A Consideration on Amplification Function in BJT Evers-Moll Model and PTT (II)
---- H Parameters in the Small Signal Amplifier Circuit----

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Abstract

The notion of PTT (Photon Transport Transistor) has been proposed in 1989 as an optical coupling device of light emitting diode (LED) and light receiving diode (Photo Diode, PD), where the carrier of the base layer is light (Photon) only. In this paper, in order to deal with the various applications of PTT circuit more theoretically, based on the approximate VI characteristic equation of LED and PD, we newly derive the h parameters in PTT emitter common circuit, while referring to the h parameter of Bipolar Junction Transistor (BJT).

Keywords: PTT, LED, PD, positive feedback circuit, amplification function, equivalent circuit, h parameters

1. Introduction

The notion of Photon Transport Transistor (PTT) has been presented by B. J. Van Zeghbroeck et al. at IBM Research Laboratories presented in 1989 {1}. The PTT consists of the optical coupling between Light Emitting Diode (LED) or Laser Diode and Photo Diode (PD). Moreover in 1996, it has been theoretically shown that the PTT can be a very low-noise transistor-like device with an amplification function in a positive feedback circuit [2].

Later, an audio amplifier prototype using PTT positive feedback circuit with optical coupling between high-brightness LED and PD was developed, and it was reported that the PTT even exhibited functions similar to those of a thyristor ([3], [4], [5], [6], [7], [8]). Also, while referencing the conventional studies on BJT ([9], [10], [11], [12]), authors have analyzed the current amplification function of the PTT based on experimental results ([13], [14]).

In this paper, in order to deal with the various applications of PTT circuit more theoretically, based on the approximate VI characteristic equation of LED and PD, we derive the h parameters in PTT emitter common circuit, while referring to the h parameter of Bipolar Junction Transistor (BJT) ([9], [10], [12]).

Moreover, from the similarities between PTT and BJT Ebers-Moll Model (EMM), we discuss the essential factors of amplification function in PTT and BJT.

2. PTT and BJT EMM in Emitter Common Circuit
2.1. VI Characteristic of PTT and BJT EMM

The PTT with fixed bias in emitter common circuit is illustrated in Fig. 1. Generally, the emitter common circuits are used for small signal voltage amplification. The VI characteristic equations for each of the PD and LED that make up the PTT are shown in Eq. (1) and Eq. (2), respectively ([13], [14]). Each meaning of the symbols in those equations is shown in Table 1. As for the BJT and BJT EMM, those circuits without fixed bias in emitter common are illustrated in Fig. 2(a) and Fig. 2(b). respectively. And, the VI characteristic equations of two diode parts of the BJT EMM are shown in Eq. (3)

and Eq. (4), respectively, where α_0 of Eq. (3) means the current amplification constant.

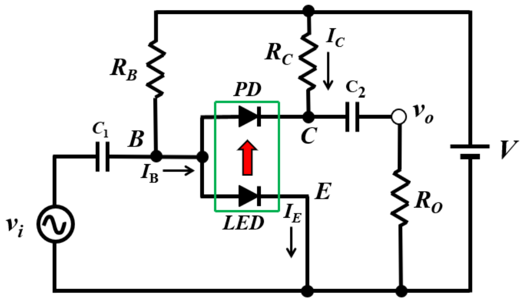


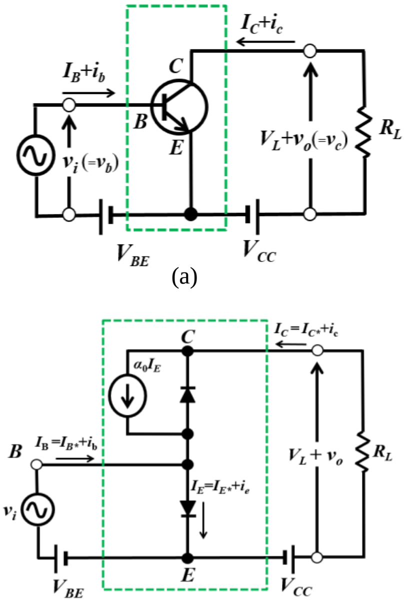
Fig.1. PTT emitter common circuit.

