An Adaptive Control Method for a Knee-Joint Prosthetic Leg Toward Dynamic Stability and Gait Optimization

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Abstract

This paper presents a hybrid control strategy that combines Model Predictive Control (MPC) and Linear Quadratic Regulator (LQR) to achieve robust and stable tracking of human knee joint motion. The state-space model of the system is discretized to facilitate real-time implementation. MPC is employed to track the knee joint trajectory during dynamic motion, while the LQR controller is activated at critical points, particularly when the joint angle approaches zero, to stabilize the system and ensure safety.

Keywords: Model Predictive Control (MPC), Linear Quadratic Regulator (LQR), Trajectory Tracking, Lower-Limb Prosthetics, Knee Joint Motion

1. Introduction

In recent decades, millions of individuals have faced challenges in using their lower limbs due to conflicts, illnesses, traffic accidents, and natural disasters. Consequently, many have lost their capacity to work and are unable to engage in normal social activities [1]. Studies have explored the potential of motorized prostheses to help patients regain their walking ability [2]. Active prostheses have inspired numerous control strategies. However, despite advancements in control strategies, challenges remain in the mechanical design of active prostheses [3].

Sup et al. (2008) [4] developed an electrically driven prosthetic knee and ankle using a ball-screw mechanism. The control strategy employed a finite state machine-based impedance control to ensure stable torque output across gait phases.

Martinez-Villalpando et al. (2009) [5] developed an agonist-antagonist active knee prosthesis featuring two parallel-series elastic actuators that mimic the biomechanical characteristics of the human knee.

Salman and Kadhim (2022) [6] proposed a dual-degree-of-freedom knee prosthesis control method based on backstepping control. Utilizing Lyapunov stability theory, they designed a recursive controller to ensure dynamic stability of the closed-loop system. The control parameters were further optimized using the bat algorithm, significantly improving trajectory tracking accuracy and robustness.

While Taherian et al. [7] explored the application of MPC and LQR for collision avoidance in autonomous driving, their approach also provides valuable insights for developing control strategies applicable to prosthetic systems. Similarly, Capron et al. [8] highlighted the stability and performance benefits of robust LQR-MPC control in industrial processes, offering a solid theoretical foundation for designing effective control strategies in dynamic environments such as prosthetic systems.

This paper presents an MPC+LQR-based approach for tracking the human knee joint angle trajectory during walking. To reduce tracking errors at critical walking points, a switching mechanism to LQR control is designed when the prosthetic knee reaches the support point (0 degrees). The leg model is developed using the Lagrange equation and transformed into a discrete state-space form for the implementation of the MPC+LQR controller. Simulation results demonstrate that compared to using MPC alone, the integrated control strategy significantly reduces trajectory errors at critical points and improves the accuracy of tracking human knee motion. The inclusion of LQR control further enhances system stability and robustness.

2. Control Objective

The objective of this study is to establish a mathematical model of the leg and design a controller to regulate knee joint angle variations, ensuring they align with the knee motion trajectory observed during human movement (Fig.1).

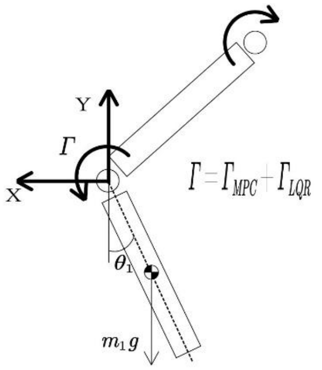

Figure.1. Mathematical Modeling

Based on the measurements conducted by Thomas Seel et al. (2014) [9], the knee joint angle variations can be approximated as a periodic motion with a duration of 1.1 seconds. In this paper, we use a sine function to fit the bending process of the knee joint and apply smoothing techniques to simulate the smooth fluctuations of the knee angle when the foot makes contact with the ground, ensuring the function remains differentiable within the period.

