Parallel acoustic analysis based on the domain decomposition method with higher-order element

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Abstract

Large-scale analyses, using numerical models with over 10 trillion elements, are required for the analysis of a large space such as a concert hall with higher-frequency bands. Large spaces are often limited to low-frequency analysis. In this study, the number of elements is reduced by wave acoustic analysis using higher-order elements. Based on the results using higher-order elements, it is shown that it is possible to analyze a real environment model such as a live music club and a concert hall.

Keywords: large-scale simulation, acoustical sound field, higher-order element

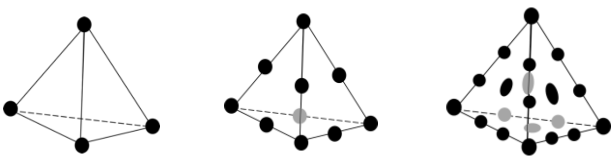
1. Introduction

Estimation of the sound field is important for improving the quality of acoustic spaces such as concert halls and live music clubs [1]. Scale model experiments and the computer simulations are used for estimation of the sound field. Scale model experiments are used in many fields [2], however, creating models requires a lot of time and is expensive. On the other hand, computer simulation creates a model and sound in a virtual space. Therefore, in the computer simulation, it is easy to change the conditions of the analysis as well as to change materials and shapes. However, a large-scale analysis, using a numerical model with over 10 trillion elements, is required for the analysis of a large space with high frequency. n a large-scale finite element steady-state acoustic analysis, the iterative domain decom-position method [3] is proposed and applied as a parallelization technique. It is shown that the large-scale analysis becomes possible by the iterative domain decomposition method [4]. In this study, higher-order elements are introduced into a parallel finite ele-ment steady-state acoustic analysis method and greatly reduce the number of elements. Higher-order elements are not actively used because the matrix expands. In particular, there are some examples of higher-order elements, e.g., higher than the 3rd order ele-ment [5], however, there are few examples of acoustic analysis. As far as we know, there is no example showing the superiority of reducing the number of necessary elements by applying higher-order elements, especially in large-scale acoustic calculations using the domain decomposition method.

2. Finite element

2.1. Higher-order elements

To reduce the number of elements, 2nd and 3rd order elements are introduced. With higher-order elements, the number of nodes increases because the nodes are placed on the sides and faces of the element. Fig. 1 shows the nodal arrangement of the tetrahedras with 1st, 2nd, and 3rd order elements. Table 1 shows the shape function of each element [6].



(a) 1st order elemenet (b) 2nd order (c) 3rd order

Fig. 1. Nodal arrangement of each element

Table 1. Shape function

1st order elm.	$N_i = L_i$	*i=0,1,…,3*
2nd order elm.	$N_i = L_i(2L_i - 1)$	*i=0, 1, …, 3*
	$N_i = 4L_jL_k$	*i=4,…,9 j, k=0,…,3*
3rd order elm.	$N_i = \frac{1}{2}(3L_i - 1)(3L_i - 2)L_i$	*i=0,…,3*
	$N_i = \frac{9}{2}(3L_j - 1)L_jL_k$	*i=4,…,15 j, k=0,…,3*
	$N_i = 27L_jL_kL_l$	*i=16,…,19*

L : Volume coordinate variable
N : Shape function
i : Number of nodes

2.2. Helmholtz equation

In the 3-dimensional sound field, the wave equation for velocity potential is expressed by the following equation:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}=q \tag{1}$$

where ϕ is the velocity potential, c is the speed of sound, and q is the distribution function.

To consider the steady-state, the velocity potential is expressed by Eq. (2). Using Eq. (1) and Eq. (2), the Helmholtz equation is obtained:

$$\phi=\Phi e^{-j\omega t} \tag{2}$$

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}+\frac{\omega^2}{c^2}\Phi=q \tag{3}$$

where ω is the angular frequency.

The velocity potential of Eq. (3) and calculate the sound pressure using the following equation are obtained:

$$p=j\omega\rho\Phi \tag{4}$$

where j is the imaginary number, and ρ is the medium density.