$$I_C=\gamma^* I_E - I_{C0}\left(\exp\left(c_{PD}V_{BC}\right)-1\right),\ c_{PD}=\frac{q}{m_2kT_{PD}} \quad (1)$$

$$I_E=I_{E0}\left(\exp\left(c_{LED}V_{BE}\right)-1\right),\ c_{LED}=\frac{q}{m_1kT_{LED}} \quad (2)$$

Table 1. The symbol list of PTT characteristic formula.

I_{E0}	Reverse saturation current in Emitter (LED)	I_{C0}	Reverse saturation current in Collector (PD)
m_1	Ideality factor (LED)	m_2	Ideality factor (PD)
q	Charge	k	Boltzmann's constant
T_{LED}	Absolute temperature of LED	T_{PD}	Absolute temperature of PD
V_{BE}	Base-Emitter voltage	V_{BC}	Base-Collector voltage
γ*	Proportional constant	---	----



(a)

(b)

Fig.2. (a) BJT emitter common circuit for small signal voltage amplification. (b) BJT EMM without fixed bias where α_0 means the current amplification constant, and $I_E = I_B+I_C$.

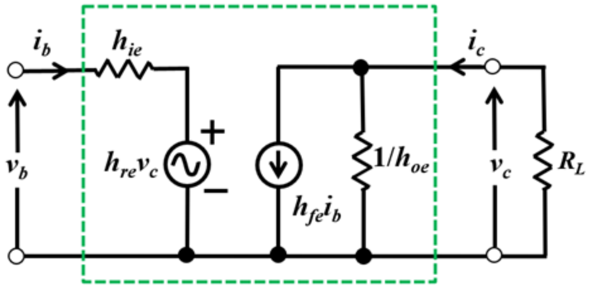
$$I_C=\alpha_0 I_E - I_{C0}\left(\exp\left(\frac{qV_{BC}}{kT}\right)-1\right) \quad (3)$$

$$I_E=I_{E0}\left(\exp\left(\frac{qV_{BE}}{kT}\right)-1\right) \quad (4)$$

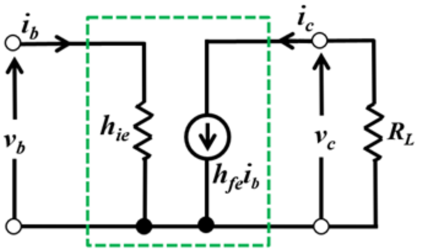
Comparing the pair of Eq. (3) and Eq. (4) with that of Eq. (1) and Eq. (2). we can expect that, the h parameters of PTT circuit using Eq. (3) and Eq. (4) will become the h parameters of BJT EMM by setting $\gamma^*=\alpha$, $m_1=1$, $m_2=1$. Then, we only have to find out the h parameters of PTT.

2.2. H Parameter for Small Signal Equivalent Equivalent Circuit

As is well known, for small signal amplification circuit of BJT in emitter common circuit, the small signal equivalent circuit with a set of h parameters (h_{ie}, h_{re}, h_{fe}, and h_{oe}) is used as shown in Fig. 3 ([9], [10], [12]).



(a) Precise equivalent circuit.



(b) Simplified equivalent circuit.

Fig. 3. Equivalent circuit of BJT emitter common with h parameters.

The equivalent relation equations in Fig. 3 are as follows.

$$\begin{bmatrix} v_b \\ i_c \end{bmatrix} = \begin{bmatrix} h_{ie}i_b + h_{re}v_c \\ h_{fe}i_b + h_{oe}v_c \end{bmatrix} = \begin{bmatrix} h_{ie}, & h_{re} \\ h_{fe}, & h_{oe} \end{bmatrix} \cdot \begin{bmatrix} i_b \\ v_c \end{bmatrix} \quad (5)$$

These h parameters are obtained from the total differential equation for V_{BE} and I_C, assuming the following functional relationship between f_1 and f_2.