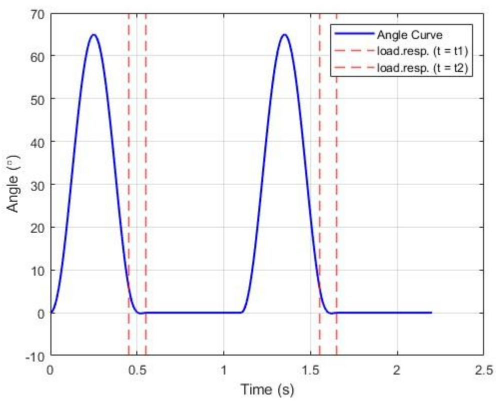

Figure.2. Angle Curve

In the figure (Fig.2), t_1 to t_2 are the start and end times of foot contact with the ground, making the knee Angle smoothly transition to zero.

3. Methodology

3.1. Model Predictive Control modeling

In the simulation of the legs, we employ two standard homogeneous rods connected by revolute joints to replicate the human leg motion mechanism. The lengths and masses of the rods are set as estimated values based on standard adult male anthropometric assumptions [10]. The schematic diagram is presented in Figure (Fig.3):

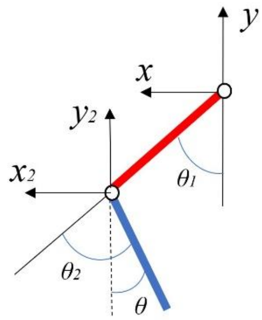

Figure.3. Coordinate diagram of leg mechanism

The coordinate system is defined with the hip joint rotation point as the origin. The X-axis is horizontal to the left. The Y axis goes straight up, and the gravity direction is the -y axis. The whole system is negative on the Y-axis.

The Angle of rotation of the joint is defined as θ_1, θ_2. θ_1 is the Angle between the thigh connecting rod and the y axis, positive in the clockwise direction. θ_2 is the Angle between the thigh extension line and the calf, positive in the counterclockwise direction. Due to the anatomical structure of the human body, θ_2 values greater than 0.

Where, θ_2 is the relative Angle, the absolute Angle θ of the calf relative to the coordinate system is:

$$\theta = \theta_1 - \theta_2 \tag{1}$$

The following Table.1 describes other physical data：

Table. 1. Physical data

Physical quantity	Value	Unit
m_1	7	kg
l_1	0.5	m
m_2	3	kg
l_2	0.45	m
c	0.1	

3.2. Lagrange equations

Equations of Lagrange dynamics [11]:

$$L = T - U \tag{2}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i} = Q_i \tag{3}$$

Analyzing the dynamic model of each component.

Centroid of thigh:

$$x_1 = \frac{l_1}{2}\sin\theta_1 \qquad y_1 = -\frac{l_1}{2}\cos\theta_1 \tag{4}$$

Thigh kinetic energy:

$$T_1 = \frac{1}{2}I_1\dot{\theta}_1^2 + \frac{1}{2}m_1\left(\frac{l_1}{2}\right)^2\dot{\theta}_1^2 \tag{5}$$

Thigh potential energy:

$$U_1 = -m_1 g\frac{l_1}{2}\cos\theta_1 \tag{6}$$

The calf movement is completed by the coupling of the calf rotation and the thigh rotation, wherein the calf rotation Angle θ_2 relative to the thigh is the relative Angle, and the absolute Angle of the calf is defined in Eq.1. The absolute position of the center of mass of the calf is:

$$\begin{aligned} x_2 &= l_1\sin\theta_1 + \frac{l_2}{2}\sin(\theta_1-\theta_2) \\ y_2 &= -l_1\cos\theta_1 - \frac{l_2}{2}\cos(\theta_1-\theta_2) \end{aligned} \tag{7}$$

Calf kinetic energy:

$$T_2 = \frac{1}{2}m_2\left(\dot{x}_2^2+\dot{y}_2^2\right) + \frac{1}{2}I_2\left(\dot{\theta}_1-\dot{\theta}_2\right)^2 \tag{8}$$

Calf kinetic energy:

$$U_2 = -m_2 g\left(l_1\cos\theta_1 + \frac{l_2}{2}\cos(\theta_1-\theta_2)\right) \tag{9}$$

