An Analysis of Translational Motion for a Mobile Robot with Line-Symmetric Rollers Arrangement

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Abstract

In fields such as logistics, robots are required to have efficient mobility. There are various types of rollers used in mobile robots. Among them, omni-rollers have excellent omnidirectional mobility and are easy to control. In this study, mechanism kinematics has been proposed that assumes arbitrary changes in the roller arrangement position on a circular mechanism and the roller arrangement has been evaluated from the viewpoint of speed efficiency. Furthermore, we aim to evaluate the mobility of mobile robots by focusing on their translational components. Moreover, we have examined the behavior of the area of the area generated by the end point of the robot velocity vector and evaluated the velocity efficiency.

Keywords: Omni-roller, Translational motion, Motion analysis of mobile robot

1. Introduction

In recent years, industries such as logistics require efficient mobile robots for object transport. Development of such mobile robot vehicles has attracted great attention among the research community. The degrees of freedom for omnidirectional movement (if holonomic properties are provided) is three, expressed as the sum of a translational component of 2 degrees of freedom and a rotational component of 1 degree of freedom. The holonomic movement mechanism is easy to control and has excellent maneuverability in all directions due to the independent drive characteristics of the wheels.

In particular, there are many moving mechanisms equipped with three omni rollers, and the kinematics for these mechanisms have been derived [1]. The three-roller arrangement of this mechanism has a basic structure of an equilateral triangle.

This basic arrangement is also used in the soccer robots of RoboCup Meddle-sized-reague, such as RV-infinity [2], Musashi 150 [3], and NuBot [4]. Thus, a regular polygon with the highest degree of symmetry among triangles was used, and no theoretical research had been conducted. However, recently, researchers have started focusing on the kinetic energy of drive rollers for moving mechanisms [5] and spherical conveyance [6].

In a movement mechanism using a sphere as wheels (or a mechanism using a sphere as a conveyance object), the optimal angle of the sphere rotation axis with respect to the movement direction [7] and the placement location of the two drive rollers that drive the sphere should be determined. The optimal location has been identified to be on the equator [8]. In a mechanism using rollers as wheels, a transformation matrix is defined that associates the input roller speed with the output robot speed (translation/rotation), and the "Image volume" and "Orthographic area" are used as an evaluation function for movement efficiency. The roller arrangement was evaluated in a previous study [9].

The present study focuses on sectional area by considering the kinetic energy of a mobile robot in the line-symmetric roller's arrangement which has one-dimensional of freedom. Focusing only on the robot's translational motion, we derive a relational expression between velocity efficiency and roller arrangement, and subsequently analyze the behavior of the end point region area of the robot velocity vector.

The rest of this study is as follows: Chapter 2 discusses the kinematics of mobile robots. Chapter 3 derive sectional area function. Chapter 4 conducted the simulation. Finally, we present the summary and future tasks.

2. Kinematics of Transfer Mechanism in case of isosceles triangle three rollers arrangement

In a previous study, we have defined "Image volume" and "Orthographic projection area" as evaluation functions analysis roller contact location ([9]).

Section 2.1 introduces only on the robot translational motion and analysis robot motion.

Section 2.2 discusses kinematics for a mechanism that adapts three omni-rollers in line symmetry roller's arrangement.

2.1. *Liner Transformation mapping for correspondence of roller speed and robot speed*

As shown in Figure 1(a), the mobile robot that has a common radius of all omni-wheels adapted the *i-th* rollers ($i = 1,2,3$) contact point $\boldsymbol{P_i}$ on a circle. $X - Y$ is the global coordinate system (origin $\boldsymbol{O}$). The robot translation speed is $\boldsymbol{V} = [V_x, V_y]^{\boldsymbol{T}}$ and φ denotes robot direction. Robot rational speed $L\dot{\phi}$ ($\dot{\phi}$: robot angular velocity) roller peripheral speed v_i are decomposed as translation and rotational components. Contact point $\boldsymbol{P_i}$ are adapted angle θ_i on the circle that has a radius L. Thus, the correspondence of $[v_1, v_2, v_3]$ and $[V_x, V_y, \dot{\phi}L]$ is represented in Figure 1(a).