Since we assume that $V_{BE} = f_1(I_B, V_{CE})$, $I_C = f_2(I_B, V_{CE})$,

$$\begin{cases} \Delta V_{BE}(\equiv v_b) = \left(\dfrac{\partial V_{BE}}{\partial I_B}\right)\Delta I_B + \left(\dfrac{\partial V_{BE}}{\partial V_{CE}}\right)\Delta V_{CE} = h_{ie}i_b + h_{re}v_c \\ \Delta I_C(\equiv i_c) = \left(\dfrac{\partial I_C}{\partial I_B}\right)\Delta I_B + \left(\dfrac{\partial I_C}{\partial V_{CE}}\right)\Delta V_{CE} = h_{fe}i_b + h_{oe}v_c \end{cases} \tag{6}$$

First of all, as for h_{re}, we find the following relationship between the other h parameter, h_{oe}.

$$\begin{aligned} h_{re} &= \left.\frac{\partial V_{BE}}{\partial V_{CE}}\right|_{I_B=const} = \left(\frac{\partial V_{BE}}{\partial I_C}\right)\left(\frac{\partial I_C}{\partial V_{CE}}\right) = \left(\frac{\partial V_{BE}}{\partial I_E}\right)\left(\frac{\partial I_E}{\partial I_C}\right)\left(\frac{\partial I_C}{\partial V_{CE}}\right) \\ &= \left(\frac{\partial I_E}{\partial V_{BE}}\right)^{-1}\cdot\left(\frac{\partial I_E}{\partial I_C}\right)\cdot h_{oe} \end{aligned} \tag{7}$$

Since

$$\begin{aligned} \frac{\partial I_E}{\partial V_{BE}} &= \frac{\partial I_{E0}\left(\exp\left(C_{LED}V_{BE}\right)-1\right)}{\partial V_{BE}} = I_{E0}\cdot C_{LED}\cdot\exp\left(C_{LED}V_{BE}\right) \\ &= C_{LED}\cdot(I_E + I_{E0}) \approx C_{LED}\cdot I_E \end{aligned} \tag{8}$$

and

$$\left(\frac{\partial I_E}{\partial I_C}\right) = \left(\frac{\partial(I_B + I_C)}{\partial I_C}\right) = 1 \tag{9}$$

then we have

$$h_{re} = \frac{h_{oe}}{C_{LED}(I_E + I_{E0})} \approx \frac{h_{oe}}{C_{LED}\cdot I_E} \tag{10}$$

Since $I_E = I_B + I_C$ in Eq. (1),

$$I_C = \gamma^*(I_B + I_C) - I_{C0}\left(\exp\left(c_{PD}V_{BC}\right)-1\right),\ \ c_{PD} = \frac{q}{m_2 kT_{PD}} \tag{11}$$

then

$$\begin{aligned} I_C &= \frac{\gamma^* I_B}{1-\gamma^*} - \frac{I_{C0}\left(\exp\left(c_{PD}V_{BC}\right)-1\right)}{1-\gamma^*} \\ &= \frac{\gamma^* I_B}{1-\gamma^*} - \frac{I_{C0}\left(\exp\left(-c_{PD}\left(V_{CE}-V_{BE}\right)\right)-1\right)}{1-\gamma^*} \end{aligned} \tag{12}$$

Therefore, we have the h_{fe} of PTT as in the following.

$$\begin{aligned} h_{fe} &= \frac{\partial I_C}{\partial I_B} = \frac{\gamma^*}{1-\gamma^*} - \frac{I_{C0}}{1-\gamma^*}\left(\frac{\partial\exp\left(C_{PD}V_{BC}\right)}{\partial I_B}\right) \\ &= \frac{\gamma^*}{1-\gamma^*} - \frac{I_{C0}}{1-\gamma^*}\left(\exp\left(-C_{PD}V_{CE}\right)\cdot\frac{\partial\exp\left(C_{PD}V_{BE}\right)}{\partial V_{BE}}\cdot\frac{\partial V_{BE}}{\partial I_B}\right) \\ &= \frac{\gamma^*}{1-\gamma^*} - \frac{I_{C0}}{1-\gamma^*}C_{PD}\cdot\exp\left(C_{PD}V_{BC}\right)\cdot h_{ie} \end{aligned} \tag{13}$$