2.3. Finite element formulation

To derive a weak form, the Galerkin method is applied to Eq. (3). By applying the finite element approximation and discretization, the following equation is obtained:

$$-k^2[M]\{\Phi\}+j\omega\rho[C]\{\Phi\}+[K]\{\Phi\}=\{q\} \tag{5}$$

where [・] is a matrix, {・} is a vector. In Eq. (5), the matrices $[M]$, $[C]$ and $[K]$ can be calculated using Eq. (6.1,2,3):

$$[M]_e=\iiint_{\Omega_e}\{N\}\{N\}^T d\Omega_e \tag{6.1}$$

$$[K]_e=\iiint_{\Omega_e}\nabla\{N\}\nabla\{N\}^T d\Omega_e \tag{6.2}$$

$$[C]_e=-\frac{1}{Z_n}\iint_{\Gamma_e}\{N\}\{N\}^T d\Gamma_e \tag{6.3}$$

where N is the shape function, $[M]_e$ and $[K]_e$ are the volume integrals, and $[C]_e$ is a surface integral to the sound-absorbing boundary surface. Z_n is a specific acoustic impedance.

2.4. Calculaion of element matrix

Let us consider the calculation $\{N\}\{N\}^T$ in Eqs. (6.1) and (6.3). This calculation uses an integration formula of the 3 or 2-dimensional finite element method. These integration formulas are shown in the following equations [7].

$$\iiint_{\Omega_e} L_1^k L_2^l L_3^m L_4^n dxdydz = 6V_e\frac{k!\,l!\,m!\,n!}{(k+l+m+n+3)!} \tag{7}$$

$$\iint_{\Gamma_e} L_1^k L_2^l L_3^m dxdy = 2A_e\frac{k!\,l!\,m!}{(k+l+m+2)!} \tag{8}$$

On the other hand, the calculation of $\nabla\{N\}\nabla\{N\}^T$ in Eq. (6.2) shown in the following equation.

$$\begin{aligned}\frac{\partial f}{\partial x}&=\frac{1}{6V_e}\left(b_1\frac{\partial f}{\partial L_1}+b_2\frac{\partial f}{\partial L_2}+b_3\frac{\partial f}{\partial L_3}+b_4\frac{\partial f}{\partial L_4}\right)\\ \frac{\partial f}{\partial y}&=\frac{1}{6V_e}\left(c_1\frac{\partial f}{\partial L_1}+c_2\frac{\partial f}{\partial L_2}+c_3\frac{\partial f}{\partial L_3}+c_4\frac{\partial f}{\partial L_4}\right)\\ \frac{\partial f}{\partial z}&=\frac{1}{6V_e}\left(d_1\frac{\partial f}{\partial L_1}+d_2\frac{\partial f}{\partial L_2}+d_3\frac{\partial f}{\partial L_3}+d_4\frac{\partial f}{\partial L_4}\right)\end{aligned} \tag{9}$$

We convert the finite element equations of (5) to a matrix form as follows:

$$Ku=f. \tag{10}$$

2.5. Hierarchical domain decomposition method

The original analysis domain is first divided into parts, which are further decomposed into smaller domains called subdomains. This is called the hierarchical domain decomposition method (HDDM) [8], [9].

3. Numerical experiment

3.1. Verification by benchmark problem

Fig.2 shows that test model for simulation. The model is AHLV100 that is known as a reference model in code_Aster. This is also described in the ADVENTURE_sound manual as a sample. This simulation was done to confirm the use of transient analysis in ADVENTURE_Sound. To evaluate the accuracy of the acoustic analysis code, an acoustic benchmark problem is used. The analysis uses the test model AHLV100 of Code_Aster [10], which is known as a representative benchmark problem among acoustic problems (Fig. 4).

This model is an acoustic tube that has a length of 1 [m], a height of 0.1 [m] and a width of 0.2 [m]. It has a vibration boundary at the left end and a sound absorption boundary at the right end. The other faces are given rigid boundaries. The specific acoustic impedance Z_n=445.9 [kg/m^3 · s] is given as a sound absorption boundary condition.

