Chaos Synchronization and Circuit Design of Chen System and Lü System with Different Structures

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Abstract

In this paper, by using nonlinear feedback control, chaos synchronization is achieved between the Chen system and the Lü system with different initial values, and the error curves and state synchronization curves of the corresponding states in the response Lü system and the drive Chen system are plotted. Finally, the simulation circuit model of the synchronization system of the drive Chen system and the response Lü system is designed by Multisim circuit simulation software. Comparing the output curves with the curves obtained by MATLAB simulation software, it can be found that the two curves achieve a good qualitative agreement. It prove that the circuit model of the synchronization system is correct and the synchronization of the drive Chen system and the response Lü system is accomplished.

Keywords: Chaos; Chaos synchronization of different systems; Nonlinear feedback control; Chaotic circuit

1. Introduction

In 1999, Professor Guanrong Chen of the University of Houston discovered a new chaotic attractor, the Chen system, using engineering feedback control [1]. which is similar to the Lorenz system but not topologically equivalent, with a more complex structure and more diverse dynamic behavior.In 2001, Jinhu Lü and Guanrong Chen discovered a new chaotic system, the Lü system, that system links the Lorenz system and the Chen system and represents a continuous evolution between the two systems [2].

In this paper, the above two systems are taken as the research objects to analyze their chaotic properties, and based on this, the two systems are synchronized to chaos by using the method of nonlinear feedback [3], and then numerical simulation is carried out by using the MATLAB software, and the simulated circuits are designed, and the circuit simulation is carried out in Multisim, which is designed to provide sufficient theoretical support for the future application of the heterostructured chaos synchronization of the Chen system and the Lü system to be successfully applied in the field of confidential communication.

2. Chaos Synchronization and Circuit Design of Chen System and Lü System with Different Structures

2.1. Modeling of the Drive Chen system

$$\begin{cases} \dot{x}_1 = a_1(y_1 - x_1) \\ \dot{y}_1 = (c_1 - a_1)x_1 - x_1 z_1 + c_1 y_1 \\ \dot{z}_1 = x_1 y_1 - b_1 z_1 \end{cases} \tag{1}$$

Where, x_1, y_1, $z_1 \in R$ is the state variable of the system and $a_1 = 35$, $b_1 = 3$, $c_1 = 28$ is the typical parameter of Lü system.

2.2. Modeling of Response Lü Systems

$$\begin{cases} \dot{x}_2 = a_2(y_2 - x_2) + u_{c1} \\ \dot{y}_2 = -x_2 z_2 + c_2 y_2 + u_{c2} \\ \dot{z}_2 = x_2 y_2 - b_2 z_2 + u_{c3} \end{cases} \tag{2}$$

Where, x_2, y_2, $z_2 \in R$ is the state variable of the system and $a_2 = 36$, $b_2 = 3$, $c_2 = 20$ is the typical parameter of Lü system. $u_c = [u_{c1}\, u_{c2}\, u_{c3}]^T$ is the required synchronization controller, through which the synchronization of two chaotic attractors with very different structures can be realized.

2.3. Design of Synchronization Controller

Subtracting the corresponding terms in the mathematical model Eq. (1) of the driving Chen system from the terms in the mathematical model Eq. (2) of the response Lü system, the error system model is shown as Eq. (3) below:

$$\begin{cases} \dot{e}_1 = \dot{x}_2 - \dot{x}_1 \\ \quad = a_2(y_2 - x_2) - \mathrm{a}_1(y_1 - x_1) + u_{\mathrm{c}1} \\ \dot{e}_2 = \dot{y}_2 - \dot{y}_1 \\ \quad = -x_2 z_2 + \mathrm{c}_2 y_2 \\ \quad -[(c_1 - \mathrm{a}_1)x_1 - x_1 z_1 + c_1 y_1] + u_{\mathrm{c}2} \\ \dot{e}_3 = \dot{z}_2 - \dot{z}_1 \\ \quad = x_2 y_2 - \mathrm{b}_2 z_2 \\ \quad -(x_1 y_1 - \mathrm{b}_1 z_1) + u_{\mathrm{c}3} \end{cases} \tag{3}$$

