

# Event-Triggered Consensus Control for Nonlinear Singular Multi-Agent Systems under Directed Topology

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## Abstract

The event-triggered consensus problem for singular multi-agent systems with Lipschitz nonlinearity under directed topology is investigated in this paper. A sampled-data-based event-triggered mechanism is constructed to decide when the current data-packet should be broadcast. The objective is to design an event-triggered protocol such that the considered multi-agent system can achieve admissible consensus. Based on graph theory and singular system theory, sufficient conditions of consensus of nonlinear singular multi-agent systems are derived. Finally, a numerical example shows the effectiveness of our proposed approach.

*Keywords:* Singular multi-agent systems, Event-triggered scheme, Consensus control, Lipschitz nonlinearity

## 1. Introduction

In recent decades, the distributed coordinated control of multi-agent systems has attracted tremendous attention due to its wide adoption in various fields, such as robot formation [1], [2], [3], sensor network [4], [5], [6], neural network [7], [8], target tracking control [9] and so on. Consensus is the basis problem of the coordinated control of multi-agent systems. In practical, each agent may be equipped by the embedded microprocessor, which is with limited computing and communication capabilities. How to utilize these limited resources to operate system efficiently becomes a concerned issue. Therefore, event-triggered consensus has attracted much attention and achieved some results [10], [11], [12]. Singular systems, can provide convenient and accurate description in economic systems, power systems and aircraft modeling from practical considerations. Some literatures have studied the event-triggered for singular systems [13], [14], [15]. However, to the best of our knowledge, there are few works regarding to the event-triggered consensus for singular multi-agent systems (SMAS), which motivates the current study.

In the present study, we focus on the event-triggered consensus control for a class of SMAS with Lipschitz nonlinearity under directed topology. An even-triggered mechanism excluding Zeno behavior is proposed, and an artificial transition time-delay is introduced to design the event-triggered consensus control protocol. Sufficient conditions that can guarantee the consensus of the considered SMAS are obtained. Throughout this paper,  $\mathbf{P} > \mathbf{0}$  ( $\mathbf{P} \geq \mathbf{0}$ ) means that  $\mathbf{P}$  is positive definite (positive semi-definite).  $\mathbf{I}_N$  is the  $N \times N$  identity matrix.  $\otimes$  and  $\|\cdot\|$  are used to represent Kronecker product and

Euclidean norm of a vector or a matrix, respectively. In a symmetric matrix,  $*$  denotes the matrix entries implied by symmetry.  $\mathbf{diag}\{\dots\}$  stands for a block-diagonal matrix.

## 2. Problem formulation and preliminaries

### 2.1. Graph theory

A directed graph  $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$  is considered in this paper, in which  $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$  is the node set,  $\mathcal{E}$  is the edge set, and  $\mathcal{A} = [a_{ij}]_{N \times N}$  is the adjacency matrix,  $a_{ij} > 0$  if  $v_i$  can receive the information from  $v_j$  ( $i, j = 1, 2, \dots, N$ ), otherwise  $a_{ij} = 0$ . The set of neighbors of the  $v_i$  is denoted as  $\mathcal{N}_i$ . Let  $d_i = \sum_{j=1}^N a_{ij}$  be the in-degree of node  $v_i$  and  $D = \text{diag}(d_1, \dots, d_N)$ . The Laplacian matrix of  $\mathcal{G}$  can be defined as  $\mathcal{L} = D - \mathcal{A}$ . Suppose the graph  $\mathcal{G}$  has a directed spanning tree.

### 2.2. Problem formulation

Consider a group of  $N$  agents, and the dynamic of  $i$ th agent is described by

$$E\dot{x}_i(t) = Ax_i(t) + Bu_i(t) + f(x_i(t)), i = 1, 2, \dots, N(1)$$

where  $x_i(t) \in \mathbb{R}^n$ ,  $u_i(t) \in \mathbb{R}^m$  denote the state and the input, respectively.  $E$ ,  $A$  and  $B$  are constant with  $\text{rank}(E) = r < n$ . The nonlinear function  $f(x_i(t))$  satisfies  $\|f(\zeta_1) - f(\zeta_2)\| \leq \mu \|\zeta_1 - \zeta_2\|$ ,  $\forall \zeta_1, \zeta_2 \in \mathbb{R}^n$  with  $\mu > 0$  is the Lipschitz constant.

