Research on Multi-Robot Formation on Two-Dimensional Plane

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Abstract

Mobile robots are playing an increasingly important role in both service and manufacturing industry. The management of multiple mobile robots is a very important issue on the research of mobile robotics. From a mathematical perspective, this paper discusses the problem of multiple robots on a two-dimensional plane reaching the designated positions in the shortest time to complete formation transformation. We improved the algorithm proposed by Hsia, Li and Su and proposed a new algorithm using a determinant and the Munkres assignment algorithm. Finally, the new algorithm is compared with the path distribution obtained by the Monte Carlo method under different numbers of robots and the excellence of the new algorithm has been verified.

Keywords: Multiple mobile robot, Pattern formation, Two-dimensional plane, Munkres assignment algorithm.

1. Introduction

In this society where time is money, saving time is an important factor that must be considered on task completion. How to complete tasks in the shortest time is what task assignment issues aim to achieve. Assignment problem [1] refers to the process of assigning specific work, tasks or responsibilities to specific people or teams. In an organization or team, assigning issues is a way of allocating work to ensure that matters are carried out efficiently and achieve desired goals. How to make an assignment that the tasks could be completed in the shortest time has always been the most important issue.

The shortest-path planning problem [2] is to find an assignment way for shortest-time running. Path planning refers to the movement path generated in order to enable one or more objects to complete a certain task in a designated area. In order to complete the task assignment problem in the shortest time, it is important to find a way to have the shortest path. The importance of the algorithm for this problem is that it can have the solution with higher efficiency, and the obtained result is repeatable.

This paper discusses the assignment problem of formation change of multiple robots. For a centralized robot system, there is a control center that makes decisions and assignments, while the other robots execute the received commands. In this paper, the multi-robot system is a centralized system and an algorithm is used to determine the destination that each robot is planning to move to. There are many types of formations in discussion, such as wild goose formation, front arrow formation, T-shaped formation, etc. The robots are asked to transfer their formation from one to another. The robots could only move laterally or vertically on a plane, so the robot is limited to a two-dimensional grid plane. In order to allow multiple robots to complete the formation in the shortest time, an algorithm can be used to find the assignment way that makes the formation transformation successfully in the shortest time.

2. Preliminary

2.1. *Problem description*

In the multi-robot formation transformation problem, there are n robots on a two-dimensional grid plane. The robots move horizontally or vertically from the starting positions to the target positions in the same speed. In order to find the fastest way to complete the formation change, we have to decide the target position for every robot and the robot with the longest distance will be assigned with a fastest way. This is a min-max problem.

2.2. *L1-norm between two points*

The L1-norm between two points is defined by the total distance of the projection of the line segment formed by each point on the axes on the Cartesian coordinate system, that is, the horizontal distance between the two points plus the longitudinal distance between them. Suppose the two points are at (x_1, y_1) and (x_2, y_2), their distance is $|x_1 - x_2| + |y_1 - y_2|$.

2.3. *Determinant*

To complete the assignment problem of *n* robots, each robot at his starting position will have a corresponding target point position. When all robots are assigned, their corresponding relationships will be presented in the distance matrix, and unassigned positions can be filled with 0. Under such circumstances, there will be *n* non-zero values in the distance matrix on different rows and columns, forming a square matrix of full rank. The fastest way to check for full rank is to check whether its determinant is 0 or not. Therefore, we use the determinant as a tool for checking the feasibility of formation transformation planning.

2.4. *Munkres assignment algorithm*

Munkres assignment algorithm [3] is an optimization algorithm for solving assignment problems. Given a cost or benefit matrix, it finds a set of assignments that minimize the total cost or maximize the total benefit, in which each row and column only can be assigned once.

2.5. *Monte Carlo method*

The Monte Carlo method is a numerical calculation method based on random sampling and statistical simulation [4]. When assigning tasks using the Monte Carlo method, one can randomly find an allocation way, and then find the maximum of the solution. One can get the number of steps for completing the tasks by using the Monte Carlo method one time. The more times of sampling, the more assignment ways there will be. By minimizing the maximum value of these, one can get a assignment way almost the most suitable.

