

# A Consideration on Amplification Function in BJT Ebers-Moll Model and PTT (I) ---- V-I Characteristics ----

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## Abstract

In 1989, the notion of Photon Transport Transistor (PTT) has been proposed by B.J. Van Zeghbroeck et al., at IBM Research Center at that time. PTT is an optical coupling device of Light Emitting Diode (LED) and light receiving diode (Photo Diode, PD) where the carrier of the base layer is light (Photon) only. Later in 1996, W. N. Cheung and Paul J. Edwards have shown that PTT can be a very low noise amplifier in a positive feedback circuit, based on theoretical calculations of the noise figure by Quantum Mechanics. In this paper we consider the amplification principle noticing the similarity of the VI characteristics between the Bipolar Junction Transistor (BJT) and PTT.

*Keywords*, PTT, LED, PD, BJT, positive feedback circuit, Ebers-Moll model, VI characteristics

## 1. Introduction

As is well known, the Bipolar Junction Transistor (BJT) has been invented in 1948. The transistor has a structure in which two diodes, each made of a P-type and an N-type semiconductor, are further joined at the base layer.

On the other hand, B. J. Van Zeghbroeck et al. at IBM Research Laboratories presented the notion of a Photon Transport Transistor (PTT) based on optical coupling between a Light Emitting Diode (LED) and a Photo Diode (PD) [1]. Later in 1996, W. N. Cheung and Paul J. Edwards have shown that PTT can be a very low noise amplifier in a positive feedback circuit, based on theoretical calculations of the noise figure by Quantum Mechanics [2].

After that, it has been reported that, using the PTT that consists of high-brightness LED and PD ([3], [4], [5]), audio amplifier can be developed and that the PTT expresses not only an amplification function but also a switching function similar to a thyristor ([6], [7], [8]).

In this paper, we discuss the amplification function of PTT in comparison with the conventional BJT.

For this discussion, firstly, we show the similarity of the VI characteristics equation of BJT Ebers-Moll Model (BJT EMM) and PTT in the emitter common (or grounded) circuit as a positive feedback circuit. This is because, when using the BJT transistor for its amplification function, a common emitter circuit is generally used.

And secondary, we show the circuit equations in order to analyze how a fixed bias PTT circuit achieves DC current amplification function.

## 2. BJT EMM and PTT

### 2.1. Circuit and Characteristic Equation

The BJT emitter common (or grounded) circuit and its BJT EMM are illustrated in Fig. 1, Fig. 2, respectively. Furthermore, a PTT emitter common circuit for small signal amplification is shown in Fig. 3. In this section, we show substantial correspondence between the two devices of BJT EMM and PTT in their emitter common circuits, by representing the VI characteristics of their devices.

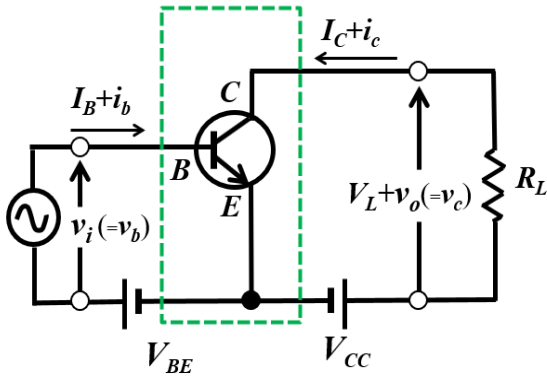


Fig. 1 BJT emitter common circuit for small signal voltage amplification.

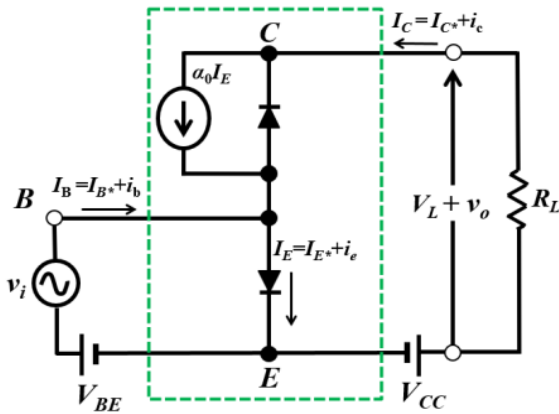


Fig. 2 BJT emitter common circuit that corresponds to the where  $I_E = I_B + I_C$ , based on Ebers-Moll model.