As shown in Figure 1(b), linear transformation mapping f_A : $[V_x, V_y, \dot{\phi}L]^{\boldsymbol{T}} \rightarrow [v_1, v_2, v_3]^{\boldsymbol{T}}$ and $f_A(W) = \mathrm{Image} f_A \in \mathbf{R}^3$ is a parallelepiped domain from cubic domain W.

In this study, we represented a sectional area of $f_{A^{-1}}(W)$ for a horizontal plane as follow:

$$f_{A^{-1}}(W) \cap \{V_x V_y - \text{plane}\} = \{(V_x, V_y, 0) \mid |v_1|, |v_2|, |v_3| \leq 1\} \quad (1)$$

Where

$$f_{A^{-1}}(W) = \{(V_x, V_y, L\dot{\phi}) \mid |v_1|, |v_2|, |v_3| \leq 1\} \quad (2)$$

$$W = \{(v_1, v_2, v_3) \mid |v_1|, |v_2|, |v_3| \leq 1\} \quad (3)$$

2.2 *Kinematics for isosceles triangle three rollers arrangement*

In our previous work [9], we derived kinematics for arbitrary roller arrangement position as $(\theta_1, \theta_2, \theta_3)$. In this study, we restrain an isosceles triangular roller arrangement (line symmetry roller's arrangement) in Eq. (6) of the previous study [9]

Substituting $\theta_1 = \theta$, $\theta_2 = 360° - \theta$ and $\theta_3 = 0°$ with $(\theta_1, \theta_2, \theta_3)$ in Eq. (6) of [9].

Inverse kinematics ($[v_1, v_2, v_3]^{\boldsymbol{T}}$ is determined from $[V_x, V_y, L\dot{\phi}]^{\boldsymbol{T}}$) is represented as follows:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -\sin\theta & \cos\theta & 1 \\ \sin\theta & \cos\theta & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ L\dot{\phi} \end{bmatrix} \quad (4)$$

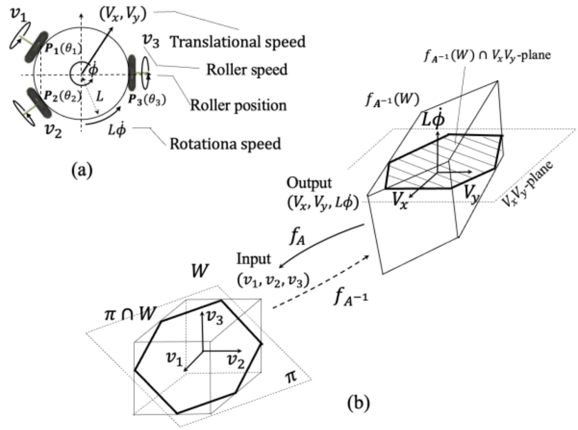

Figure 1 Correspondence between three omni-rollers speed $[v_1, v_2, v_3]^{\boldsymbol{T}}$ and $[V_x, V_y, \dot{\phi}L]^{\boldsymbol{T}}$ (robot mobile speed (V_x, V_y) rotational speed $\dot{\phi}$). (a) mobile robot and (b) translation mapping.

Forward kinematics ($[V_x, V_y, L\dot{\phi}]^{\boldsymbol{T}}$ is determined from $[v_1, v_2, v_3]^{\boldsymbol{T}}$) is represented as follows:

$$\begin{bmatrix} V_x \\ V_y \\ L\dot{\phi} \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{2\sin\theta} & \dfrac{1}{2\sin\theta} & 0 \\ -\dfrac{1}{2-2\cos\theta} & -\dfrac{1}{2-2\cos\theta} & \dfrac{1}{1-\cos\theta} \\ \dfrac{1}{2-2\cos\theta} & \dfrac{1}{2-2\cos\theta} & -\dfrac{\cos\theta}{1-\cos\theta} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad (5)$$

3. Derive sectional area function

In this section, we calculate area using Eq. (1). Section 3.1 shows analysis of roller's speed space, while Section 3.2 shows analysis of robot speed space.