**Definition 1:** The SMAS (1) is said to achieve admissible consensus if it is regular, impulse free while satisfying

$$\lim_{t \rightarrow \infty} \|x_i(t) - x_j(t)\| = 0, \forall i, j = 1, 2, \dots, N.$$

In this paper, we assume that the state of the multi-agent system (1) is periodically sampled at a constant sampling period  $h > 0$ . The sampling sequence is described by the set  $\mathbb{S}_1 = \{0, h, 2h, \dots, kh, \dots\}$ . The transmission event is determined by an event-triggered scheme. The transmission sequence of agent  $i$  is described by the set  $\mathbb{S}_2 = \{0, t_1^i h, t_2^i h, \dots, t_k^i h, \dots\}$ .

The event detector is described as follows

$$\begin{cases} t_{k+1}^i h = \inf \{kh | kh > t_k^i h, \psi_i(kh) \geq 0\} \\ \psi_i(kh) = \chi_i^T(t_k^i h + l_i h) \Phi \chi_i(t_k^i h + l_i h) - \\ \delta_i \lambda_i^T(t_k^i h + l_i h) \Phi \lambda_i(t_k^i h + l_i h) \end{cases} \quad (2)$$

where  $kh = t_k^i h + l_i h$ ,  $l_i \in \mathbb{N}$ ,  $\delta_i > 0$  is the threshold parameter,  $\Phi$  is a positive definite matrix, and

$$\begin{cases} \chi_i(t_k^i h + l_i h) = x_i(t_k^i h + l_i h) - x_i(t_k^i h) \\ \lambda_i(t_k^i h + l_i h) = \sum_{j \in \mathcal{N}_i} a_{ij} (x_i(t_k^i h) - x_j(t_{k_j}^j h)) \end{cases}$$

with  $k_j^j = \operatorname{argmin}_p \{t_k^i + l_i - t_p^j | t_k^i + l_i > t_p^j, p \in \mathbb{N}\}$ .

Under the event-triggered scheme (2),  $t_{k+1}^i h - t_k^i h$  denotes the transmission period of the Event generators. Let  $\tau_k^i$  denote the signal transmission delay of  $i$ th agent. Suppose  $\tau_k^i$  is bound, that is,  $\tau_k^i \in (0, \tau_M]$ , where  $\tau_M$  is a positive integer. In view of the effect of signal transmission delay, the released states  $x_i(t_k^i h)$  will reach the controller at the time instants  $t_k^i h + \tau_k^i$ . Obviously,  $t_k^i h + \tau_k^i < t_{k+1}^i h + \tau_{k+1}^i$  ( $k = 1, 2, \dots$ ). Figure 1 shows the above-mentioned event-triggered transmission scheme.

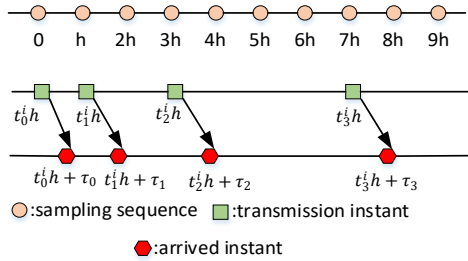


Figure 1. Example of time evolution of the sampling and transmission series of agent  $i$

In system (1), based on the event-triggered mechanism (2), consider the following control protocol

$$u_i(t) = -K \sum_{j \in \mathcal{N}_i} a_{ij} \left( x_i(t_k^i h) - x_j(t_{k_j}^j h) \right) \quad (3)$$

where  $t \in [t_k^i h + \tau_k^i, t_{k+1}^i h + \tau_{k+1}^i)$ ,  $i = 1, 2, \dots, N$ , and  $K$  is the control gain matrix to be determined.

Define the function  $\tau_i(t)$  as

$$\tau_i(t) = \begin{cases} t - t_k^i h, & t \in [t_k^i h + \tau_k^i, t_k^i h + h + \tau_M) \\ t - t_k^i h - \Delta h, & t \in [t_k^i h + \Delta h + \tau_M, t_k^i h + \Delta h + h + \tau_M) \\ t - t_k^i h - dh, & t \in [t_k^i h + dh + \tau_M, t_{k+1}^i h + \tau_{k+1}^i) \end{cases}$$

where  $\tau_M = \max \{\tau_k^i\}$ ,  $\Delta$  is a positive integer satisfying  $\Delta \geq 1$ . It is easy to see that

