# Frequency Dependence Performance Limit of Vibration Absorbers

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**Abstract**

Optimal design of vibration absorbers has been extensively investigated. Most of the design methods are approached by optimizing certain performance indices, resulting in a set of optimal parameters that are independent of exogenous forcing frequencies. In practical designs, however, it is often desirable to know the performance limits over a frequency band of interest. This problem is tackled in the present paper where both lower and upper bounds are obtained. Numerical examples are given to validate the corresponding designs.

*Keywords*: Vibration absorbers, optimal design, performance limit

## 1. Introduction

Tuned mass dampers (TMD) are widely used for vibration attenuation either at a frequency or over a frequency range. Many configurations have been proposed [1], [2], [3] while one typical TMD consists of a secondary mass-damper-spring system, attached to a vibrating primary system. The secondary system is also called an absorber whose parameters are to be designed. Besides the classical “equal height” methods, many approaches have been proposed for optimal design of TMD parameters. Most of the developments are preceded with formulating optimization problems by optimizing a properly chosen performance index. For example, H2/H∞ forms of performance indices can be optimized where even analytical solution can be found for optimal parameter selections [4], [5]. A comparison between different optimization criteria is given in [6], while more elaborated examples can be found in [7]. As a result of the optimization, optimal parameters can be obtained which is usually represented as optimal damping and frequency ratios as a function of the mass ratio.

In review of the available results in the literature, it is seen that the optimization of performance indices leads to the results that only optimize the pre-designated indices, yet performance limits, particularly the frequency-dependence performance bounding information cannot be obtained. This frequency-dependence performance limit of vibration absorbers is considered in the paper. It is organized as follows: section 2 formulates the problem to be considered. Section 3 proceeds to develop the lower and upper bounds. Numerical examples are also provided before a conclusion in section 4.

## 2. Problem Formulation

The model can be represented as:

$$\begin{aligned}
&m_1\ddot{x}_1+(c_1+c_2)\dot{x}_1-c_2\dot{x}_2+(k_1+k_2)x_1-k_2x_2=d\\
&m_2\ddot{x}_2-c_2\dot{x}_1+c_2\dot{x}_2-k_2x_1+k_2x_2=0\\
&F=c_1\dot{x}_1+k_1x_1
\end{aligned}\tag{1}$$

where the dependence on time has been omitted for easy reference. The frequency response property for the transmission force can then be written down:

$$\frac{F(j\omega)}{D(j\omega)} = \frac{(k_1 + jc_1\omega)(-m_2\omega^2 + k_2 + jc_2\omega)}{[-m_1\omega^2 + k_1 + k_2 + j(c_1 + c_2)\omega](-m_2\omega^2 + k_2 + jc_2\omega) - (k_2 + jc_2\omega)^2} \quad (2)$$

where $F(j\omega)$ and $D(j\omega)$ are the Fourier transforms of $F(t)$ and $d(t)$, respectively.

The aim of the TMD design is to design the absorber parameters $m_2$, $c_2$, and $k_2$ in such a way, so that certain appropriate performance indices such as energy/magnitude defined by H2/H∞ norms are optimized. Although the optimization "routines" exist for obtaining feasible solutions to the corresponding optimization problems, it is still of great significance and interest to seek the boundary or limit of performance. As the limits will dictate the achievable performance, on the one hand, *they are not to be conquered by any form of performance index to be optimized; on the other hand, they will actually provide guidance to the designers* if a choice of performance index is suitable by knowing its "distance" to the limits. Henceforth, performance limits should be treated as a benchmarking that any design through any optimization method with any performance index must compare with. These issues are treated in the following sections.