3.1. *Analysis of roller's speed space*

(A) Setup for Cubic domain

As shown in Figure 2(a), the cubic domain is composed of the following eight apexes: $\boldsymbol{B_1} = (1, 1, 1)$, $\boldsymbol{B_2} = (-1, 1, 1)$, $\boldsymbol{B_3} = (-1, -1, 1)$, $\boldsymbol{B_4} = (1, -1, 1)$, $\boldsymbol{B_5} = (1, 1, -1)$, $\boldsymbol{B_6} = (-1, 1, -1)$, $\boldsymbol{B_7} = (-1, -1, -1)$, $\boldsymbol{B_8} = (1, -1, -1)$. Additionally, sectional domain denoted by π is symmetrical with respect to the origin $\boldsymbol{O}$. For the sectional shape to be hexagonal like Figure 2(a), six dots $\boldsymbol{P}(p, -1, 1)$, $\boldsymbol{Q}(-1, q, 1)$, $\boldsymbol{R}(-1, 1, r)$, $\boldsymbol{S}(s, 1, -1)$,

$\boldsymbol{T}\,(1, t, -1)$, $\boldsymbol{U}\,(1, -1, u)$ (real parameter : $-1 \leq p, q, r, s, t, u \leq 1$) should satisfy the following condition.

$$p = s, q = t, r = u \tag{6}$$

As the two faces, $\boldsymbol{B_1 B_2 B_3 B_4}$ and $\boldsymbol{B_5 B_6 B_7 B_8}$ (face to face), are parallel, side $\boldsymbol{PQ}$ and $\boldsymbol{TS}$ are parallel and have same length.

(B) Plane representation as roller speed existent set

Focusing on $L\dot{\phi}$ -component of Eq. (5), only transrational motion is equivalent to $\dot{\phi} = 0$.

$$v_1 \sin\theta + v_2 \sin\theta - 2\, v_3 \sin 2\theta = 0 \tag{7}$$

Eq. (8) represents a plane equation including origin by three-dimensional combinate $[v_1, v_2, v_3]^{\boldsymbol{T}}$.

$$\pi = \{(v_1, v_2, v_3) | v_1 \sin\theta + v_2 \sin\theta - 2\, v_3 \sin 2\theta = 0\} \tag{8}$$

(C) Normal vector of plane

Eq. (7) is equivalent to following expression.

$$\sin\theta \left\langle \begin{bmatrix} 1 \\ 1 \\ -2\cos\theta \end{bmatrix}, \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \right\rangle = 0 \tag{9}$$

Thus. $[1, 1, -2\cos\theta]^{\boldsymbol{T}}$ is normal vector of plane π. Also, $\sin\theta > 0$ and $-2\cos\theta > 0$ for all $90° \leq \theta < 180°$.

Figure 2(b) shows a section of the cubic domain with respect to rectangle ($\boldsymbol{B_1 B_3 B_7 B_5}$). It can be observed that normal vectors shift counterclockwise direction from v_3-axis (when $\theta = 90°$) to line $\boldsymbol{B_3 O}$ (when $\theta = 180°$). Thus. $\pi \cap W$ satisfies the following property.

Property

(i) Sectional shape is symmetrical with respect to origin

(a) $\theta_1 = 90°$ ⇔ Rhomb

(b) $90° < \theta_1 < 180°$ ⇔ Hexagon

(ii) When $\theta_1 = 120°$ it has minimal area.

3.2. *Analysis of robot speed space*

Using Eq. (5), image of $\pi \cap W$ by transformation mapping $f_{A^{-1}}$ can be represented as follows:

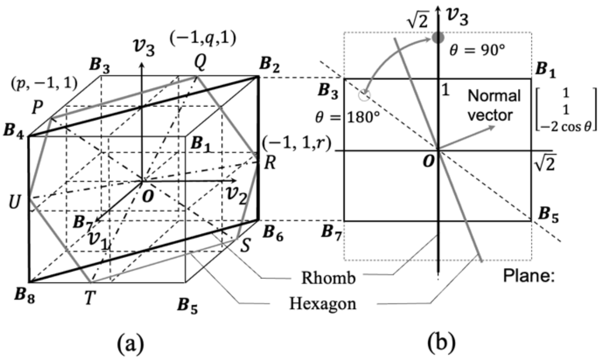

Figure 2 Sectional shape of roller's speed space $\pi \cap W$ in case of an isosceles triangle roller arrangement. (a) Isometric view. (b) Sectional view.