Similarly, we have h_{oe} as shown in the following.

$$\begin{aligned} h_{oe} &= \frac{\partial I_C}{\partial V_{CE}} = \frac{\partial\left(\dfrac{\gamma^*}{1-\gamma^*}I_B - \dfrac{I_{C0}}{1-\gamma^*}\left(\exp(C_{PD}V_{BC})-1\right)\right)}{\partial V_{CE}} \\ &= \frac{\left(\dfrac{-I_{C0}}{1-\gamma^*}\right)\partial\exp\left(C_{PD}(V_{BE}-V_{CE})\right)}{\partial V_{CE}} \\ &= \frac{\left(\dfrac{-I_{C0}}{1-\gamma^*}\right)\cdot\partial\left(\exp\left(C_{PD}V_{BE}\right)\cdot\exp\left(-C_{PD}V_{CE}\right)\right)}{\partial V_{CE}} \\ &= \left(\frac{-I_{C0}}{1-\gamma^*}\right)\cdot\left\{\begin{aligned} &\frac{\partial\left(\exp\left(C_{PD}V_{BE}\right)\right)}{\partial V_{CE}}\cdot\exp\left(-C_{PD}V_{CE}\right) \\ &+\exp\left(C_{PD}V_{BE}\right)\cdot\frac{\partial\left(\exp\left(-C_{PD}V_{CE}\right)\right)}{\partial V_{CE}} \end{aligned}\right\} \\ &= \left(\frac{C_{PD}\cdot I_{C0}}{1-\gamma^*}\right)\cdot\left\{1-\left(\frac{\partial V_{BE}}{\partial V_{CE}}\right)\right\}\cdot\exp\left(C_{PD}V_{BC}\right) \\ &= \left(\frac{C_{PD}\cdot I_{C0}}{1-\gamma^*}\right)\cdot\left\{1-h_{re}\right\}\cdot\exp\left(C_{PD}V_{BC}\right) \end{aligned} \tag{14}$$

As for h_{ie}, from Eq. (6), we have

$$\begin{aligned} h_{ie} &= \frac{\partial V_{BE}}{\partial I_B} = \left(\frac{\partial V_{BE}}{\partial I_E}\right)\left(\frac{\partial I_E}{\partial I_B}\right) = \left(\frac{\partial I_E}{\partial V_{BE}}\right)^{-1}\left(\frac{\partial(I_C + I_B)}{\partial I_B}\right) \\ &= \left(\frac{\partial I_E}{\partial V_{BE}}\right)^{-1}\left(h_{fe}+1\right) = \frac{\left(h_{fe}+1\right)}{C_{LED}(I_E + I_{E0})} \approx \frac{\left(h_{fe}+1\right)}{C_{LED}\cdot I_E} \end{aligned} \tag{15}$$

So far, we have obtained the following equations for the four parameters, h_{ie}, h_{fe}, h_{re} and h_{oe}.

$$\begin{cases} h_{ie} = \dfrac{\partial V_{BE}}{\partial I_B} = \dfrac{\left(h_{fe}+1\right)}{C_{LED}(I_E+I_{E0})} \\ h_{fe} = \dfrac{\partial I_C}{\partial I_B} = \dfrac{\gamma^*}{1-\gamma^*} - \dfrac{I_{C0}}{1-\gamma^*} C_{PD}\cdot\exp\left(C_{PD}V_{BC}\right)\cdot h_{ie} \\ h_{oe} = \dfrac{\partial I_C}{\partial V_{CE}} = \left(\dfrac{C_{PD}\cdot I_{C0}}{1-\gamma^*}\right)\cdot\{1-h_{re}\}\cdot\exp\left(C_{PD}V_{BC}\right) \\ h_{re} = \dfrac{\partial V_{BE}}{\partial V_{CE}} = \dfrac{h_{oe}}{C_{LED}(I_E+I_{E0})} \end{cases} \tag{16}$$