$$e_i(t_k^i h + \Delta h) = 0, \quad t \in [t_k^i h + \tau_k^i, t_k^i h + h + \tau_M),$$

$$e_i(t_k^i h + \Delta h) = x_i(t_k^i h + \Delta h) - x_i(t_k^i h),$$

$$t \in [t_k^i h + \Delta h + \tau_M, t_k^i h + \Delta h + h + \tau_M)$$

$$e_i(t_k^i h + \Delta h) = x_i(t_k^i h + dh) - x_i(t_k^i h),$$

$$t \in [t_k^i h + dh + \tau_M, t_{k+1}^i h + \tau_{k+1}^i)$$

Then, we get

$$x_i(t_k^i h) = x_i(t - \tau_i(t) - e_i(t_k^i h + \Delta h))$$

Then, the event-triggered scheme (2) can be rewritten as

$$\chi_i^T(t_k^i h + l_i h) \Phi \chi_i(t_k^i h + l_i h) < \delta_i \left[ \sum_{j \in \mathcal{N}_i} a_{ij} (x_i(t_k^i h) - x_j(t_{k_j}^j h)) \right]^T \Phi \left[ \sum_{j \in \mathcal{N}_i} a_{ij} (x_i(t_k^i h) - x_j(t_{k_j}^j h)) \right]$$

Substituting (3) into the system (1) yields

$$(I_N \otimes E) \dot{x}(t) = (I_N \otimes A) x(t) - (\mathcal{L} \otimes BK) x(t - \tau(t)) + (\mathcal{L} \otimes BK) e(kh) + F(x(t))$$

Let  $z_i(t) = x_i(t) - x_{i+1}(t)$ ,  $\varepsilon_i(kh) = e_i(kh) - e_{i+1}(kh)$ ,  $z(t) = [z_1^T(t), z_2^T(t), \dots, z_{N-1}^T(t)]^T = (T_1 \otimes I_n) x(t)$ ,  $\varepsilon(kh) = [\varepsilon_1^T(kh), \varepsilon_1^T(kh), \dots, \varepsilon_{N-1}^T(kh)] = (T_2 \otimes I_n) e(kh)$ ,  $T_1 = [1, -I_{N-1}] \in \mathbb{R}^{(N-1) \times N}$ ,  $T_2 = [0, -I_{N-1}]^T \in \mathbb{R}^{N \times (N-1)}$ . The following system is immediate

$$(I_{N-1} \otimes E) \dot{z}(t) = (I_{N-1} \otimes A) z(t) - (T_1 \mathcal{L} T_2 \otimes BK) z(t - \tau(t)) + (T_1 \mathcal{L} T_2 \otimes BK) \varepsilon(kh) + (T_1 \otimes I_n) F(x(t)) \quad (4)$$

Now, the consensus problem of SMAS (1) is transformed to the admissible of system (4).

### 3. Main results

**Theorem 1:** Given  $h$ ,  $\tau_m$ ,  $\bar{\tau}_M$ , and  $\mu$ , the SMAS (1) achieves admissible consensus if there exist real matrices  $P$ , positive definite matrices  $Q, R_1, R_2, Z_1, Z_2$  such that the following matrix inequalities hold

$$\bar{E}^T P \bar{E} \geq 0 \quad (5)$$

$$\Omega_1 = \begin{bmatrix} \Psi_0 + Y + Y^T & \sqrt{\tau_m} \Psi_1 & \sqrt{\tau_{Mm}} \Psi_2 & \sqrt{\tau_m} L & \sqrt{\tau_{Mm}} M \\ * & Z_1 & 0 & 0 & 0 \\ * & * & Z_2 & 0 & 0 \\ * & * & * & Z_1 & 0 \\ * & * & * & * & Z_2 \end{bmatrix} < 0 \quad (6)$$

$$\Omega_2 = \begin{bmatrix} \Psi_0 + Y + Y^T & \sqrt{\tau_m} \Psi_1 & \sqrt{\tau_{Mm}} \Psi_2 & \sqrt{\tau_m} L & \sqrt{\tau_{Mm}} N \\ * & Z_1 & 0 & 0 & 0 \\ * & * & Z_2 & 0 & 0 \\ * & * & * & Z_1 & 0 \\ * & * & * & * & Z_2 \end{bmatrix} < 0 \quad (7)$$

