Terminal Synergetic Controller for Car’s Active Suspension System Using Dragonfly Algorithm

Tinnakorn Kumsaen
Department of Chemical Engineering, Faculty of Engineering, Khon Kaen University, Khon Kaen, 40002, Thailand

Sorn Simatrang
Nacres Co., Ltd, Bangkok 10150, Thailand

Arsit Boonyaprapasorn
Department of Mechanical Engineering, Chulachomklao Royal Military Academy, Nakhon-Nayok, 26000, Thailand

Thunyaseth Sethaput
Sirindhorn International Institute of Technology, Thammasat University, Pathum Thani, 12120, Thailand
Email: tinnakor@kku.ac.th, urarl.urarl9@gmail.com, robosorn@gmail.com, thunyaseth@siit.tu.ac.th

Abstract

This research introduces a terminal synergetic controller (TSC) designed for the active suspension system of automobiles through the implementation of the dragonfly algorithm (DA). The proposed controller aims to enhance the dynamic performance of a car's suspension using the DA in tuning the system parameters. The stability of the designed controller is proved through the application of Lyapunov stability theory. Through iterative optimization processes, the TSC approach seeks to achieve an optimal balance between ride comfort and vehicle handling. The simulation results demonstrate that the proposed controller enhances convergence properties and alleviates the presence of chattering. The results indicate that the proposed approach with the optimal parameters provided insights into its potential application in improving the overall suspension system.

Keywords: Active suspension, Synergetic control, Feedback control, Metaheuristic; Swarm-based optimization

1. Introduction

Since the primary objective of controlling active suspension systems is disturbance rejection, in this control application, it is imperative to select a potential feedback control method that can effectively counteract disturbances. Even though the robustness and enhancement of the convergence of the control system can be achieved by using the concept of sliding mode control (SMC), the method has a key drawback from the chattering phenomenon. This drawback can be avoided by using the synergetic control (SC) which was proposed by Kolesikov et al. [1], [2]. The SC method allows the designer to achieve the desired characteristics of the control system including parameter insensitivity and robustness if the macro variables are selected properly. Sliding surfaces in sliding mode control can be utilized as macro variables in the SC method. Consequently, the convergence time of the control system under the SC method can be improved by choosing macro variables and/or dynamic evolutions with terminal attractors [3]. Swarm-based optimization algorithms, including the dragonfly algorithm (DA) [4], enable the optimal selection of controller parameters. The benefit of achieving an improved convergence rate is obtained when a feedback controller is combined with the DA [5].

In this study, we present the TSC method specifically designed for the active suspension system of automobiles. Based on the quarter-car model, the primary objective of the proposed controller is to improve the stability of the car by utilizing the DA for the precision tuning of system parameters.

2. Methodology

Lemma 1: Consider a nonlinear system in Eq. (1):

$$\dot{\mathbf{x}}(t)=\mathbf{f}(\mathbf{x}(t)), \tag{1}$$

where $\mathbf{x}(t)$ denotes a state vector and $\mathbf{x}(\cdot):\mathbb{R}\rightarrow\mathbb{R}^n$,. $\mathbf{f}(\cdot):\mathbb{R}^n\rightarrow\mathbb{R}^n$ is sufficiently smooth nonlinear mapping. If there exists a positive-definite and continuous Lyapunov function $V(\cdot,\cdot):\mathbb{R}^n\times\mathbb{R}\rightarrow[0,\infty)$ satisfied with Eq. (2)

$$\dot{V}(\mathbf{x}(t),t)\le-\lambda V^{\eta}(\mathbf{x}(t),t),\ \forall t\ge t_0,\ V(t_0)\ge 0, \tag{2}$$

where $\lambda>0$, η is a constant exponent such that $0<\eta<1$, and $t_0>0$ is the initial time. Then,

$$V(\mathbf{x}(t),t)=0,\ \forall t\ge T_s, \tag{3}$$

where T_s is so call the settling time and determined as

$$T_s=t_0+\frac{V^{1-\eta}(\mathbf{x}(t_0),t_0)}{\lambda(1-\eta)} \tag{4}$$

2.1. Mathematical model

In this study, the model of the suspension system derived from Newton's law presented in [6], [7] was used. Fig. 1 shows a quarter-car model of an active suspension which includes the hydraulic actuator in the suspension system. The model can be extended by considering the hydraulic dynamics can be found in [6]. However, in this study, the model views the hydraulic actuator in the suspension system as the control input u_a and ignores the hydraulic dynamics as shown in Eq. (5):

