t & -1+\cos\dot{\phi}t \end{bmatrix} \begin{bmatrix} V_y^o \\ -V_x^o \end{bmatrix}$$

where

$$\phi = \dot{\phi}t \quad (6)$$

As shown in Figure 3, Robot trajectories are classified by $v_1 + v_2 + v_3$ and Following property can be valid.

[Property 1]
By the sign of $v_1 + v_2 + v_3$, the following holds.

(i) For $v_1 + v_2 + v_3 > 0$ ⇨ The movement is counter-clockwise rotation.

(ii) For $v_1 + v_2 + v_3 < 0$ ⇨ It moves in a clockwise rotation.

(B) Case 2 of $v_1 + v_2 + v_3 = 0$
In this case, since $\dot{\phi} = 0$ (translational motion only), $[x_T, y_T]^{\boldsymbol{T}}$ is represented as following line trajectory.

$$\begin{bmatrix} x_T \\ y_T \end{bmatrix} = \frac{1}{\dot{\phi}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_x^o \\ V_y^o \end{bmatrix} + \frac{1}{\dot{\phi}} \begin{bmatrix} V_y^o \\ -V_x^o \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} x_T \\ y_T \end{bmatrix} = t \begin{bmatrix} V_x^o \\ V_y^o \end{bmatrix} \quad (8)$$

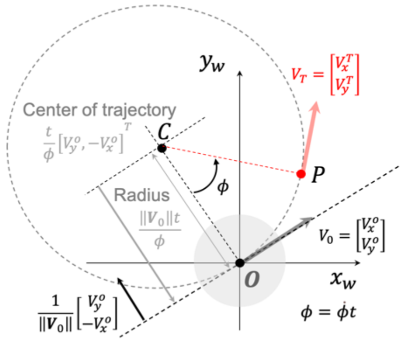


Figure 2 Property of robot trajectory in case of constant roller's speed

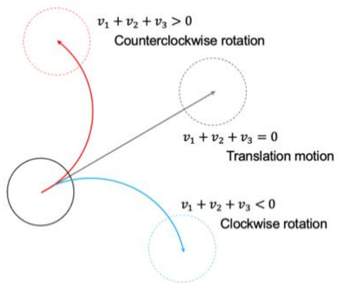


Figure 3 Classification of robot trajectory by input condition

Table 1 Parameter List

Parameter	Name
L	Radius of robot
r	Radius of roller
$v_i = r\omega_i$	Relationship roller speed and rotational speed
ω_i	Rotational speed
$N_i = 60\omega_i/360$	Number of revolutions [RPM]
$N_i = v_i/6r$	Relationship roller speed and rotational speed

3. Simulation

Using the derived Eq. (5) and Eq. (8), simulations on the mobile robot's movement trajectory were carried out. Furthermore, in order to verify whether the derived equations represent the intended conceptual model, simulations on the movement trajectory were conducted using Motion in the 3D CAD software SolidWorks. The results of the two simulations are shown in Figure 4. As shown in Table 2, The three-wheeled assembly of the mechanism with omni-rollers, giving a radius of L for the airframe, r for the omni-rollers and $(\theta_1, \theta_2, \theta_3)$ for the location of the three rollers and the friction is set by giving the contact constraint of the passive longs of the floor and the omni rollers as the analysis conditions.

For the error rate, the following is defined as an evaluation of the closeness of the theoretical value $\mathbb{r}_t(t) = (X_W^m(t), Y_W^m(t))$ to the experimental value $\mathbb{r}_e(t) = (X_W^e(t), Y_W^e(t))$.

Table 2 Parameter adjustment for simulation

Parameter	Value
L	0.108[m]
r	0.023[m]
$(\alpha_1, \alpha_2, \alpha_3)$	$(\pi/6,\ 5\pi/6,\ 3\pi/2)$ [rad]

$$e(t) = \frac{1}{T}\int_0^T \frac{\|\mathbb{r}_t(t) - \mathbb{r}_e(t)\|}{\|\mathbb{r}_t(t)\|} dt \qquad (9)$$

The theoretical equation calculated from Eq. (5) and Eq. (8) are shown by the orange line and the experimental values by the blue line.

(A) Simulation 1 (circular motion)

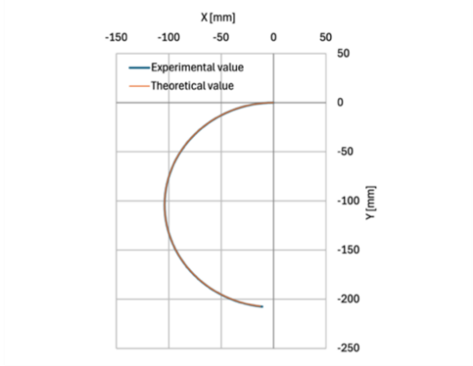
Assume that the origin is the start and the target of the initial velocity vector $\boldsymbol{V}$ such that $\varphi = 180°$ and $\|V\| = 0.022$ [m/s]. In this case, an input $(N_1, N_2, N_3) = (10, 10, 0)$[rpm] is given and time $t = 20$[s].

From $N_1 + N_2 + N_3 = 20 \neq 0$, a circular trajectory is theoretically drawn; in fact, the theoretical line shows a circular orbit close to the experimental line. The error rate was $|1 - L_W^e/L_W^m| = 0.037$.

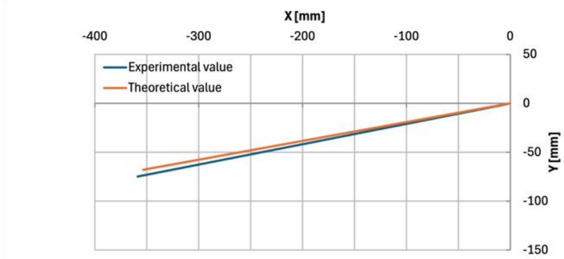
(B) Simulation 1(linear motion)

Assume that the origin is the start and the target of the initial velocity vector $\boldsymbol{V}$ such that $\varphi = 190.9°$ and $\|V\| = 0.10$ [m/s]. In this case, an input $(N_1, N_2, N_3) = (10, 20, -30)$[rpm] is given and time $t = 5$ [s].

From $N_1 + N_2 + N_3 = 0$, the nature of the trajectory implies a translational linear motion, but in fact, the theoretical line is close to the experimental line, indicating a linear trajectory. The error rate was $|1 - L_W^e/L_W^m| = 0.057$. The accuracy of the kinematics is therefore demonstrated.



(a) Case of circle



(b) Case of straight line

Figure 4 Trajectory of mobile robot in case of constant roller's speed

4. Conclusion

In this study, a trajectory equation for a mobile robot for a constant roller speed was derived. The theoretical equation was successfully verified by simulation. The theoretical equation proves that the trajectory of the robot is divided into cases (circles and lines) according to the sum of the input values, which is also confirmed by experiments.

As future work, we would like to study the speed efficiency of the mobile robot in all directions and the mobility efficiency of the mobile robot when the position and direction of the rollers are taken as parameters.

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