where

$$\Phi_0 = \bar{A}^T P \bar{P}^T + P \bar{A} + R_1 + R_2 + \mu^2 I,$$

$$\Phi_1 = -Q + \lambda (T_2^T L^T \Lambda L T_2) \otimes \Phi,$$

$$\Phi_2 = -\lambda (T_2^T L^T \Lambda L T_2) \otimes \Phi,$$

$$\Phi_3 = -(I_{N-1} \otimes \Phi) + \lambda (T_2^T L^T \Lambda L T_2) \otimes \Phi,$$

$$Y = [L\bar{E} \quad \mathbf{0} \quad (N-M)\bar{E} \quad (M-N)\bar{E} \quad -N\bar{E} \quad \mathbf{0}],$$

$$\Psi_1 = [Z_1\bar{A} \quad \mathbf{0} \quad Z_1\bar{L} \quad \mathbf{0} \quad \mathbf{0} \quad -Z_1\bar{L}]^T,$$

$$\Psi_2 = [Z_2\bar{A} \quad \mathbf{0} \quad Z_2\bar{L} \quad \mathbf{0} \quad \mathbf{0} \quad -Z_2\bar{L}]^T,$$

$$\mathbb{P} = I_{N-1} \otimes (E^T P + TS^T).$$

$$\Psi_0 = \begin{bmatrix} \Phi_0 & \mathbb{P} & -\mathbb{P}\bar{L} & \mathbf{0} & \mathbf{0} & \mathbb{P}\bar{L} \\ * & -I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & \Phi_1 & \mathbf{0} & \mathbf{0} & \Phi_2 \\ * & * & * & -R_1 & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -R_2 & \mathbf{0} \\ * & * & * & * & * & \Phi_3 \end{bmatrix}$$

**Proof:** Firstly, similar to the analysis in [14], it can be proved that LMIs (6) and LMIs (7) can guarantee that system (4) is regular and impulse-free.

Next, we will prove that the system (4) is asymptotically stable. Construct the Lyapunov functional

$$V(\mathbf{t}) = V_1(\mathbf{t}) + V_2(\mathbf{t}) + V_3(\mathbf{t})$$

$$V_1(\mathbf{t}) = \mathbf{z}^T(\mathbf{t})\bar{E}^T P \bar{E} \mathbf{z}(\mathbf{t}) + (\bar{\tau}_M - \tau(\mathbf{t})) \mathbf{z}^T(\mathbf{t} - \tau(\mathbf{t})) Q \mathbf{z}(\mathbf{t} - \tau(\mathbf{t}))$$

$$V_2(\mathbf{t}) = \int_{t-\tau_m}^t \mathbf{z}^T(\alpha) E R_1 \mathbf{z}(\alpha) d\alpha + \int_{t-\bar{\tau}_M}^t \mathbf{z}^T(\alpha) R_2 \mathbf{z}(\alpha) d\alpha$$

$$V_3(\mathbf{t}) = \int_{t-\tau_m}^t \int_{t+\beta}^t \dot{\mathbf{z}}^T(\alpha) \bar{E}^T Z_1 \bar{E} \dot{\mathbf{z}}(\alpha) d\alpha d\beta + \int_{t-\bar{\tau}_M}^t \int_{t+\beta}^t \dot{\mathbf{z}}^T(\alpha) \bar{E}^T Z_2 \bar{E} \dot{\mathbf{z}}(\alpha) d\alpha d\beta$$

From  $\tau_m \leq \tau(\mathbf{t}) \leq \bar{\tau}_M$ . And taking the derivative of  $V(\mathbf{t})$  with respect to  $\mathbf{t}$  along the solution of (4) yields