The VI characteristics equations of two diode parts in Fig. 2 are shown as Eq. (1) and Eq. (2). The each meaning of symbols such as  $I_{C0}$ ,  $I_{E0}$ ,  $q$ ,  $k$ ,  $T$  is shown in Table 1

$$I_C = \alpha_0 I_E - I_{C0} \left( \exp\left(\frac{qV_{BC}}{kT}\right) - 1 \right) \quad (1)$$

$$I_E = I_{E0} \left( \exp\left(\frac{qV_{BE}}{kT}\right) - 1 \right) \quad (2)$$

Table 1 The symbol list of the VI characteristic equation of the two diode parts in BJT Ebers-Moll Model.

$I_{E0}$	Reverse saturation current in Emitter	$I_{C0}$	Reverse saturation current in Collector
$q$	Charge	$k$	Boltzmann's constant
$T$	Absolute temperature	$\alpha_0$	Current amplification factor
$V_{BE}$	Base-Emitter voltage	$V_{BC}$	Base-Collector voltage

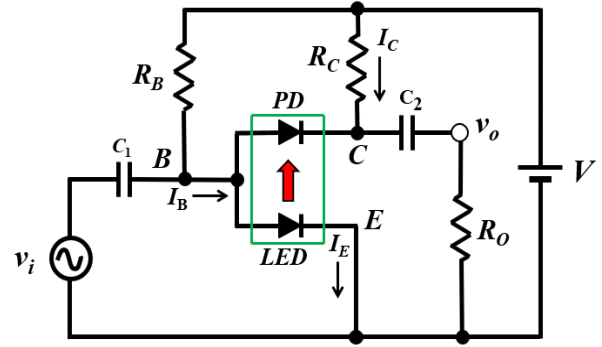


Fig. 3 Fixed bias PTT emitter common circuit for small signal voltage amplification.

The approximate VI characteristic equations for PD part and LED part of the PTT circuit in Fig. 3 are shown in Eq. (3) and Eq. (4), respectively. The meaning of each symbol is shown in Table 2.

We can point out that the pair of Eq. (3) and Eq. (4) is corresponding to that of Eq. (1) and Eq. (2) as the similarity between the BJT EMM and PTT. The difference of the two sets of pair equations is as follows.

$$I_C = \gamma^* I_E - I_{C0} \left( \exp(c_{PD} V_{BC}) - 1 \right), \quad c_{PD} = \frac{q}{m_2 k T_{PD}} \quad (3)$$

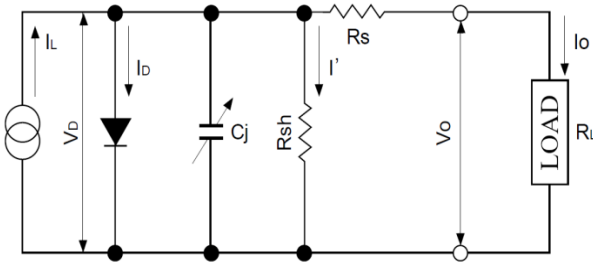
$$I_E = I_{E0} \left( \exp(c_{LED} V_{BE}) - 1 \right), \quad c_{LED} = \frac{q}{m_1 k T_{LED}} \quad (4)$$

Table 2 The symbol list of the VI characteristic equation of PTT circuit.

