

# Disturbance Observer-based Anti-unwinding Control for Flexible Spacecrafts

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## Abstract

The anti-unwinding control problem for the six-degrees-of-freedom (6-DOF) motion of the flexible spacecraft is studied in this paper. Firstly, the translation-rotation-vibration coupling motion of the flexible spacecraft is described by dual quaternion. Then, a nonlinear disturbance observer (NDO) is applied to estimate and compensate the lumped disturbances including the flexible vibration and unknown external disturbances. An anti-unwinding controller is designed based on the sliding mode technology. The stability of the closed-loop system is verified via Lyapunov method. Finally, numerical simulations indicate the effectiveness of the designed controller.

*Keywords:* flexible spacecraft, dual quaternion, disturbance observer, anti-unwinding control

## 1. Introduction

The flexible spacecraft is a complex dynamic system of rotation-translation-vibration coupling. Many researches have been focused on modeling and control for the 6-DOF motion of the flexible spacecraft. Dual quaternion has been gradually applied to describe the rotational and translational motion of the spacecraft. In this formalism, an actual physical attitude corresponds to two mathematical descriptions. Hence, there are also two equilibrium points  $\pm \hat{\mathbf{1}}$ , where  $\hat{\mathbf{1}} = [1, 0, 0, 0]^T + \varepsilon[0, 0, 0, 0]^T$ . Only one of them is considered in most researches, which may lead to the unwinding phenomenon[1]. This phenomenon may cause unnecessary fuel consumptions and should be avoided. There are few relevant investigations about this and it is worthy to be studied.

Vibration suppression is a key issue in flexible spacecraft control. Neural network[2], adaptive control[3] and disturbance observer[4] are three commonly used methods to estimate the flexible vibration which is assumed as disturbances. However, in the initial stage of the neural network approximation, the estimation error is relatively large, which limits its applications. The adaptive control usually estimates the upper bounds of the disturbances. Thus, the disturbance observer is employed in our study.

In this paper, an NDO is firstly proposed to attenuate the influence of unknown external disturbances and the flexible vibration. Then, a sliding mode controller (SMC) is developed, which is of the anti-unwinding performance and guarantees the accuracy and robustness of the flexible spacecraft 6-DOF stability control simultaneously.

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## 2. Kinematic and Dynamic Model for the Flexible Spacecraft Based on Dual Quaternion

The spacecraft is assumed to be a central rigid body with a flexible appendage, which is shown in Fig. 1.

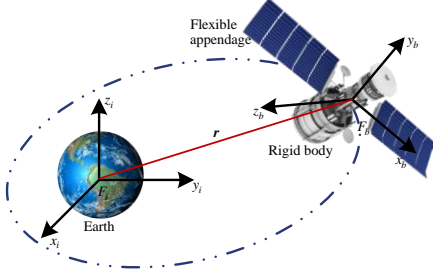


Fig. 1 Coordinate system for the flexible spacecraft

The 6-DOF motion of the flexible spacecraft in  $F_i$  can be expressed as  $\hat{q} = [\hat{q}_0, \hat{q}_v]^T = \mathbf{q} + \varepsilon(1/2 \mathbf{q} \circ \mathbf{r})$ , where  $\mathbf{q}$  and  $\mathbf{r}$  are its rotational and translational motion. Noting that  $\hat{q}$  is a unit dual quaternion which satisfies  $\|\hat{q}\| = \hat{1}$  and  $q_0 q'_0 + \mathbf{q}_v^T \mathbf{q}'_v = 0$ . The kinematic model is given as

$$\dot{\hat{q}}_0 = -\frac{1}{2} \hat{q}_v^T \hat{\omega}, \quad \dot{\hat{q}}_v = \frac{1}{2} (\hat{q}_0 \mathbf{I} + \hat{q}_v^\times) \hat{\omega}, \quad (1)$$

where  $\hat{\omega} = \omega + \varepsilon \mathbf{v} = \omega + \varepsilon(\dot{\mathbf{r}} + \omega \times \mathbf{r})$  is the dual angular velocity.  $\omega$  and  $\mathbf{v}$  are the angular velocity and velocity expressed in  $F_i$ , and  $\mathbf{I} \in \mathbb{R}^{3 \times 3}$  is the identity matrix. The dynamic model is denoted as