$$\begin{aligned} \dot{V}(\mathbf{t}) &= \dot{\mathbf{z}}^T(\mathbf{t}) \bar{E}^T (I_{N-1} \otimes P) \bar{E} \mathbf{z}(\mathbf{t}) + \\ &\mathbf{z}^T(\mathbf{t}) \bar{E}^T (I_{N-1} \otimes P) \bar{E} \dot{\mathbf{z}}(\mathbf{t}) - \mathbf{z}^T(\mathbf{t} - \tau(\mathbf{t})) Q \mathbf{z}(\mathbf{t} - \tau(\mathbf{t})) + \\ &\mathbf{z}^T(R_1 + R_2) \mathbf{z}(\mathbf{t}) - \mathbf{z}^T(\mathbf{t} - \tau_m) R_1 \mathbf{z}(\mathbf{t} - \tau_m) - \mathbf{z}^T(\mathbf{t} - \\ &\bar{\tau}_M) R_2 \mathbf{z}(\mathbf{t} - \bar{\tau}_M) + \tau_m \dot{\mathbf{z}}^T(\mathbf{t}) \bar{E}^T Z_1 \bar{E} \dot{\mathbf{z}}(\mathbf{t}) + (\bar{\tau}_M - \\ &\tau_m) \mathbf{z}^T(\mathbf{t}) \bar{E}^T Z_2 \bar{E} \dot{\mathbf{z}}(\mathbf{t}) - \int_{t-\tau_m}^t \dot{\mathbf{z}}^T(\alpha) \bar{E}^T Z_1 \bar{E} \dot{\mathbf{z}}(\alpha) d\alpha - \\ &\int_{t-\bar{\tau}_M}^t \dot{\mathbf{z}}^T(\alpha) \bar{E}^T Z_2 \bar{E} \dot{\mathbf{z}}(\alpha) d\alpha. \end{aligned}$$

For any free-weighting matrices  $L$ ,  $M$  and  $N$ , we have

$$2\eta^T(\mathbf{t}) L \left[ \bar{E} \mathbf{z}(\mathbf{t}) - \bar{E} \mathbf{z}(\mathbf{t} - \tau_m) - \int_{t-\tau_m}^t \bar{E} \dot{\mathbf{z}}(\alpha) d\alpha \right] = \mathbf{0}$$

$$2\eta^T(\mathbf{t}) M \left[ \bar{E} \mathbf{z}(\mathbf{t} - \tau_m) - \bar{E} \mathbf{z}(\mathbf{t} - \tau(\mathbf{t})) - \int_{t-\tau(\mathbf{t})}^{t-\tau_m} \bar{E} \dot{\mathbf{z}}(\alpha) d\alpha \right] = \mathbf{0}$$

$$2\eta^T(\mathbf{t}) N \left[ \bar{E} \mathbf{z}(\mathbf{t} - \tau(\mathbf{t})) - \bar{E} \mathbf{z}(\mathbf{t} - \bar{\tau}_M) - \int_{t-\bar{\tau}_M}^{t-\tau(\mathbf{t})} \bar{E} \dot{\mathbf{z}}(\alpha) d\alpha \right] = \mathbf{0}$$

where  $\eta(\mathbf{t}) = [\mathbf{z}^T(\mathbf{t}), F^T(\mathbf{z}(\mathbf{t})), \mathbf{z}^T(\mathbf{t} - \tau(\mathbf{t})), \mathbf{z}^T(\mathbf{t} - \tau_m), \mathbf{z}^T(\mathbf{t} - \bar{\tau}_M), \varepsilon^T(kh)]^T$ . Then, we have

$$\begin{aligned} -2\eta^T(\mathbf{t}) L \int_{t-\tau_m}^t \bar{E} \dot{\mathbf{z}}(\alpha) d\alpha &\leq \tau_m \eta^T(\mathbf{t}) L Z^{-1} L^T \eta(\mathbf{t}) \\ &+ \int_{t-\tau_m}^t \dot{\mathbf{z}}^T \bar{E}^T Z_1 \bar{E} \dot{\mathbf{z}}(\alpha) d\alpha \end{aligned}$$

$$-2\eta^T(\mathbf{t}) M \int_{t-\tau(\mathbf{t})}^{t-\tau_m} \bar{E} \dot{\mathbf{z}}(\alpha) d\alpha \leq (\tau(\mathbf{t}) - \tau_m) \eta^T(\mathbf{t})$$

$$\begin{aligned} &M Z_1^{-1} M^T \eta(\mathbf{t}) + \int_{t-\tau(\mathbf{t})}^{t-\tau_m} \dot{\mathbf{z}}^T \bar{E}^T Z_2 \bar{E} \dot{\mathbf{z}}(\alpha) d\alpha \\ &- 2\eta^T(\mathbf{t}) N \int_{t-\bar{\tau}_M}^{t-\tau(\mathbf{t})} \bar{E} \dot{\mathbf{z}}(\alpha) d\alpha \leq (\tau_m - \tau(\mathbf{t})) \eta^T(\mathbf{t}) \\ &N Z^{-1} N^T \eta(\mathbf{t}) + \int_{t-\bar{\tau}_M}^{t-\tau(\mathbf{t})} \dot{\mathbf{z}}^T(\alpha) \bar{E}^T Z_2 \bar{E} \dot{\mathbf{z}}(\alpha) d\alpha \end{aligned}$$