Find the Lagrange equation for the whole system:

$$\frac{d}{dt}\begin{bmatrix}\frac{\partial L}{\partial\dot{\theta}_1} \\ \frac{\partial L}{\partial\dot{\theta}_2}\end{bmatrix} - \begin{bmatrix}\frac{\partial L}{\partial\theta_1} \\ \frac{\partial L}{\partial\theta_2}\end{bmatrix} = \begin{bmatrix}\tau_1 \\ \tau_2\end{bmatrix} \tag{10}$$

From an energy perspective, the Lagrange method offers an efficient and systematic approach to modeling the entire system. This method not only effectively handles the dynamic modeling of complex systems, particularly those with rigid multi-links, but also preserves the physical consistency and energy conservation properties of the system.

3.3. State-space equation

For calf movements, define state variables:

$$x = \begin{bmatrix}\theta_2 \\ \dot{\theta}_2\end{bmatrix},\ u = \tau_2 \tag{11}$$

The state space model is:

$$\dot{x} = Ax + Bu \tag{12}$$

Obtained from the Lagrange equations:

$$\ddot{\theta}_2 = \frac{1}{M_{22}}(\tau_2 - C_2) \tag{13}$$

Where M_{22} is the equivalent inertial term, derive from $\frac{\partial^2 T}{\partial\dot{\theta}_2^2}$. C_2 represents the equivalent Coriolis, centrifugal, and gravitational terms.

The state space equation is derived as follows:

$$\begin{bmatrix}\dot{\theta}_2 \\ \ddot{\theta}_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ -\frac{\partial C_2}{\partial\theta_2}/M_{22} & -\left(\frac{\partial C_2}{\partial\dot{\theta}_2}+c\right)/M_{22}\end{bmatrix}\begin{bmatrix}\theta_2 \\ \dot{\theta}_2\end{bmatrix} + \begin{bmatrix}0 \\ \frac{1}{M_{22}}\end{bmatrix}u \tag{14}$$

Calculated by MATLAB:

$$\begin{bmatrix}\dot{\theta}_2 \\ \ddot{\theta}_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ -\frac{g\,l_2\,m_2\sin(\theta_1-\theta_2)}{2\left(\frac{m_2\,l_2^2}{4}+I_2\right)} & -\frac{c}{\frac{m_2 l_2^2}{4}+l_2}\end{bmatrix}\begin{bmatrix}\theta_2 \\ \dot{\theta}_2\end{bmatrix} + \begin{bmatrix}0 \\ \frac{1}{\frac{m_2\,l_2^2}{4}+I_2}\end{bmatrix}u \tag{15}$$

Where c is the damping coefficient, representing the resistance to be overcome by rotating the knee joint. The damping coefficient is related to the angular velocity, and the equation is

$$f = c\cdot\dot{\theta}_2 \tag{16}$$

4. MPC+LQR trajectory tracking

4.1. MPC modeling

By introducing Model Predictive Control (MPC) into the mathematical model, the controller can predict the system's behavior over a finite time horizon and continuously optimize the control input to achieve the optimal solution at the current time step. MPC achieves this by solving an online optimization problem at each control interval, which considers system dynamics, constraints, and performance objectives [12]. This predictive capability allows MPC to handle multi-variable systems, incorporate input and state constraints, and adapt to disturbances, making it particularly suitable for complex and dynamic systems.

From the previous Eq.12, it is concluded that the state-space equation of the system can be rewritten into discrete form:

$$x_{k+1} = A_d x_k + B_d u_k \tag{17}$$

Where x_{k+1} is the state of the next moment, determined jointly by the state of this moment x_k and the control input of this moment u_k, with A_d representing the state transition matrix and B_d representing the input matrix in the discrete-time system.

The discrete state space equation is incorporated into MATLAB for simulation and numerical computation. To reduce computational load, the equation is linearized around the thigh lift action. Taking $\theta_1 = \pi/2$ and $\theta_2 = \pi/3$.