$I_{E0}$	Reverse saturation current in Emitter (LED)	$I_{C0}$	Reverse saturation current in Collector (PD)
$m_1$	Ideality factor (LED)	$m_2$	Ideality factor (PD)
$q$	Charge	$k$	Boltzmann's constant
$T_{LED}$	Absolute temperature of LED	$T_{PD}$	Absolute temperature of PD
$V_{BE}$	Base-Emitter voltage	$V_{BC}$	Base-Collector voltage
$\gamma^*$	Proportional constant	---	----

The Eq. (3) is an approximated VI characteristic equation of PD, based on the Eq. (5) that can be obtained from the PD equivalent circuit as shown in Fig. 4 ([4], [5]).

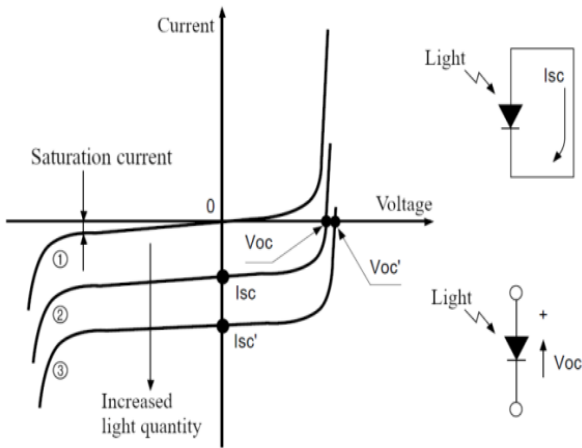
$$\begin{cases} I_O = I_L - I_S \left( \exp\left(\frac{qV_D}{kT}\right) - 1 \right) - I' \\ V_D = V_{BC} - I_O R_S \\ I' = \frac{V_{BC} - I_O R_S}{R_{sh}} \end{cases} \quad (5)$$



$I_L$  : Generated current by incident light,  
 $I_D$  : Current of diode,  $C_j$  : Junction capacitance,  
 $R_{sh}$  : Shunt resistance,  $R_s$  : Series resistance,  
 $I'$  : Current of  $R_{sh}$ ,  $V_D$  : Voltage of diode,  
 $I_o$  : Output current,  $V_o$  : Output voltage.

Fig. 4. Equivalent circuit of PD ([4], [5]), where the upper part corresponds to Base side and the lower Collector side.

The internal resistance  $R_s$  in Eq. (5) and in Fig. 4 is approximated as 0 because it is a small resistance, and  $V_D = V_{BC} = -V_{CB}$ . The resistance  $R_{sh}$  is a large resistance, so that  $I'$  is approximated as zero. Furthermore,  $I_L$  corresponds to  $I_{sc}$  in Fig. 5 and represents the photocurrent proportional to the amount of light incident on the PD (amount of light received by the PD).



$V_{oc}$  : open circuit voltage,  $I_{sc}$  : Photocurrent

Fig.5. V-I characteristic of the equivalent circuit as shown in Fig. 4

In the PTT of Fig. 3, the LED and the PD are physically facing each other at a certain distance, so the amount of light received by the PD is considered to be proportional to the amount of light emitted from the LED. Since the amount of light emitted from the LED is proportional to the current  $I_E$  flowing in the LED,  $I_L$  can

be expressed as  $I_L = \gamma^* I_E$ , where  $\gamma^*$  is the proportionality constant. Then, we obtain the Eq. (3) as the result of introducing a correction term into the approximate equation derived from Eq. (5).

## 2.2. Similarity and Difference in Diode Equation

The similarity points between BJT and PTT: Both devices are composed of two diode parts that are collector-side diode and emitter-side one, and while the collector-side diode is in a state where the current  $I_C$  flows in the opposite direction of the diode and includes the generated current as proportional to the current  $I_E$  of the emitter-side diode.

The differences in the equations for the two diodes in BJT and PTT: Absolute temperature  $T$  of two diodes is same in BJT EMM, while it is different in PTT. And the Ideality factors are 1 in BJT EMM, while they are different in PTT.