By using the Kronecker product, the event-triggered scheme can be written in the compact form as follows:

$$\begin{aligned} &\varepsilon^T(kh) (I_{N-1} \otimes \Phi_1) \varepsilon(kh) \\ &= e^T(kh) (T_1^T I_{N-1} T_1 \otimes \Phi_1) e(kh). \end{aligned}$$

It is obviously that

$$\begin{aligned} &\varepsilon^T(kh) (I_{N-1} \otimes \Phi_1) \varepsilon(kh) \leq \lambda e^T(kh) (I_N \otimes \Phi_1) e(kh) \\ &\leq \lambda [z(kh) - \varepsilon(kh)]^T [(T_1^T L^T \Lambda L T_2) \otimes \Phi_2] [z(kh) - \varepsilon(kh)] \end{aligned}$$

where  $\lambda$  is the largest eigenvalue of  $T_1^T T_1$ .  $\Lambda = \text{diag}\{\delta_1, \delta_2, \dots, \delta_N\}$ .

Since  $\|f(x_i(t)) - f(x_j(t))\| \leq \mu \|x_i(t) - x_j(t)\|$ , Then,  $F^T(z(t))F(z(t)) \leq \mu^2 z^T(t)(I_{N-1} \otimes I_n)z(t)$ . To sum up, we can get

$$\dot{V}(\mathbf{t}) \leq \eta^T(\mathbf{t}) \Omega \eta(\mathbf{t})$$

where

$$\begin{aligned} \Omega &= \Psi + Y + Y^T + \tau_m \Psi_1 Z_1^{-1} \Psi_1^T + (\bar{\tau}_M - \tau_m) \Psi_2 Z_2^{-1} \Psi_2^T \\ &+ \tau_m L Z_1^{-1} L^T + (\tau(\mathbf{t}) - \tau_m) M Z_1^{-1} M^T \\ &+ ((\bar{\tau}_M - \tau(\mathbf{t})) N Z_1^{-1} N^T. \end{aligned}$$

The inequalities (6) and (7) are equivalent to  $\Omega < \mathbf{0}$ , that is  $\dot{V}(\mathbf{t}) < \mathbf{0}$ , which implies system (4) is asymptotically stable. Hence, system (4) is admissible. From Definition 1, the multi-agent system (1) achieves admissible consensus. This completes the proof.

#### 4. Simulation example

Consider a SMAS with six agents shown in Figure 2. There is a directed spanning tree in this graph.

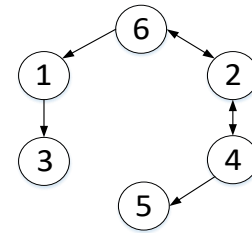


Figure 2. The interaction topology of agents

The dynamics of each agent described by (1) with  $f(x_i(t)) = [\mathbf{0} \quad \mathbf{0} \quad -\mu \sin(x_{i1})]^T$ , and

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -2 & 1 & 0.5 \\ 0 & 0 & -1 \\ 2 & 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Let  $h = 0.05$ ,  $\tau_m = 0.02$ ,  $\bar{\tau}_M = 0.07$ ,  $\mu = 0.5$  and  $x_i = [x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5}, x_{i6}]^T$ . The initial state is selected as

$$\begin{bmatrix} x_1(0) & x_2(0) & x_3(0) & x_4(0) & x_5(0) & x_6(0) \end{bmatrix} = \begin{bmatrix} 1.5 & -1 & 1 & 1.5 & -1.5 & 0.5 \\ 1 & -1.5 & 1.5 & 1 & -1 & -1.5 \\ -1.5 & 0.5 & -1 & 1.5 & 1 & -0.5 \end{bmatrix}$$

Solve the matrix inequalities (5), (6), (7), we can obtain  $K = [0.0445 \quad -0.2153 \quad 0.1489]$ , and

$$\Phi = \begin{bmatrix} 29.4174 & 9.0284 & 9.4232 \\ 9.0284 & 43.4806 & 32.8987 \\ 9.4232 & 32.8987 & 48.9758 \end{bmatrix}$$

The state trajectories of system (1) are shown in Figure 3, 4, 5. One can clearly see that all agent's states can indeed reach consensus.