Concurrently, to simulate the knee joint constraints during real walking motion, the range of θ_2 is designed between 0-70 degrees, while the target trajectory angle range is set between 0-60 degrees. In addition, to model the errors and external disturbances encountered in real-world scenarios, the disturbance function is introduced as:

$$d(t) = A_d dsin(\omega_d t) \tag{18}$$

MPC possesses the capability to track nonlinear motion trajectories, which can be achieved by inputting the trajectory data into the MPC controller. By incorporating the target trajectory into the MPC controller, the tracking plot can be obtained as figure.

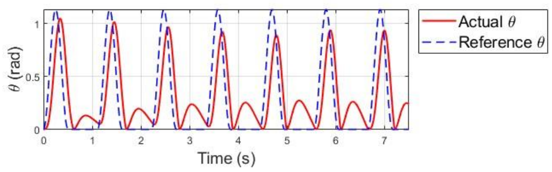

Figure. 4.A MPC Angle Tracking Curve

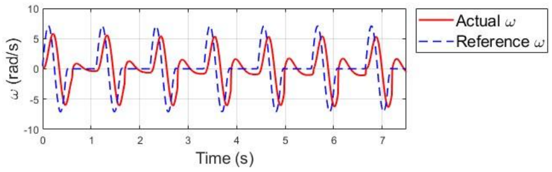

Figure. 4.B MPC Velocity Tracking Curve

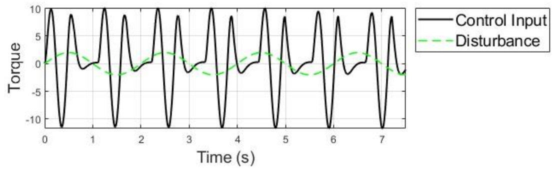

Figure. 4.C Control Input and Disturbance

Figure. 4. MPC tracking curve

The green dashed line represents the disturbance applied to the system. As shown in the figure (Fig.4), with output weights set to [25, 0.5], the MPC achieves most of the trajectory tracking under the combined influence of the disturbance function and system damping, with a calculated Root Mean Square (RMS) Error of 0.28851 rad. However, significant errors occur at the point where the system angle equals zero.

4.2. MPC+LQR Control scheme

To reduce tracking errors to the point where the angle equals 0, we introduced LQR control and designed a control logic to switch to LQR when the tracking trajectory approaches 0 degrees.

Linear Quadratic Regulator (LQR) is a linear control scheme where the optimal gain matrix is computed off-line and applied in real-time for system regulation [13]. By introducing the performance evaluation equation as:

$$J = \sum_{k=0}^{\infty} (x_k^T Q x_k + u_k^T R u_k) \tag{19}$$

By setting the values of the state weight matrix Q and the disturbance weight matrix R, the feedback gain matrix K is determined to satisfy:

$$u_k = -Kx_k \tag{20}$$

After introducing the LQR control with the state weight matrix $Q = \begin{bmatrix} 300 & \\ & 1 \end{bmatrix}$ and input weight $R = 0.05$ at the point where the angle is zero, the simulation results are shown in the figure (Fig.5) below:

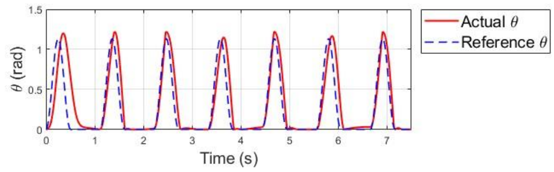

Figure. 5.A MPC+LQR Angle Tracking Curve

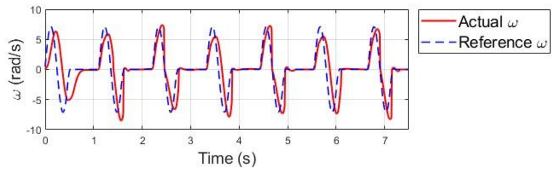

Figure. 5.B MPC+LQR Velocity Tracking Curve

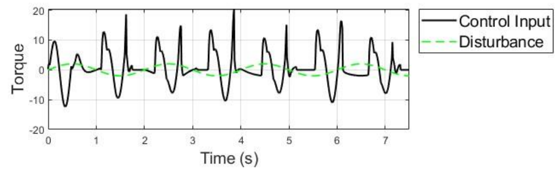

Figure. 5.C Control Input and Disturbance

Figure. 5. MPC + LQR tracking curve

As shown in the figure (Fig.5), with the same tuning parameters, the combined MPC and LQR controller reduced the RMS Error to 0.20437 rad, and the simulation scheme demonstrates better tracking performance when the knee joint angle is at 0 degrees.