From Eq. (16), we can see the recursive relation between h_{ie} and h_{fe}, and another between h_{re} and h_{oe}. So, we derive those parameters in the details, as follows.

$$\begin{aligned} h_{fe} &= \frac{\gamma^*}{1-\gamma^*} - \frac{I_{C0}}{1-\gamma^*} C_{PD}\cdot\exp\left(C_{PD}V_{BC}\right)\cdot h_{ie} \\ &= \frac{\gamma^*}{1-\gamma^*} - \frac{I_{C0}}{1-\gamma^*} C_{PD}\cdot\exp\left(C_{PD}V_{BC}\right)\cdot\frac{\left(h_{fe}+1\right)}{C_{LED}(I_E+I_{E0})} \end{aligned}$$

So, we define the following K.

$$K \triangleq \frac{I_{C0}}{1-\gamma^*} C_{PD}\cdot\exp\left(C_{PD}V_{BC}\right)\cdot C_{LED}(I_E+I_{E0})^{-1}$$

Then we have

$$\begin{aligned} h_{fe} &= \left(\frac{\gamma^*}{1-\gamma^*} - K\right)\cdot\left(1+K\right)^{-1} \\ &= \frac{\dfrac{\gamma^*}{1-\gamma^*} - \dfrac{I_{C0}}{1-\gamma^*} C_{PD}\cdot\exp\left(C_{PD}V_{BC}\right)\cdot\left(C_{LED}(I_E+I_{E0})\right)^{-1}}{1+\dfrac{I_{C0}}{1-\gamma^*} C_{PD}\cdot\exp\left(C_{PD}V_{BC}\right)\cdot\left(C_{LED}(I_E+I_{E0})\right)^{-1}} \\ &= \frac{\dfrac{\gamma^* C_{LED}(I_E+I_{E0})}{1-\gamma^*} - \dfrac{I_{C0}}{1-\gamma^*} C_{PD}\cdot\exp\left(C_{PD}V_{BC}\right)}{C_{LED}(I_E+I_{E0}) + \dfrac{I_{C0}}{1-\gamma^*} C_{PD}\cdot\exp\left(C_{PD}V_{BC}\right)} \end{aligned} \tag{17}$$

Similarly, as for h_{re}, we have the following equation.

$$h_{re} = \frac{\left(\dfrac{C_{PD}\cdot I_{C0}}{1-\gamma^*}\right)\cdot\exp\left(C_{PD}V_{BC}\right)}{C_{LED}\cdot I_{E0}\cdot\exp\left(C_{LED}V_{BE}\right) + \left(\dfrac{C_{PD}\cdot I_{C0}}{1-\gamma^*}\right)\cdot\exp\left(C_{PD}V_{BC}\right)} \tag{18}$$

If we consider the case when V_{BC} approximately equals 0, then the term exp($C_{PD}V_{BC}$) nearly becomes 1.0. In this case we have the more simplified four parameters as follows that are corresponding to Fig. 3(b).

$$\begin{aligned} & h_{ie} = \frac{\left(\dfrac{1}{1-\gamma^*}\right)}{C_{LED}(I_E+I_{E0})} = \frac{\left(\dfrac{1}{1-\gamma^*}\right)}{C_{LED}\cdot I_{E0}\exp(C_{LED}V_{BE})}, \\ & h_{fe} \approx \frac{\gamma^*}{1-\gamma^*},\ h_{re} \approx 0,\ h_{oe} \approx 0 \end{aligned} \tag{19}$$

3. Conclusion

We have newly derived the four h-parameters in the PTT small signal circuit, based on the VI characteristic equation of LED and PD that composes the PTT. From those computed h parameters, we find out that PPT functions the amplifier, because approximated and simplified h parameters are very similar to the case of BJT.

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