## 3. Performance Limit: Lower & Upper Bounds

From equation (1), it is known that the following frequency responses relationship holds:

$$F(j\omega) = (k_1 + j\omega c_1)X_1(j\omega) \quad (3)$$

For any particular configuration, the primary system parameters $c_1$ and $k_1$ are known, hence optimizing $F(j\omega)/D(j\omega)$ is equivalent to optimizing $X_1(j\omega)/D(j\omega)$. From equation (2), it is known:

$$\frac{X_1(j\omega)}{D(j\omega)} = \frac{(-m_2\omega^2 + k_2 + jc_2\omega)}{[-m_1\omega^2 + k_1 + k_2 + j(c_1 + c_2)\omega](-m_2\omega^2 + k_2 + jc_2\omega) - (k_2 + jc_2\omega)^2} \quad (4)$$

Thus the objective of TMD design can be re-stated to reduce the transmission magnitude $|X_1(j\omega)/D(j\omega)|$ through the optimal selection of the absorber parameters $m_2$, $c_2$, and $k_2$. In the following, the performance bounds for attenuation of the magnitude of $X_1(j\omega)/D(j\omega)$ through the to-be-designed parameters of $m_2$, $c_2$, and $k_2$ will be sought. This is preceded by boldly stating the results.

**Theorem 1 (Lower Bound)**: The performance of $|X_1(j\omega)/D(j\omega)|$ is bounded from below by:

$$\left|\frac{X_1(j\omega)}{D(j\omega)}\right| > \frac{1}{\sqrt{(k_1 - m_1\omega^2)^2 + c_1^2\omega^2} + \omega\sqrt{\frac{m_2(k_2^2 + c_2^2\omega^2)}{2}}}, \ \forall\omega \quad (5)$$

The relationship is strictly "greater than" implying that the lower bound is absolute and never to be attained.

**Theorem 2**: The performance of $|X_1(j\omega)/D(j\omega)|$ is bounded from above by:

$$\left|\frac{X_1(j\omega)}{D(j\omega)}\right| < \frac{\sqrt{K^2 + M^2\omega^4 + C^2\omega^2}}{h(\omega)}, \ \forall\omega \quad (6)$$

where: $h(\omega)$ is a positive function dependent on frequency.

From the above result, a series of observations follow:

(1) While the minimum lower bound is attained at natural frequency of the primary system $\omega_1 = \sqrt{k_1/m_1}$, the minimum upper bound is achieved at the natural frequency of the absorber $\omega_2 = \sqrt{k_2/m_2}$.

(2) As the lower bound, the upper bound is also inversely proportional to the natural frequency $\omega_2$ —to increase the bound, $\omega_2$ needs to be decreased!

(3) Yet one of the most important applications for upper bound is the assertion that the performance $|X_1(j\omega)/D(j\omega)|$ will always be attenuated over the frequency bands where the upper bound is less than unity. This can be developed into a very useful and powerful design methodology.

For example, assume $c_1$ and $m_2$ are unities, then the bound becomes:

$$\frac{1}{\omega + \frac{c_2\omega^5}{(k_2 - \omega^2)^2 + c_2^2\omega^2}} \quad (7)$$

Then a calculation for cubic unities of $k_2$, $c_2$, and $\omega$ with a grid of $10\times10\times10$ shows that a set of solutions exist for satisfying (7). This is shown in Figure1.

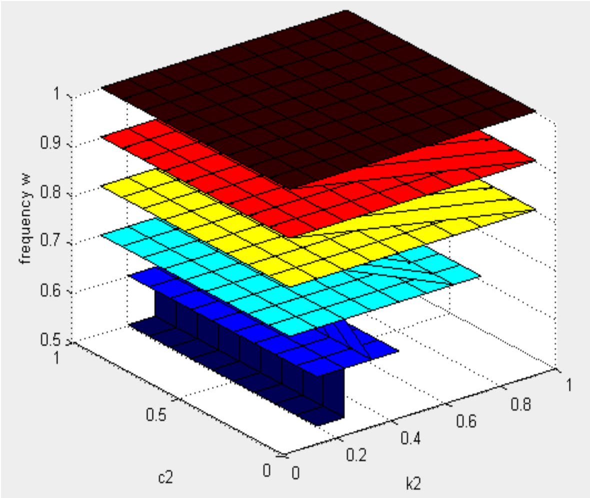


Figure 1: feasible choices of parameters $k_2$, $c_2$, and $\omega$ within a $10 \times 10 \times 10$ cubic unity grid

## 4. Conclusion

Frequency-dependent performance limits of tuned mass dampers have been considered. It has been demonstrated that the performance is not only bounded from below, but also bounded from above. The existence of the lower bound is very useful providing guidance upon best performance to be expected. Combining with the concept of attenuation by bounding from above, it has been shown that this can lead to design methods with such features as guaranteed performance.

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## Authors Introduction

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