3. Proposed new algorithm

3.1. *Deviation of the new algorithm*

We refer to the algorithm proposed by Hsia, Li and Su [5], but instead of finding the assignment way from the entries of the distance matrix, we find the minimum path from the positional relationship of the values in the matrix, and use the determinant to make the decision. The program searches sequentially from 0, and fills the searched position into another decision matrix. When a value is searched in the distance matrix, the corresponding position in the decision matrix is filled with prime numbers from small to large, and the rest are not searched. The reached position is represented by 0 in the judgment matrix. When the search is completed, find the determinant of the judgement matrix. If the result is 0, add 1 to the search value and search again. Repeat the above steps until the determinant value is not 0, and then use the Munkres assignment algorithm to assign tasks.

3.2. *The algorithm*

Step 1: Calculate the L1-norm of all starting points to target points, and form the results into a matrix named distance matrix. The columns of the matrix correspond to the starting points, and the rows correspond to the target points.

Step 2: Initialize the judgement matrix of same dimension as distance matrix with all entries 0.

Step 3: Set the search value as 0.

Step 4: Fill in prime numbers in sequence to the corresponding position in the judgment matrix where the entry of the distance matrix is equal to the search value. The selection of prime numbers starts from 2, and each prime number filled in is the smallest prime number larger than the previous one.

Step 5: If the determinant of the decision matrix is not 0, jump to step 7.

Step 6: Increase the search value and back to step 4.

Step 7: Replace all positions of the distance matrix by a sufficiently large number, e.g. 999, if the corresponding position of the judgment matrix is 0.

Step 8: For a distance matrix, subtract the minimum value of that row from each row and subtract the minimum value of that column from each column. Thus at least one element in each row and each column will be zero.

Step 9: Use the least straight line to pass through all 0s, so that each 0 has at least one line passing through it.

Step 10: When the number of lines equals the number of robots, the position of 0 in the matrix can be used for task assignment. Starting from the row with the fewest 0s, only one 0 can be circled in each row and column. If the number of circles is equal to the number of robots, the assignment of the task is completed. If not, go to the next step.

Step 11: Subtract the minimum value in the undrawn row /column, and add the minimum value to the intersection position of the lines. After completion, back to step 9.

4. Comparison to Monte Carlo method

This comparison was performed on Matlab 2021, simulating the formation transformation situations of 10, 30, 50, 100, 500 and 1000 robots respectively. The starting and target points of the robots are randomly generated each time. Then the proposed algorithm and the Monte Carlo method are used for task assignment. The Monte Carlo method takes the best allocation method out of 10,000 times as the representative value. After comparison, the smaller one is the better assignment in the simulation. For each number of robots, we have 1000 simulations for both methods. This comparison index is to count the number of simulations that the proposed algorithm's result is not greater than the Monte Carlo method's result in 1000 simulations with different numbers of robots. The simulation results are summarized in Table 1. It is clear from Table 1 that the proposed algorithm can always have the same or better assignment way. Hence the proposed algorithm is better than Monte Carlo method for the formation transformation problem.

Table 1. Comparison results.

Number of robots	10	30	50	100	500	1000
Number of the results of proposed algorithm not worse than Monte Carlo method	1000	1000	1000	1000	1000	1000

5. Illustrated Examples

This paper takes three formation transformations as examples, namely the flying goose formation, the front vector formation and the T-shaped formation, and MATLAB is used for simulation. The robot is located on a 7x7 grid plane. The configuration of each formation is shown in Fig. 1. For the flying goose formation (V shape), the robots are at (4, 1), (4, 7), (5, 2), (5, 6), (6, 3), (6, 5), and (7, 4). For the front vector formation (Q shape), the robots are at (2, 4), (3, 4), (4, 4), (5, 4), (6, 3), (6, 5), and (7, 4). For the T-shaped formation (T shape), the robots are at (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (3, 3), and (4, 3).