$$M_b\ddot{x}_s + K_a(x_s - x_w) + C_a(\dot{x}_s - \dot{x}_w) - u_a = 0$$

$$M_{us}\ddot{x}_w + K_a(x_w - x_s) + C_a(\dot{x}_w - \dot{x}_s) + K_t(x_w - r) + u_a = 0 \tag{5}$$

where M_b represents the car body mass and M_{us} the wheel. The displacement of car body and wheel are denoted by x_s and x_w .The stiffness of the active suspension system are defined by K_a and K_t . The damping coefficient is represented by C_a . The force u_a is the actuator force from the hydraulic system. The road disturbance is defined by r and it is assumed to be bounded.

According to [6], [7] by letting $x_1 = x_s$ $x_2 = \dot{x}_s$ $x_3 = x_w$ and $x_4 = \dot{x}_w$, the corresponding state space representation is shown in Eq. (6):

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{1}{M_b}\left[K_a(x_1 - x_3) + C_a(x_2 - x_4) - u_a\right] \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{1}{M_{us}}\left[K_a(x_1 - x_3) + C_a(x_2 - x_4) - K_t(x_3 - r) + u_a\right]
\end{aligned} \tag{6}$$

The control objective is to control the error in Eq. (7) to converge to zero:

$$e = x_1 - \bar{x}_3, \tag{7}$$

where $\bar{x}_3$ is the filtered signal of the wheel displacement, x_3. The filter is defined as:

$$\dot{\bar{x}}_3 = \varepsilon\left(x_3 - \bar{x}_3\right). \tag{8}$$

where $\varepsilon > 0$ is the parameter to be determined later. The output is defined as:

$$y \triangleq e = x_1 - \bar{x}_3. \tag{9}$$

It is worth noting that the reasons for introducing ε and additional dynamics of Eq. (8) were elaborated in detail in [6]. To summarize, introducing ε can provide us the degree of freedom to guarantee the stability of the zero dynamics corresponding to the chosen output in Eq. (9) as we will further discuss at the end of section 2.3. In addition, it will play an important role in the tradeoff between ride quality and rattle space usage.

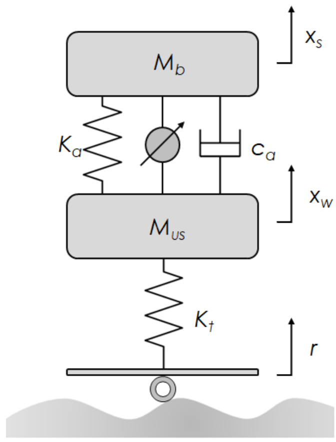

Fig. 1. A quarter-car model with active suspension system

2.2. *Controller design*

According to [1], [2], [3], the terminal synergetic controller design procedure can be presented as follows: Firstly, define the macro variable of the controller.

$$\psi = e + \frac{(\dot{e})^{\frac{p}{q}}}{\alpha}. \tag{10}$$

where p and q are positive odd numbers such that $p > q$ Second, define the dynamic evolution of the macro variable as Eq. (11):

$$(\beta\dot{\psi})^{\frac{m}{n}} + \psi = 0 \tag{11}$$

where $1 < {}^{m}\!/_{n} < 2$, m and n are positive odd numbers, $\beta > 0$ is parameters to be determined by the designer and affect the convergence time of the macro variable to converge to zero. Finally, determine the control input as follows. By Eq. (10), the derivative of the macro variable is,

$$\dot{\psi} = \dot{e} + \frac{p(\dot{e})^{\frac{p}{q}-1}\ddot{e}}{q\alpha} \tag{12}$$

Recall that $e = x_1 - \bar{x}_3$, then.

$$\dot{e} = \dot{x}_1 - \dot{\bar{x}}_3 = x_2 + \varepsilon(\bar{x}_3 - x_3) \tag{13}$$

And,

$$\ddot{e} = -\frac{[K_a(x_1 - x_3) + C_a(x_2 - x_4)]}{M_b} - \varepsilon^2(\bar{x}_3 - x_3) - \varepsilon x_4 + \frac{u_a}{M_b} \tag{14}$$