$$\hat{M} \dot{\hat{\omega}} = -\hat{\omega} \times (\hat{M} \hat{\omega} + \hat{B} \ddot{\eta}) - \hat{B} \ddot{\eta} + \hat{g} + \hat{u} + \hat{d}_0, \quad (2)$$

where  $\hat{M}$  is given as  $\hat{M} = (d/d\varepsilon) m \mathbf{I} + \varepsilon \mathbf{J}$ .  $m$ ,  $\mathbf{J}$  are the mass and inertia matrix. Dual vectors  $\hat{g} = \mathbf{f}_g + \varepsilon \boldsymbol{\tau}_g$ ,  $\hat{u} = \mathbf{f}_u + \varepsilon \boldsymbol{\tau}_u$  and  $\hat{d}_0 = \mathbf{f}_{d0} + \varepsilon \boldsymbol{\tau}_{d0}$  represent the gravity force, control input and unknown external disturbances, respectively.  $\eta \in \mathbb{R}^n$  denotes the modal coordinate vector of the flexible appendage and  $n$  is the modal number. The dynamics of the modal coordinate is presented as

$$\ddot{\eta} + 2\xi \Omega \dot{\eta} + \Omega^2 \eta + \mathbf{B}_r^T \omega + \mathbf{B}_i^T \mathbf{v} = \mathbf{0}, \quad (3)$$

where  $\xi = \text{diag}\{\xi_i\}$  ( $i = 1, \dots, n$ ) is the damping ratio matrix and  $\Omega = \text{diag}\{\Omega_i\}$  is the natural frequency matrix. The dual vector  $\hat{\eta}$  holds the definition of  $\hat{\eta} = \eta + \varepsilon \boldsymbol{\eta}$ .  $\hat{B} = (d/d\varepsilon) \mathbf{B}_i + \varepsilon \mathbf{B}_r$  is the rigid-flexible coupling dual matrix with  $\mathbf{B}_i, \mathbf{B}_r \in \mathbb{R}^{3 \times n}$ .

Then, denote  $\hat{d} = \mathbf{d} + \varepsilon \mathbf{d}' = -\hat{\omega} \times \hat{B} \ddot{\eta} - \hat{B} \ddot{\eta} + \hat{d}_0$  as the lumped disturbances, which are assumed to be bounded and their time derivative is approximated to zero. The dynamic equation (2) can be rewritten as

$$\hat{M} \dot{\hat{\omega}} = -\hat{\omega} \times \hat{M} \hat{\omega} + \hat{g} + \hat{u} + \hat{d}. \quad (4)$$

## 3. Disturbance Observer-based Anti-unwinding Controller Design

### 3.1. Nonlinear Disturbance observer design

The NDO is design as

$$\begin{cases} \hat{\mathbf{D}} = \hat{\mathbf{z}}_d + \hat{\boldsymbol{\rho}}(\hat{\omega}), \\ \dot{\hat{\mathbf{z}}}_d = -\hat{\boldsymbol{\lambda}} \hat{\mathbf{z}}_d + \hat{\boldsymbol{\lambda}} [\hat{\omega} \times \hat{M} \hat{\omega} - \hat{g} - \hat{u} - \hat{\boldsymbol{\rho}}(\hat{\omega})], \end{cases} \quad (5)$$

where the dual function  $\hat{\boldsymbol{\rho}}(\hat{\omega}) = \hat{\boldsymbol{\lambda}} \hat{\boldsymbol{\rho}}(\hat{\omega})$  and the dual matrix  $\hat{\boldsymbol{\lambda}} = \boldsymbol{\lambda} + \varepsilon \boldsymbol{\lambda}' = \text{diag}\{\lambda_i\} + \varepsilon \text{diag}\{\lambda'_i\}$  ( $i = 1, 2, 3$ ) with  $\lambda_i, \lambda'_i > 0$ .  $\hat{\mathbf{D}}$  is the estimation of  $\mathbf{d}$ .  $\square$  is the corresponding product.

**Theorem 1.** Considering the dynamic equation (4) of the flexible spacecraft, the estimation error  $\hat{\mathbf{d}}_e = \hat{\mathbf{d}} - \hat{\mathbf{D}}$  can converge to the origin with the proposed NDO (5).