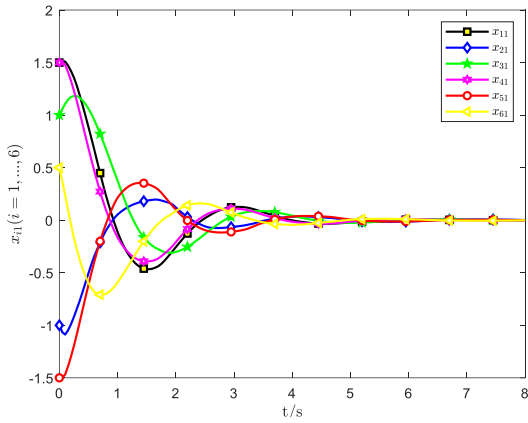


Figure 3. State trajectories  $x_{i1}, (i = 1, 2 \dots 6)$

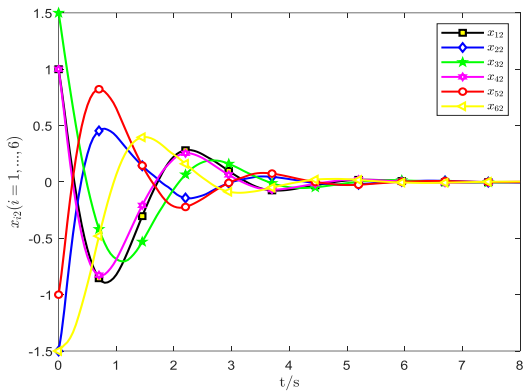


Figure 4. State trajectories  $x_{i2}, (i = 1, 2 \dots 6)$

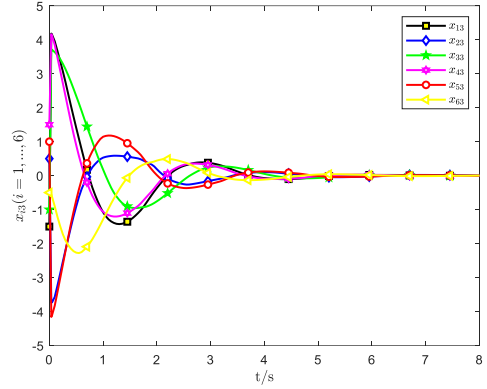


Figure 5. State trajectories  $x_{i3}, (i = 1, 2 \dots 6)$

The transmission instants and release intervals are illustrated in Figure 6, which shows that the number of the sampled-data transmission is significantly decreased.

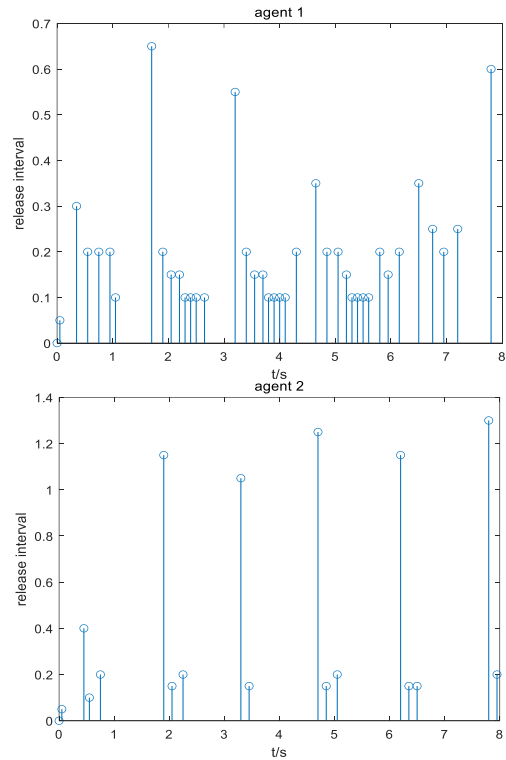


Figure 6. Transmission instants and release interval of agent  $i (i = 1, 2)$

## 5. Conclusion

Event-triggered consensus problem of nonlinear SMAS with directed topologies has been investigated in this paper. The conditions of achieving event-triggered consensus were obtained, while consensus control gain matrix is designed. The effectiveness of the proposed method was verified by a numerical example. It would be important and more practical to study consensus for SMAS with leaders or under switching topologies, which will be our further analysis.

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## Authors Introduction

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