By using the Eq. (11) – Eq. (12) and Eq. (14), we can determine the controller u_a as

$$u_a = M_b\left(\left(\frac{(-\psi)^{\frac{n}{m}}}{\beta} - \dot{e}\right)\frac{q\alpha}{p(\dot{e})^{\frac{p}{q}-1}} + \varepsilon^2(\bar{x}_3 - x_3) + \varepsilon x_4\right) + [K_a(x_1 - x_3) + C_a(x_2 - x_4)] \tag{15}$$

Note that to guarantee that the term $(\dot{e})^{\frac{p}{q}-1}$ will not cause the singularity, the additional condition must be imposed on the parameters p, q, m, n, which the following i) $\dfrac{pn + m(q-p)}{qn} > 0$ and ii) $\dfrac{m(p-q)}{pn} < 1$.

2.3. *Proof of stability*

The Lyapunov function is defined in terms of the macro variable as

$$V(\psi) = \frac{1}{2}\psi^2 \tag{16}$$

By Eq. (12), the derivative of Eq. (16) is determined as

$$\dot{V}(\psi) = \psi\dot{\psi} = \psi\left(\dot{e} + \frac{p(\dot{e})^{\frac{p}{q}-1}\ddot{e}}{q\alpha}\right) \tag{17}$$

By substituting the controller in Eq. (15) into Eq. (17), we yield,

$$\dot{V}(\psi) = \psi\frac{(-\psi)^{\frac{n}{m}}}{\beta}$$

Recall that m and n are positive odd number such that $0.5 < n/m < 1$ and let define $\eta = \dfrac{n+m}{2m}$ and $\lambda = \dfrac{2^{\eta}}{\beta}$. Note that $0.75 < \eta < 1$ and $\lambda > 0$ by construction, then,

$$\dot{V}(\psi) = -\psi\frac{(\psi)^{\frac{n}{m}}}{\beta} = -\frac{1}{\beta}\left(2\frac{\psi^2}{2}\right)^{\eta} = -\lambda V^{\eta} \tag{18}$$

Based on Lemma 1, the settling time T_1 is determined as

$$T_1 = t_0 + \frac{(\psi(t_0))^{2-2\eta}}{2^{1-\eta}\lambda(1-\eta)}. \tag{19}$$

Then, this implies that the manifold $\psi = 0$, $\forall t > T_1$. From Eq. (10),

$$\dot{e} = -\bar{\alpha}e^{\bar{\eta}} \tag{20}$$

whenever the manifold $\psi = 0$, where $\bar{\alpha} \triangleq \alpha^{\frac{q}{p}}$ and $\bar{\eta} \triangleq q/p$. Furthermore, notice that $\alpha > 0$ and both of p, q are positive odd numbers such that $p > q$, then $\bar{\alpha} > 0$ and $0 < \bar{\eta} < 1$ by construction. Again, by Lemma 1, the error $e(t)$ converges to zero within the time:

$$T_2 = T_1 + \frac{(e(T_1))^{2-2\bar{\eta}}}{2^{1-\bar{\eta}}\bar{\alpha}(1-\bar{\eta})} \tag{21}$$

Hence, $x_1(t)$ converges to $\bar{x}_3(t)$ within $T = t_0 + T_1 + T_2$ which is a finite real number. Note that, since we choose the output $y \triangleq x_1 - \bar{x}_3$, we can obtain the zero dynamics in the matrix form as Eq. (22) [6] by setting the output identically equal to zero.

$$\dot{z} = A_z z + B_z r \tag{22}$$

where $z \triangleq [\bar{x}_3 \quad x_3 \quad x_4]^T$

$$A_z \triangleq \begin{bmatrix} -\varepsilon & \varepsilon & 0 \\ 0 & 0 & 1 \\ -\varepsilon^2\dfrac{M_b}{M_{us}} & \left(\dfrac{\varepsilon^2 M_b - K_t}{M_{us}}\right) & -\varepsilon\dfrac{M_b}{M_{us}} \end{bmatrix}$$ and

$$B_z \triangleq \begin{bmatrix} 0 & 0 & K_t/M_{us} \end{bmatrix}^T.$$

and r is assumed to be a bounded road disturbance i.e., $|r| \le M_r$ where M_r is a positive constant.