**Proof.** Select the following Lyapunov function candidate

$$V_1 = \frac{1}{2} \langle \hat{\mathbf{d}}_e, \hat{\mathbf{d}}_e \rangle, \quad (6)$$

where  $\langle \square, \square \rangle$  is the inner product. By (5), we have

$$\dot{V}_1 = -\langle \hat{\mathbf{d}}_e, \hat{\boldsymbol{\lambda}} \hat{\mathbf{d}}_e \rangle = -\sum_{i=1}^3 (\lambda_i d_{ei}^2 + \lambda'_i d_{ei}'^2). \quad (7)$$

It can be derived that  $\dot{V}_1 < 0$  when  $\hat{\mathbf{d}}_e \neq \hat{\mathbf{0}}$ . By LaSalle's invariance principle, the designed NDO can track the lumped disturbances  $\mathbf{d}$  with asymptotic convergence.  $\square$

### 3.2. Anti-unwinding Controller Design

A sliding mode surface  $\hat{s}$  is presented as

$$\hat{s} = \mathbf{s} + \varepsilon \mathbf{s}' = \hat{\omega} + \hat{\boldsymbol{\mu}} \square (\chi \hat{q}_v), \quad (8)$$

where  $\hat{\boldsymbol{\mu}} = \text{diag}\{\mu_i\} + \varepsilon \text{diag}\{\mu'_i\}$  ( $i = 1, 2, 3$ ).  $\mu_i$  and  $\mu'_i$  are all positive factors. The parameter  $\chi$  is defined as

$$\chi = \begin{cases} 1, & q_0(0) \geq 0 \\ -1, & q_0(0) < 0 \end{cases} \quad (9)$$

where  $q_0(0)$  is the initial value of the real part of  $\hat{q}_0$ .

**Theorem 2.** Under the condition of  $\hat{s} = \hat{\mathbf{0}}$ , state variables  $\hat{\omega}$  and  $\hat{q}$  finally converge to the state  $\{\hat{q} = \pm \hat{\mathbf{1}}, \hat{\omega} = \hat{\mathbf{0}}\}$ . Meanwhile, the unwinding phenomenon are avoided.

**Proof.** Select the Lyapunov function candidate

$$V_2 = 2(1 - \chi q_0) + \frac{1}{4} \mathbf{r}^T \mathbf{r}. \quad (10)$$

The time derivative of  $V_2$  is given as

$$\dot{V}_2 = -2\chi\dot{q}_0 + \frac{1}{2}\mathbf{r}^T\dot{\mathbf{r}} = \chi\mathbf{q}_v^T\boldsymbol{\omega} + (\mathbf{q}^* \circ \mathbf{q}')^T \mathbf{v}. \quad (11)$$

When  $\hat{\mathbf{s}} = \hat{\mathbf{0}}$ ,  $\hat{\boldsymbol{\omega}}$  can be expressed as  $\hat{\boldsymbol{\omega}} = -\hat{\boldsymbol{\mu}} \square (\chi\hat{\mathbf{q}}_v)$ . Applying the properties of the unit dual quaternion yields

$$\begin{aligned} \dot{V}_2 &= -|\chi|^2 \mathbf{q}_v^T (\boldsymbol{\mu} \mathbf{q}_v) - (q_0 \mathbf{q}'_v - q'_0 \mathbf{q}_v)^T (\chi \boldsymbol{\mu}' \mathbf{q}'_v) \\ &< -\mu_m |\chi|^2 \|\mathbf{q}_v\|^2 - \mu'_m |q_0| \|\mathbf{q}'_v\|^2 - \frac{\mu'_m}{|q_0|} \|\mathbf{q}_v\|^2 \|\mathbf{q}'_v\|^2, \end{aligned} \quad (12)$$

where  $\mu_m = \min\{\mu_i\}$ ,  $\mu'_m = \min\{\mu'_i\}$  ( $i=1,2,3$ ). It can be obtained that  $\dot{V}_2 < 0$  when  $\hat{\mathbf{q}}_v \neq \hat{\mathbf{0}}$ . Thus,  $\hat{\boldsymbol{\omega}}$  and  $\hat{\mathbf{q}}$  can converge to the state  $\{\hat{\mathbf{q}} = \pm \hat{\mathbf{1}}, \hat{\boldsymbol{\omega}} = \hat{\mathbf{0}}\}$ . In this case, the kinematic model of the rotation can be written as

$$\dot{\mathbf{q}}_0 = -\frac{1}{2}\mathbf{q}_v^T \boldsymbol{\omega} = \frac{\chi}{2} \mathbf{q}_v^T \boldsymbol{\mu} \mathbf{q}_v. \quad (13)$$