$$f_{A^{-1}}(W) \cap \{V_x V_y - \text{plane}\} = \tag{10}$$

$$\{(V_x, V_y) | \, v_1 \boldsymbol{\acute{A}_1} + v_2 \boldsymbol{\acute{A}_2} + v_3 \boldsymbol{\acute{A}_3}, |v_1|, |v_2|, |v_3| \leq 1\}$$

where

$$\boldsymbol{\acute{A}_1} = \begin{bmatrix} -\frac{1}{2\sin\theta} \\ -\frac{1}{2-2\cos\theta} \\ \frac{1}{2-2\cos\theta} \end{bmatrix}, \boldsymbol{\acute{A}_2} = \begin{bmatrix} \frac{1}{2\sin\theta} \\ -\frac{1}{2-2\cos\theta} \\ \frac{1}{2-2\cos\theta} \end{bmatrix}, \boldsymbol{\acute{A}_3} = \begin{bmatrix} 0 \\ \frac{1}{1-\cos\theta} \\ -\frac{1}{1-\cos\theta} \end{bmatrix} \tag{11}$$

From Eq. (6) and property of transformation mapping $f_{A^{-1}}$, $f_{A^{-1}}(\pi \cap W)$ is decomposed as three parts.

$$f_{A^{-1}}(\pi \cap W) = 2 f_{A^{-1}}(\triangle\, \boldsymbol{OPQ}) \tag{12}$$

$$+ 2 f_{A^{-1}}(\triangle\, \boldsymbol{OQR}) + 2 f_{A^{-1}}(\triangle\, \boldsymbol{ORP})$$

Where

$$\pi \cap W = 2(\triangle\, \boldsymbol{OPQ} + \triangle\, \boldsymbol{OQR} + \triangle\, \boldsymbol{ORP}) \tag{13}$$

Thus. Sectional area of $f_{A^{-1}}(W) \cap \{V_x V_y - \text{plane}\}$ is represented as following.

$$D_{Sec}(\theta) = \| f_{A^{-1}}(\boldsymbol{P}) \times f_{A^{-1}}(\boldsymbol{Q}) \|$$

$$+ \| f_{A^{-1}}(\boldsymbol{Q}) \times f_{A^{-1}}(\boldsymbol{R}) \| + \| f_{A^{-1}}(\boldsymbol{R}) \times -f_{A^{-1}}(\boldsymbol{P}) \| \tag{14}$$

Where $\boldsymbol{P}(p, -1, 1)$, $\boldsymbol{Q}\,(-1, q, 1)$, $\boldsymbol{R}(-1, 1, 0)$

$$p = \frac{\sin\theta - \sin 2\theta}{\sin\theta}, q = \frac{-\sin\theta - \sin 2\theta}{\sin\theta} \qquad (14)$$

4. Simulation

This section presents the simulation findings, including the evaluation values of "horizontal cross-sectional area: D_{Sec}"

Simulations were performed at the various symmetric roller arrangement (Eq. (14) in $90° \le \theta < 180°$).

As shown in Figure 3, $D_{Sec}(90°) = 4.05[m/s]^2$ and $D_{Sec}(180°) = \infty$ $[m/s]^2$. minimum value is $D_{Sec}(120°)$ =3.85$[m/s]^2$. Thus. The worst efficiency roller arrangement in only translational motion is the equilateral triangular shape.

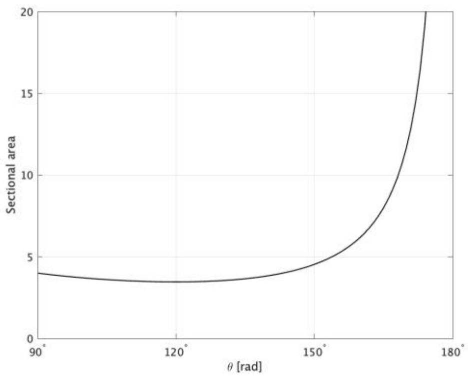

Figure 3 Behavior of Area of domain of end point of robot speed $D_{Sec}(\theta)$ $(90° \le \theta < 180°)$.

5. Conclusion

In this research, we focused on the translational motion of mobile robots. Furthermore, we derived the area of the region where the end point of the robot's velocity vector exists as an evaluation function, investigated its behavior, and determined the minimum value. We get fact that worst efficiency roller arrangement is the equilateral triangular shape.

This is a one-parameter problem because the roller arrangement is given as an isosceles triangle.

Acknowledgements

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