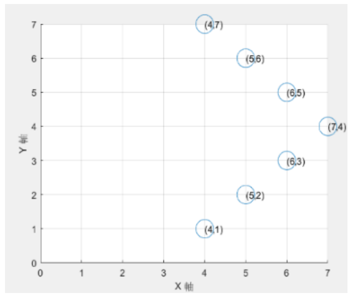
The 7 robots form the flying goose formation initially. We would like to make the robots changed to the front vector formation, and then changed to the T-shaped formation. The assignment results are summarized in Table 2, and the routes for the robots are shown in Fig. 2.

Table 2. The assignment results.

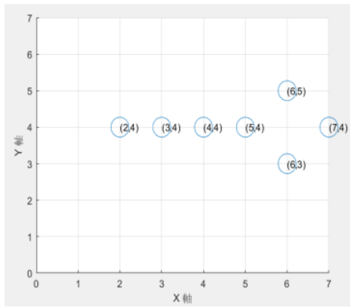
Pts of V-shape	(4, 1)	(4, 7)	(5, 2)	(5, 6)	(6, 3)	(6, 5)	(7, 4)
Pts of Q-shape	(2, 4)	(4, 4)	(3, 4)	(5, 4)	(6, 3)	(6, 5)	(7, 4)
Distance for #1	5	3	4	2	0	0	0
Pts of T-shape	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(3, 3)	(2, 5)	(4, 3)
Distance for #2	3	4	2	3	3	4	4
Pts of V-shape	(4, 1)	(5, 2)	(6, 3)	(4, 7)	(5, 6)	(6, 5)	(7, 4)
Distance for #3	2	3	4	5	5	4	4

6. Conclusion

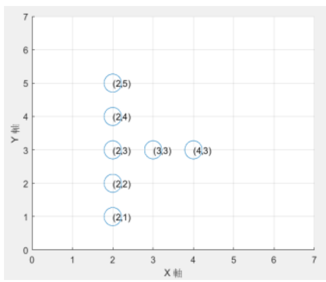
This paper proposed a new algorithm to solve the task assignment problem for a multi-robot system on formation transformation. Each mobile robot is limited to horizontal and vertical movement and starts at the same time. We'd like the transformation to be completed in the shortest time. The proposed algorithm can be used to systematically find out how to assign the target points of each robot so that the robot's formation can be transformed in the shortest time. Unlike previous works, we use determinant of a matrix to find out the shortest time required to complete the formation change, and use the Munkres assignment algorithm to obtain the shortest time. This work can ensure the best assignment result.



(a) flying goose formation



(b) front vector formation



(c) T-shaped formation

Fig. 1. Formations considered in this paper

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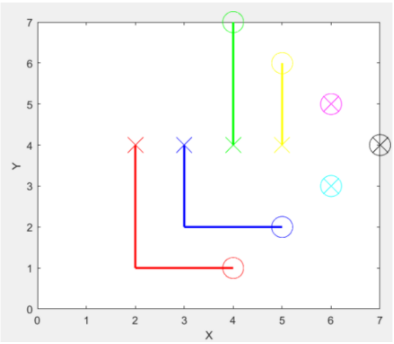
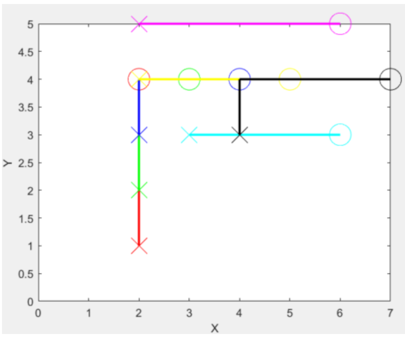
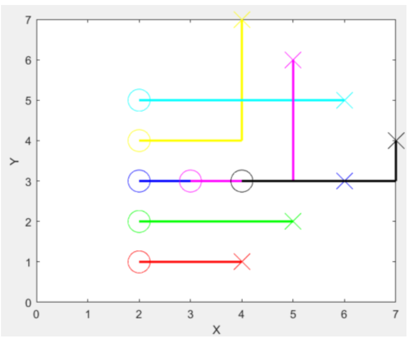


(a) flying goose formation -> front vector formation



(b) front vector formation -> T-shaped formation



(c) T-shaped formation -> flying goose formation

Fig. 2 Formation transformation simulation

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