Thus, if  $q_0(0) \geq 0$ ,  $\chi=1$  and  $\dot{q}_0 \geq 0$  are ensured, the state variables finally reaches the equilibrium point  $\hat{\mathbf{1}}$ . And  $q_0(0) < 0$  leads the trajectory reach to the state  $-\hat{\mathbf{1}}$ . Hence, the anti-unwinding property is guaranteed.  $\square$

Subsequently, the control law based on the NDO (5) and the sliding mode surface (8) is proposed as

$$\begin{aligned} \hat{\mathbf{u}} &= \hat{\boldsymbol{\omega}} \times \hat{\mathbf{M}} \hat{\boldsymbol{\omega}} - \frac{1}{2} \chi \hat{\boldsymbol{\mu}} \square \hat{\mathbf{M}} (\hat{q}_0 \mathbf{I} + \hat{\mathbf{q}}_v^\times) \hat{\boldsymbol{\omega}} - \hat{\mathbf{g}} - \hat{\mathbf{D}} \\ &\quad - \hat{\boldsymbol{\kappa}}_1 \square \hat{\mathbf{s}}^s - \hat{\boldsymbol{\kappa}}_2 \square \text{sgn}(\hat{\mathbf{s}}^s) \end{aligned} \quad (14)$$

where  $\hat{\boldsymbol{\kappa}}_x = \text{diag}\{\kappa_{xi}\} + \varepsilon \text{diag}\{\kappa'_{xi}\}$  ( $x=1,2; i=1,2,3$ ).  $\kappa_{xi}$ ,  $\kappa'_{xi}$  are all positive parameters.  $\hat{\mathbf{s}}^s$  is the swap operation of  $\hat{\mathbf{s}}$ .

**Theorem 3.** Considering the kinematic and dynamic model of the spacecraft (1) and (4), the trajectory of the closed-loop system will asymptotically converge to the manifold  $\hat{\mathbf{s}} = \hat{\mathbf{0}}$  by the control law (14).

**Proof.** Consider the Lyapunov candidate function

$$V_3 = \frac{1}{2} [\hat{\mathbf{s}}^T \hat{\mathbf{M}} \hat{\mathbf{s}}] + \frac{1}{2} \langle \hat{\mathbf{d}}_e, \hat{\mathbf{d}}_e \rangle, \quad (15)$$

where  $\langle \square \square \rangle$  is the switched inner product. Differentiating  $V_3$  yields

$$\dot{V}_3 = \left[ \hat{\mathbf{s}}^T \left[ -\hat{\boldsymbol{\omega}} \times \hat{\mathbf{M}} \hat{\boldsymbol{\omega}} + \hat{\mathbf{g}} + \hat{\mathbf{u}} + \hat{\mathbf{d}} + \frac{1}{2} \chi \hat{\boldsymbol{\mu}} \square \hat{\mathbf{M}} (\hat{q}_0 \mathbf{I} + \hat{\mathbf{q}}_v^\times) \hat{\boldsymbol{\omega}} \right] + \langle \hat{\mathbf{d}}_e, \dot{\hat{\mathbf{d}}}_e \rangle \right], \quad (16)$$

Substituting (14) into (16), it can be derived that

$$\begin{aligned} \dot{V}_3 &= \sum_{i=1}^3 (d'_{ei} s_i + d_{ei} s'_i) - \sum_{i=1}^3 (\kappa'_{li} s_i^2 + \kappa_{li} s_i'^2) \\ &\quad - \sum_{i=1}^3 (\kappa'_{2i} |s_i| + \kappa_{2i} |s'_i|) - \sum_{i=1}^3 (\lambda_i d_{ei}^2 + \lambda'_i d_{ei}'^2). \end{aligned} \quad (17)$$

Moreover, let  $a_{1i} = 1/2\sqrt{\lambda_i}$ ,  $a'_{1i} = 1/2\sqrt{\lambda'_i}$ ,  $a_{2i} = 1/4\lambda_i$ , and  $a'_{2i} = 1/4\lambda'_i$ , we have

$$\begin{aligned} \dot{V}_3 &= -\sum_{i=1}^3 \left[ (a'_{1i} s_i - \sqrt{\lambda'_i} d'_{ei})^2 + (a_{1i} s'_i - \sqrt{\lambda_i} d_{ei})^2 \right] \\ &\quad - \sum_{i=1}^3 \left[ (\kappa'_{li} - a'_{2i}) s_i^2 + (\kappa_{li} - a_{2i}) s_i'^2 \right] - \sum_{i=1}^3 (\kappa'_{2i} |s_i| \\ &\quad + \kappa_{2i} |s'_i|) \leq 0. \end{aligned} \quad (18)$$