The accuracy is calculated from the average error of four points on the sound absorption boundary. The formula for calculating the theoretical solution is as follows:

$$p(x,y,z)=\rho c V_n exp(-ikx) \tag{11}$$

where ρ is the medium, c is the speed of sound, and V_n is the particle velocity.

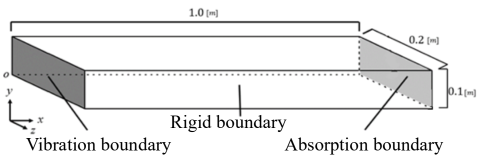


Fig. 2. Acoustic benchmark problem AHLV100

3.2. Performance evaluation

The performance of higher-order elements is evaluated based on the number of elements, accuracy rate, and memory usage. The performance evaluation conditions are: 4.0 [kHz] for frequency, air for medium, and 343 [m/s] for sound velocity. The analysis uses a PC cluster composed of 5 PCs (40 cores) equipped with a multicore CPU (Intel Core i7-9700K, 3.6GHz 8core, 32GB of memory).

Numerical results for the error rate, plotted against the number of elements, are shown in Fig. 3. Results for the memory, plotted against the number of elements, are shown in Fig. 4.

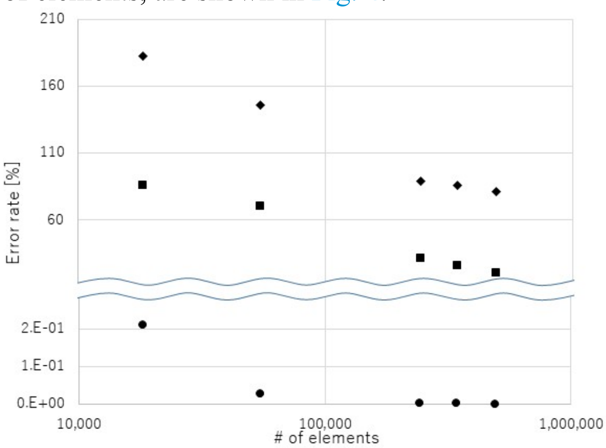


Fig. 3. Error rate plotted against the number of elements

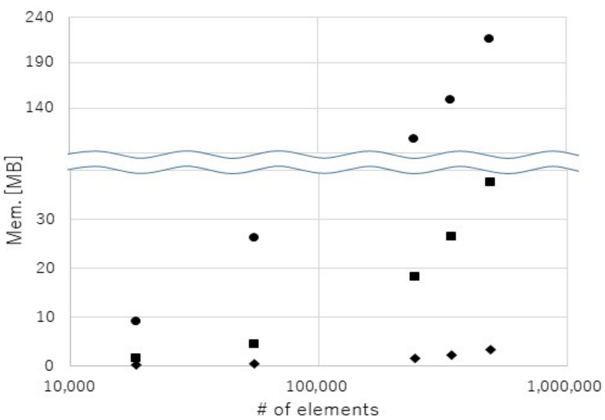
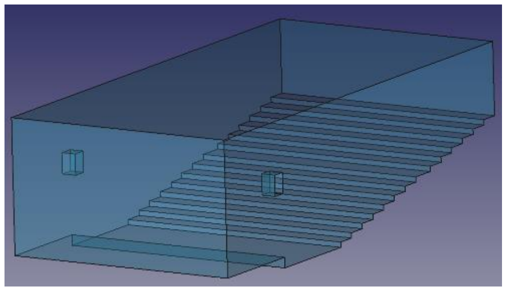


Fig. 4. Memory usage plotted against the number of elements

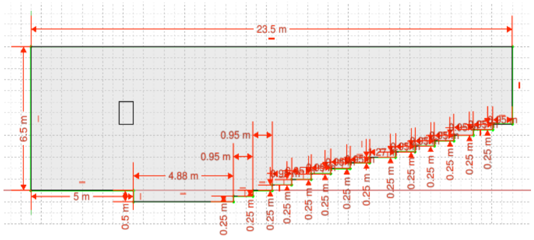
The accuracy increases dramatically when a higher-order element is applied. In particular, the accuracy changes more rapidly as the order increases.