One can easily realize that the matrix A_z is Hurwitz if and only if $\varepsilon > 0$ by using the Ruth-Hurwitz criterion. It is also worth noting that there is no finite escape time phenomenon that occurs no matter how large the magnitude of road disturbance $|r|$ is since the zero dynamics Eq. (22) is a linear system with z is the state vector of the system and r is the disturbance. From the design perspective, one can choose a parameter ε such that the matrix A_z is sufficiently robust enough to dominate the bounded road disturbance r to guarantee the convergence of the state z to zero. Then, the state x_1 and x_2 converge to zero for the following reasons. Since x_1 converges to $\bar{x}_3$ in finite time by Eq. (18) and the argument from Eq. (20) i.e., $\dot{e} = -\bar{\alpha} e^{\bar{\eta}}$ when the manifold $\psi = 0$. Then, whenever $z \triangleq [\bar{x}_3 \quad x_3 \quad x_4]^T$ is identically equal to zero, it implies that x_1 is identically equal to zero and then it also implies that $x_2 = 0$ i.e., $x_2 \triangleq \dot{x}_1$. Hence, the proof of locally terminal time stability is completed $\square$

2.4. *Controller parameter tuning*

By using the algorithm of DA, the controller parameters of the design control input as shown in Eq. (15) are selected optimally to minimize the cost function or performance index which is defined as

$$J = \sqrt{u_a^{\,T} u_a}$$

3. Simulation Results and Discussions

To show the capability of the designed controller, the model in Eq. (6) with the system parameters from [8], [9] are used as the simulation example. The system parameters are defined as follows: $M_b = 290$ kg, $M_{us} = 59$ kg, $K_a = 16{,}812$ N/m, $K_t = 190{,}000$ N/m, $C_a = 1000$ Ns/m. The controller parameters are given by DA with 40 search agents and 20 iterations. The controller parameters before tuning are arbitrarily defined as $\alpha_1 = 0.228$, $\beta_1 = 0.9431$, $\varepsilon_1 = 1.3356$, and the controller parameters after tuning are $\alpha_2 = 3.9648$ $\beta_2 = 0.10942$, $\varepsilon_2 = 3.7505$. Both cases are using $p = 5$, $q = 3$, $m = 7$, $n = 5$. We formulate the road disturbance based on the principles outlined in [6]. This disturbance is mathematically described as a single bump with a height of 5 cm ($a = 0.025$), defined as:

$$r = \begin{cases} a(1 - \cos 8\pi t), & 3.5 \le t \le 3.75 \\ 0, & otherwise \end{cases}$$

Fig. 2 and Fig. 3 illustrate that the proposed TSC method can effectively guide all state responses to the desired levels of displacement and velocity. Nevertheless, upon comparing the controller with and without optimal parameters obtained from DA, it is evident that the TSC with optimal parameters exhibits a superior convergence rate performance. Fig. 4 demonstrates that the TSC with optimal parameters can alleviate the presence of chattering.

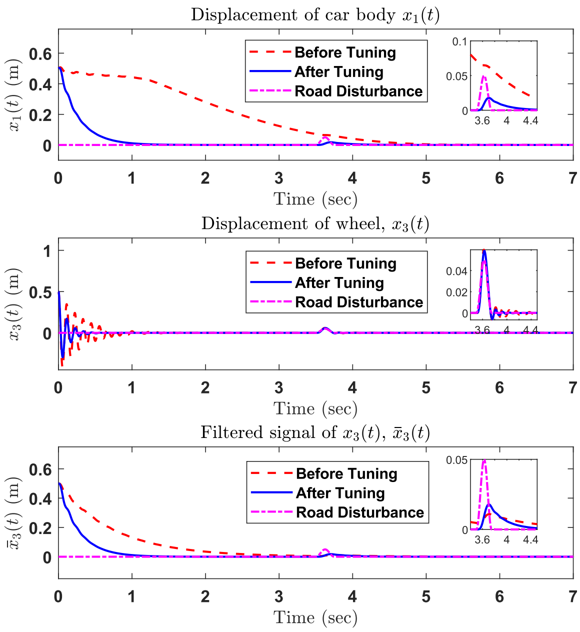

Fig. 2. Comparison of state response x_1, x_3, and $\bar{x}_3$ of the control system before and after DA tuning