Where $e = [e_1 \quad e_2 \quad e_3]^T = [x_2 - x_1 \quad y_2 - y_1 \quad z_2 - z_1]^T$, e_1, e_2, e_3 is the state variable of the error system. The synchronization controller is designed as:

$$\begin{cases} u_{c1} = (a_1 - a_2)(y_1 - x_1) - k_1 e_1 \\ u_{c2} = x_2 z_2 - x_1 z_1 + (c_1 - c_2)y_1 \\ \quad +(c_1 - a_1)x_2 - k_2 e_2 \\ u_{c3} = -x_2 y_2 + x_1 y_1 + (b_2 - b_1)z_1 \\ \quad -k_3 e_3 \end{cases} \tag{4}$$

Where k_1, k_2, $k_3 \geq 0$, Eq. (4) is substituted into Eq. (3), and the mathematical model of the error system is obtained as shown in Eq. (5).

$$\begin{cases} \dot{e}_1 = \dot{x}_2 - \dot{x}_1 \\ \quad = a_2(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 = \dot{y}_2 - \dot{y}_1 \\ \quad = c_2 e_2 + (c_1 - a_1)e_1 - k_2 e_2 \\ \dot{e}_3 = \dot{z}_2 - \dot{z}_1 = -b_2 e_3 - k_3 e_3 \end{cases} \tag{5}$$

The error system is then written in the form of $\dot{e} = f(e,t)$. If the equilibrium state of error system at the origin is uniformly asymptotic and stable in a large range, it indicates that the chaos synchronization of different structure has been realized between Chen system and Lü system.

The value of the synchronization controller parameter $k = [k_1 \quad k_2 \quad k_3]^T$ is determined by the Lyapunov second method.

According to Lyapunov's second method, firstly take the positive definite Lyapunov function $V(\tilde{e}) = 1/2[(a_1 - c_1)e_1^2/a_2 + e_2^2 + e_3^2]$, and then take its derivative:

$$\begin{aligned} \mathrm{V}(\dot{\tilde{e}}) &= \frac{a_1 - c_1}{a_2} e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3 \\ &= (a_1 - c_1)e_1 e_2 - (a_1 - c_1)e_1^2 \\ &\quad - \frac{k_1(a_1 - c_1)}{a_2} e_1^2 + c_2 e_2^2 \\ &\quad +(c_1 - a_1)e_1 e_2 - k_2 e_2^2 \\ &\quad -b_2 e_3^2 - k_3 e_3^2 \\ &= -\left[\frac{k_1(a_1 - c_1)}{a_2} + (a_1 - c_1)\right] e_1^2 \\ &\quad -(k_2 - c_2)e_2^2 - (k_3 + b_2)e_3^2 \end{aligned} \tag{6}$$

According to Lyapunov's second method, if $V(\tilde{e})$ is positively definite and $\dot{V}(\tilde{e})$ is negatively definite, then the error system is uniformly asymptotically stable in a large range at the origin. At this time, the variable coefficients of the error system should satisfy:

$$\begin{cases} -[\frac{k_1(a_1 - c_1)}{a_2} + (a_1 - c_1)] < 0 \\ -(k_2 - c_2) < 0 \\ -(k_3 + b_2) < 0 \end{cases} \tag{7}$$

Solved:

$$\begin{cases} k_1 > -a_2 = -36 \\ k_2 > c_2 = 20 \\ k_3 > -b_2 = -3 \end{cases} \tag{8}$$

If $(k_1, k_2, k_3) = (0,25,0)$, the synchronous controller u_c is as follows:

$$\begin{cases} u_{\mathrm{cl}} = -(y_1 - x_1) \\ u_{c2} = x_2 z_2 - x_1 z_1 + 33y_1 \\ \quad -7x_2 - 25y_2 \\ u_{c3} = -x_2 y_2 + x_1 y_1 \end{cases} \tag{9}$$

So far the synchronization controller has been designed.