3.3. Analyses using real environment models

To confirm the effectiveness of higher-order elements, the sound field of the real environment model is analyzed. The model used for analysis is concert hall model that is shown in Fig. 5.



(a) Concert hall 3D CAD model



(b) Concert hall model dimensions

Fig. 5. Concert hall model

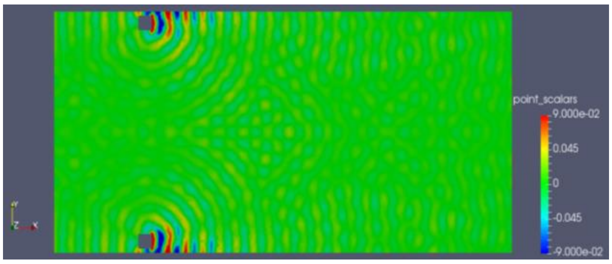
The concert hall model is constructed based on real spaces [11].

The concert hall model is 11.5 [m] wide, 7 [m] high, and 23.5 [m] deep. The sound source is set as a pair of speakers at either end of the stage. Oakwood flooring, a wooden stage, and a glass wool wall at the back of the hall are used as sound-absorbing boundary conditions. The sound field is analyzed by applying a 400 [Hz] sound to these models.

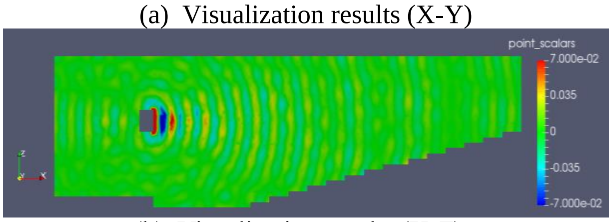
The results of these analyses are shown in Table 2. The visualization results are shown in Fig. 6, and the convergence histories of the iterative method (COCG) are shown in Fig. 7. The concert hall analysis using the 1st and the 2nd order elements are excluded from the evaluation because the number of elements exceeds 100 million. The data I/O library currently applied is 32 bit. This library cannot use data sizes that exceed 100 million elements. A 64-bit I/O library is currently under development.

Table 2. Numerical results of the concert hall model

Element type	Number of elements	Number of nodes	Elapsed Time [sec]	Memory requirement [MB/core]
1st order	823,285,162			
2nd order	102,910,645			
3rd order	878,624	4,051,235	53.97	391.97



(a) Visualization results (X-Y)



(b) Visualization results (X-Z)

Fig. 6. Concert hall model

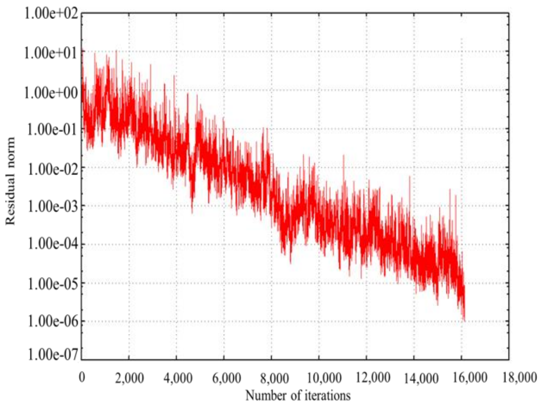


Fig. 7. Converegence history of the COCG

4. Conclusion

This paper described a large-scale acoustic analysis method using a domain decomposition method and the introduction of higher-order elements. The performance of proposed method was evaluated with higher-order elements using of AHLV100. The error rate, number of elements, and memory usage were the considered as evaluation criterions. It was shown that the calculation efficiency improved in higher-order elements. In particular, in the 3rd order element, the calculation efficiency was vastly improved. Furthermore, the accuracy of higher-order elements was verified. The 2nd order element and the 3rd order element were compared in terms of the number of elements and calculation time. Additionally, real environment model was analyzed, namely, a small concert hall model. It was shown that the real environment models could be successfully analyzed by using higher-order elements.

Acknowledgements

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