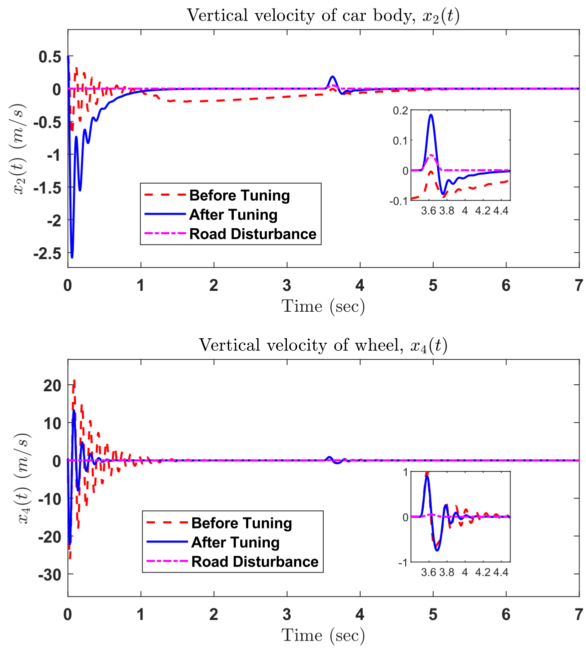

Fig. 3. Comparison of state response x_2 and x_4 of the control system before and after DA tuning

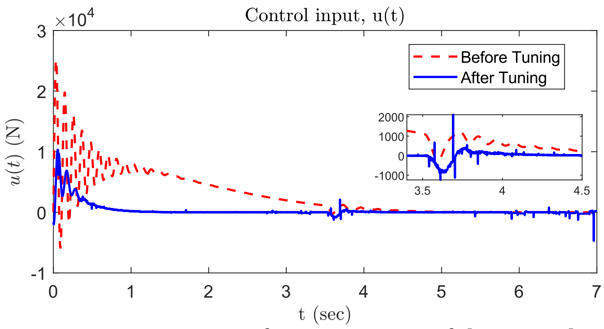

Fig. 4. Comparison of state response of the control input before and after DA tuning

4. Conclusion

In this study the TSC method was applied for controlling of the active suspension system. The controller parameters are selected optimally based on the algorithm of DA. The proposed controller effectively stabilizes the active suspension system by Lyapunov stability analysis. Simulation of the control system was carried out to validate the efficacy of the proposed controller. The simulation results demonstrate that the proposed controller enhances convergence properties. Additionally, the observed decrease in chattering in the control input signals, a significant issue in sliding mode control, further supports the potential application of the proposed TSC method in enhancing the overall performance of the suspension system.

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Authors Introduction

Dr. Tinnakorn Kumsaen

He received his B.S degree in chemical engineering from Khon Kaen University, Thailand, M.S. degree in system engineering from Case Western Reserve University (CWRU), and Ph.D. in chemical engineering from CWRU. He is currently a faculty member at chemical engineering, Khon Kaen University. His research interests are control system, process design simulations & integrations, and renewable energy.

Mr. Sorn Simatrang

He received a B.S. in electrical engineering from Chulalongkorn University, Thailand, and an M.S. in systems and control engineering from Case Western Reserve University, Ohio, USA in 2006 and 2015, respectively. Currently, he works as a control systems engineer at Nacres Co., Ltd, Bangkok, Thailand. His main research interests include output feedback control problems, adaptive control problems for nonlinear systems, and applications of nonlinear control theory to systems biology and mechanical systems.

Dr. Arsit Boonyaprapasorn

He received a B.S. in Mechanical Engineering from King Mongkut 's University of Technology (KMUTT), Thailand, in 1998. He continued his postgraduate studies at Case Western Reserve University, Cleveland, Ohio, U.S.A., obtaining an M.S.in Systems and Control Engineering in 2003 and a Ph.D. in Mechanical Engineering in 2009.Since then, he has served as a lecturer at Chulachomklao Royal Military Academy, Nakhon Nayok, Thailand. His research interests revolve around nonlinear feedback control and robotics.

Dr. Thunyaseth Sethaput

He received his B.S. degree in mechanical engineering from Sirindhorn International Institute of Technology, Thammasat University, Thailand and Ph.D. in systems and control engineering, from Case Western Reserve University, Ohio, USA. He is an assistant professor and chairperson of mechanical engineering program at Sirindhorn International Institute of Technology, Thailand. His research interests are control systems, systems biology, biomedical mechanics, simulation modeling.