The numerical simulation diagram of the chaotic synchronization controller of Chen system and Lü system with different structures built in Simulink is shown in Fig. 1.

After setting up the simulation environment, click the run button, and the corresponding state error curve is shown in Fig. 2.

The corresponding state synchronization curve is shown in Fig. 3.

Through careful observation of the images, it can be seen that the controller makes the corresponding state error curve gradually converge to zero with the passage of time, and the corresponding state synchronization curve follows the running trajectory of Chen driving system. This is enough to indicate that all the above derivations of all equations on the different structure chaos synchronization of Chen system and Lü system are correct, and Chen system and Lü system can achieve different structure chaos synchronization.

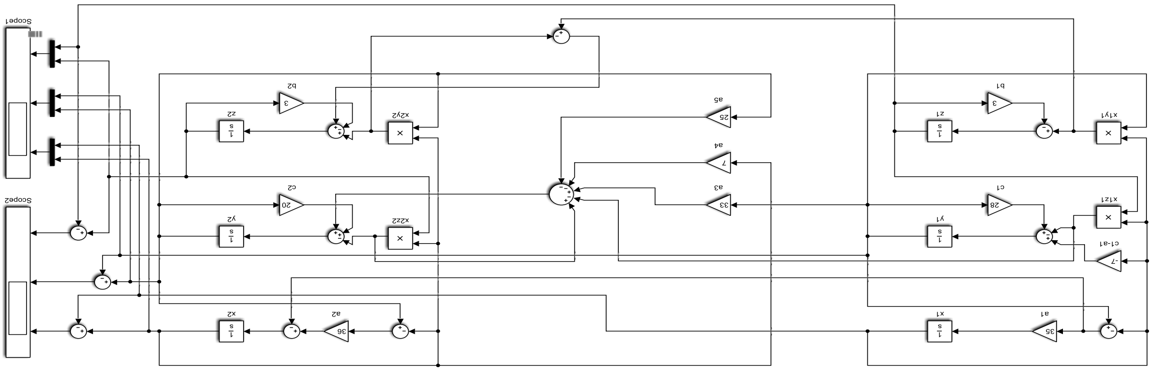

Fig. 1 Numerical simulation of chaos synchronization of Chen system and Lü system with different structure