5. Discussion

5.1. Interpretation of result

Both MPC and LQR controllers are derived from the state-space equations, ensuring that the transitions of knee joint angle and angular velocity align with human motion patterns during movement. This is also evident from the simulation results, where both the MPC controller (Fig.4) and the MPC+LQR controller (Fig.5) successfully track the trajectory for most of the time.

However, as shown in the simulation results of the standalone MPC controller (Fig.4), a significant error occurs in the predicted model when the angle approaches zero. The Root Mean Square (RMS) error for the standalone MPC controller is 0.28851 rad, reflecting this limitation.

This is primarily due to the combined effects of MPC prediction errors and the physical limitations of the knee joint. When the knee angle reaches zero, corresponding to full knee extension in physical terms, the angular velocity reduces to zero due to physical impacts. This reduction introduces significant errors in the MPC's predicted future control path, leading to inaccuracies in the actual trajectory at the zero-degree position.

These errors are inherent to the predictive control principle of MPC. While adjusting control parameters can reduce the amplitude of fluctuations, residual oscillations will still exist. Importantly, when the angle equals zero, the prosthetic knee acts as the stance leg during human walking, a critical phase where even minor oscillations in the knee joint can destabilize the wearer's center of mass.

To address this issue, LQR control is introduced when the knee angle approaches zero to stabilize the system and ensure safe and reliable gait performance for the wearer. From the simulation parameters (Fig.5), it is evident that the combined MPC+LQR control strategy not only conforms to the human motion trajectory but also reduces the RMS error to 0.20437 rad, ensuring that the knee joint is controlled to a safe position at critical points and maintaining motion stability.

5.2. Reliability and Scalability

In LQR control, the optimal gain K is typically precomputed offline by solving the Algebraic Riccati Equation, thereby avoiding real-time computational burden during implementation [14]. Compared to MPC, which excels in handling constraints and trajectory tracking tasks, LQR provides a computationally efficient solution and performs well in systems requiring fast and robust disturbance rejection [15]. Studies have demonstrated that while MPC offers smoother control signals, LQR's simple structure and reduced computational demand make it suitable for applications with limited real-time resources [16].

Although the LQR controller is limited to tracking linear tasks, the combined MPC and LQR control scheme enables the system to not only track nonlinear trajectories but also switch to linear tasks at critical points, ensuring system stability. This approach enhances the overall robustness and scalability of the system. Regardless of the type of motion trajectory being tracked, the LQR controller maintains stability when the prosthesis transitions to the supporting leg, providing greater safety for the wearer. Additionally, the system allows for the customization of MPC tracking paths by measuring the wearer's unique motion trajectories, offering more walking modes for powered prostheses.

6. Conclusion

In this paper, we have successfully developed a hybrid control strategy that integrates Model Predictive Control (MPC) and Linear Quadratic Regulator (LQR) to address the challenges of controlling the knee joint angle in a prosthetic system. By achieving the challenge of tracking the target trajectory, we also established a reliable control logic that ensures stability at critical points during movement. Through Lagrangian modeling of the human leg mechanism, we derived the state-space equations for the lower limb, enabling precise mathematical representation of its dynamics.

Simulation results demonstrate that this control scheme exhibits better performance in both trajectory tracking and maintaining stability at critical points. The combined LQR+MPC control strategy not only improved the overall trajectory tracking performance but also enhanced system robustness and safety by minimizing angular oscillations at critical phases. Furthermore, the transition between MPC and LQR control was shown to align well with natural human motion, providing smooth and physically consistent angle and velocity changes.

Future work will focus on applying this control strategy to real lower-limb prosthetics. By measuring the user's gait, the MPC tracking trajectory can be customized, enabling the prosthetic to better adapt to the user's individual movement patterns.

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