## 2.3. Direct Current Amplification from Equation of PTT Circuit

Under the assumption that the input signal voltage  $v_i=0$  in Fig. 3, applying the Kirchhoff's law to the closed circuit 1 ( $V \rightarrow A \rightarrow B \rightarrow E \rightarrow V$ ) and closed circuit 2 ( $A \rightarrow B \rightarrow C \rightarrow A$ ) in the PTT circuit of Fig. 3, we obtain the equation on direct current circuit as shown in Eq. (6).

$$\begin{cases} R_B I_B + V_{BE} = V \\ R_B I_B + V_{BC} - R_C I_C = 0 \end{cases} \quad (6)$$

On the other hand, if the ratio of current  $I_C$  and  $I_E$  is  $\alpha$ , then  $I_C = \alpha I_E$ . This is a variable commonly used in BJT, and from  $I_E = I_B + I_C$ ,  $I_B = (1-\alpha) I_E$ , and  $I_C/I_B = \alpha/(1-\alpha)$  (current amplification factor: hFE,  $\beta$ ). By rewriting Eq. (6) using this  $\alpha$ , we have the following Eq. (7).

$$\begin{cases} R_B (1-\alpha) I_E + V_{BE} = V \\ (R_B - \alpha(R_B + R_C)) I_E + V_{BC} = 0 \end{cases} \quad (7)$$

On the other hand, by transforming the Eq. (3) and Eq. (4), we have another form of VI characteristic equation as in the following ([13], [14]).

$$\begin{cases} V_{BC} = \left( \frac{1}{C_{PD}} \right) \ln \left( \frac{\gamma^* I_E - I_C}{I_{C0}} + 1 \right) = \left( \frac{1}{C_{PD}} \right) \ln \left( \frac{\gamma^* - \alpha}{I_{C0}} I_E + 1 \right) \\ V_{BE} = \left( \frac{1}{C_{LED}} \right) \ln \left( \frac{I_E}{I_{E0}} + 1 \right) \end{cases} \quad (8)$$

Assume that, when the power supply  $V$  increases to  $V+\Delta V$ ,  $\alpha$  and  $I_E$  change to  $\alpha+\Delta\alpha$ ,  $I_E+\Delta I_E$ . Then, from the Eq. (7) and Eq. (8), we have the following Eq. (9) [13].

$$\begin{cases} \Delta\alpha = \left(\frac{K_2}{K_1}\right)\Delta I_E \\ \Delta I_E = \frac{\Delta V}{\left(R_B(1-\alpha) + \frac{1}{c_{LED}I_E} - (R_B I_E)\left(\frac{K_2}{K_1}\right)\right)} \end{cases}$$

where

$$\begin{cases} K_1 = \left(R_B + R_C\right) + \left(\frac{1}{c_{PD}}\right)\frac{1}{(\gamma^* - \alpha)I_E + I_{C0}} I_E \\ K_2 = \left(R_B - (R_B + R_C)\alpha + \left(\frac{1}{c_{PD}}\right)\frac{(\gamma^* - \alpha)}{(\gamma^* - \alpha)I_E + I_{C0}}\right) \end{cases} \quad (9)$$

Eq. (9), shows that the amplification factor  $\alpha$  and emitter direct current  $I_E$  will increase according to the increase of the DC voltage  $V$ . However, we find out that when the relation Eq. (10) holds,  $\Delta\alpha=0$ , even if  $\Delta V > 0$  and  $\Delta I_E > 0$ .

$$\alpha = \gamma^* = \frac{R_B}{R_B + R_C} \quad (10)$$

At that time, from Eq. (8),  $V_{BC} = 0$ . When  $V_{BC} = 0$ , from our experiments for PTT with positive feedback circuit as shown in Fig. 3, we have obtained the direct current amplification factor  $\beta = 999$  ( $\alpha=0.999$ ) [13], [14].

We consider that the same situation can happen in the fixed bias emitter common circuit of BJT EMM, because of the similarity of VI characteristic equation.

### 3. Conclusion

In this paper, from the similarity of the VI characteristics between the BJT EMM and PTT, we have discussed the expression of direct current amplification function of PTT. As a further study, we will analyze the voltage amplification function for input AC signal of the PTT circuit based on the VI characteristics.

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