Fig. 2 State error curve: (a) e_1; (b) e_2; (c) e_3

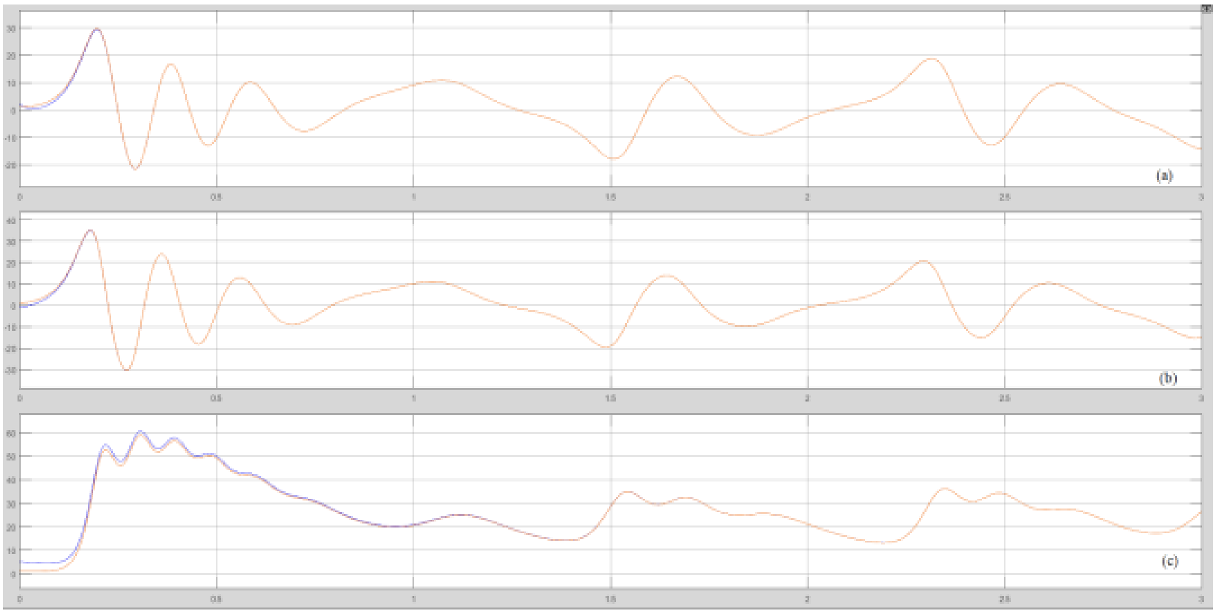

Fig. 3 State synchronization curve: （a）x1-x2；（b）y1-y2；（c）z1-z2

2.4. Design and construction of an analog circuit model for heterostructure chaotic synchronous control

The circuit model of the synchronous controller is designed and constructed, and the circuit of the synchronous controller is designed by the method of improved modular circuit design.

The state expression of the control circuit is shown in Eq. (10).

$$\begin{aligned}\tau_0 u_{c1} &= \tau_0[-(y_1 - x_1)]\\ &= -\tau_0 y_1 - \tau_0(-x_1)\\ &= -\frac{1}{\mathrm{R}_{14}\mathrm{C}_4} y_1 - \frac{1}{\mathrm{R}_{15}\mathrm{C}_4}(-x_1)\\ \tau_0 u_{c2} &= \tau_0[10x_2z_2 - 10x_1z_1\\ &\quad +33y_1 - 7x_2 - 25y_2]\\ &= -10\tau_0(-x_2z_2) - 10\tau_0 x_1 z_1\\ &\quad -33\tau_0(-y_1) - 7\tau_0 x_2 - 25\tau_0 y_2\\ &= -\frac{1}{10\mathrm{R}_{16}C_5}(-x_2z_2)\\ &\quad -\frac{1}{10\mathrm{R}_{17}C_5}x_1z_1 - \frac{1}{\mathrm{R}_{13}C_5}(-y_1)\\ &\quad -\frac{1}{\mathrm{R}_{19}C_5}x_2 - \frac{1}{\mathrm{R}_{20}C_5}y_2\\ \tau_0 u_{c_3} &= \tau_0(-10x_2y_2 + 10x_1y_1)\\ &= -10\tau_0 x_2 y_2 - 10\tau_0(-x_1y_1)\\ &= -\frac{1}{10\mathrm{R}_{21}\mathrm{C}_6}x_2y_2\\ &\quad -\frac{1}{10\mathrm{R}_{22}\mathrm{C}_6}(-x_1y_1)\end{aligned} \tag{10}$$

Substitute $\tau_0 = 100$ into Eq. (10) and take $C_4 = C_5 = C_6 = 10\mathrm{nF}$, and the calculation is as follows:

$$\begin{aligned}\mathrm{R}_{14} &= \mathrm{R}_{15} = \frac{1}{\tau_0 C_4} = \frac{1}{100 \times 10 \times 10^{-9}}\\ &= 1\mathrm{M}\Omega\\ \mathrm{R}_{18} &= \frac{1}{33\tau_0 C_5} = \frac{1}{33 \times 100 \times 10 \times 10^{-9}}\\ &= 30.3\mathrm{k}\Omega\\ \mathrm{R}_{19} &= \frac{1}{7\tau_0 C_5} = \frac{1}{7 \times 100 \times 10 \times 10^{-9}}\\ &= 142.8\mathrm{k}\Omega\\ \mathrm{R}_{20} &= \frac{1}{25\tau_0 C_5} = \frac{1}{25 \times 100 \times 10 \times 10^{-9}}\\ &= 40\mathrm{k}\Omega\\ \mathrm{R}_{16} &= R_{17} = R_{21} = R_{22} = \frac{1}{100\tau_0 C_5}\\ &= \frac{1}{100 \times 100 \times 10 \times 10^{-9}} = 10\mathrm{k}\Omega\end{aligned} \tag{11}$$

The analog circuit for the synchronous control of Chen system and Lü system is shown in Fig. 4.

Build the analog circuit shown in the above figure in Multisim software. Wait for the oscilloscope to display the waveform, observe the corresponding waveform, and obtain the corresponding synchronization curve shown in Fig. 5 and the corresponding error curve shown in Fig. 6.

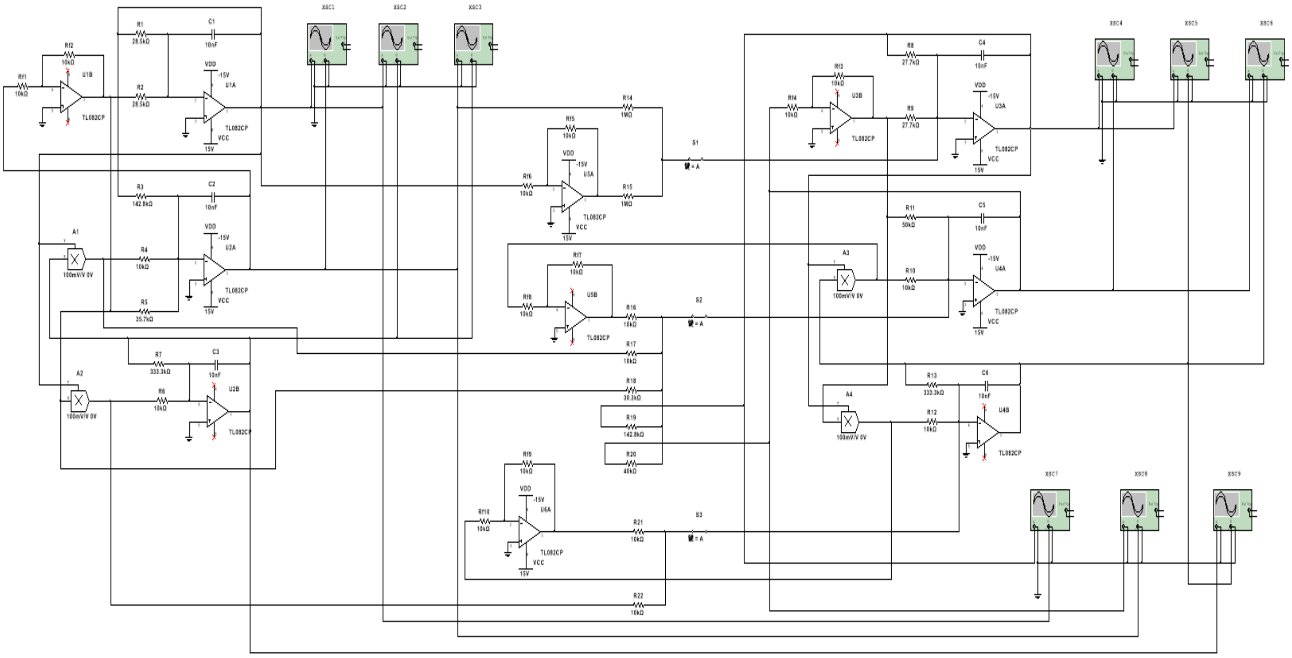

Fig. 4 Analog Circuit for Synchronous Control of Chen System and Lü System

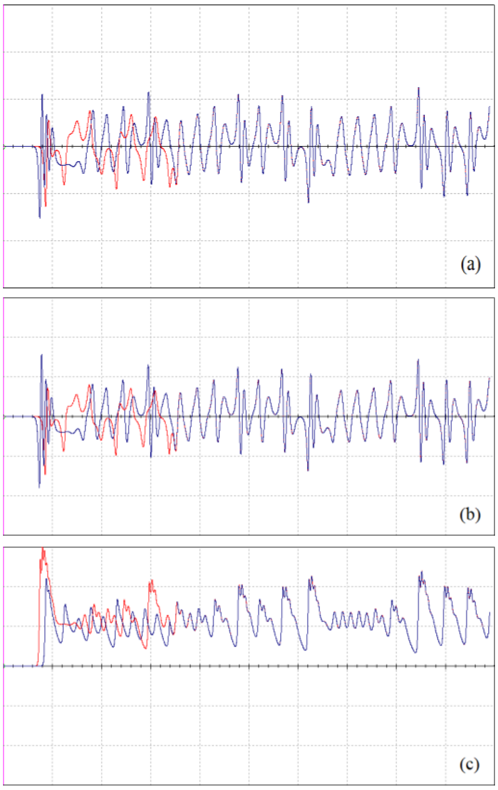

Fig. 5 Synchronization state curve:（a）x_1-x_2；（b）y_1-y_2；（c）z_1-z_2

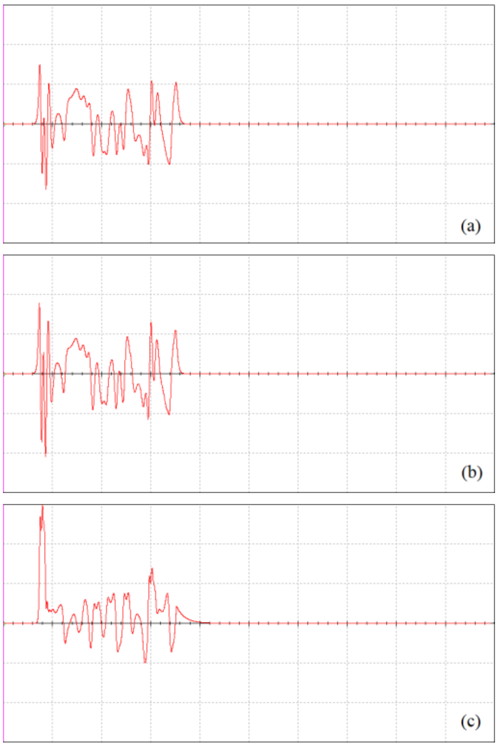

Fig. 6 Error curve:（a）e_1；（b）e_2；（c）e_3

3. Conclusion

In this paper, the driving Chen system and the responding Lü system are first mathematically modeled separately, and then the synchronous controllers are designed to achieve the purpose of removing all nonlinear terms. The parameters of the controller are solved according to Lyapunov's second method. The controller allows the numerical simulation of the system in MATLAB to show that the corresponding state error curves gradually converge to zero over time and the corresponding state synchronization curves follow the trajectory of the Chen drive system.

The simulation circuit model of the Chen chaotic system and the Lü chaotic system with heterostructure chaotic synchronization is designed and constructed with the help of Multisim software using a modified modular approach. The installation of separate switches for the structural compensator and the controller to ensure individual control is a critical step in the design of the system, and the observation of the oscilloscope showing the corresponding error curves reveals that the curves quickly converge to 0 after the switches are closed, and the whole experiment shows that the designed synchronization controllers perform the control role. The heterostructured chaotic synchronization control of Chen system with Lü system and its circuit design are implemented.

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Authors Introduction

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