It is obvious that  $V_3 \leq V_3(0)$  and  $V_3$  is bounded. The definition of the boundedness of a dual quaternion is that both its real and dual part are bounded. It is derive that  $\hat{\mathbf{s}}$ ,  $\hat{\mathbf{d}}_e$ ,  $\hat{\mathbf{q}}$ ,  $\hat{\boldsymbol{\omega}}$  and  $\hat{\mathbf{u}}$  are bounded. Then, we can also obtain that  $\dot{V}_3$  is bounded. By Barbalat's Lemma, as time goes to infinity,  $V_3$  finally converges to zero. Thus, the sliding mode variable  $\hat{\mathbf{s}}$  converges to  $\hat{\mathbf{0}}$  asymptotically.  $\square$

#### 4. Numerical Simulations

This section simulates the hovering mission. The parameters of the flexible spacecraft are the same as Ref.[5]. The initial pose is set as  $\mathbf{q}_0 = [-0.6403, -0.5, -0.3, 0.5]^T$  and  $\mathbf{r}_0 = [5, -5, 5]^T$  m. The initial value of the angular velocity and velocity are set as  $\boldsymbol{\omega}_0 = [0.1, -0.1, 0.1]^T$  rad/s and  $\mathbf{v}_0 = [0.1, -0.1, 0.1]^T$  m/s. Control parameters are denoted as:

$$\begin{aligned} \hat{\boldsymbol{\lambda}} &= \text{diag}(0.7, 0.7, 0.7) + \varepsilon \text{diag}(0.7, 0.7, 0.7), \\ \hat{\boldsymbol{\mu}} &= \text{diag}(0.1, 0.1, 0.1) + \varepsilon \text{diag}(1, 1, 1), \\ \hat{\boldsymbol{\kappa}}_1 &= \hat{\boldsymbol{\kappa}}_2 = \text{diag}(0.5, 0.5, 0.5) + \varepsilon \text{diag}(3, 3.5, 3). \end{aligned}$$

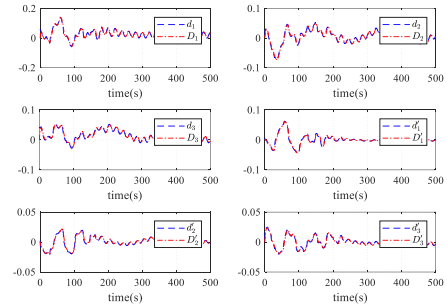


Fig. 2 Time responses of the lumped disturbances and their estimations

Fig. 2 describes the time response of the lumped disturbances and their estimations. The variations of the attitude, angular velocity, position and velocity are mentioned in Fig. 3 and Fig. 4. It can be obtained that when  $q_0(0) < 0$ , the attitude finally converges to the state  $\mathbf{q} = [-1, 0, 0]^T$  and the unwinding problem is handled from Fig. 3. The time responses of the control force and

torque are shown in Fig. 5. Thus, the accuracy and robustness of the controller (14) can be verified.

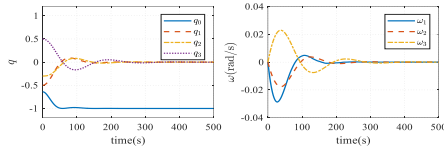


Fig. 3 Time responses of attitude and angular velocity

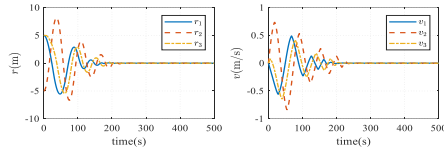


Fig. 4 Time responses of position and velocity

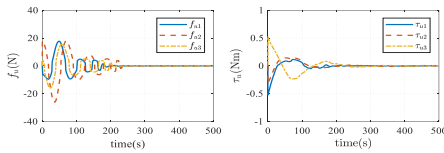


Fig. 5 Time responses of control force and torque

## 5. Conclusions

This paper considers the stabilization control for the flexible spacecraft's 6-DOF motion. The NDO is constructed to estimate the lumped disturbances. The unwinding phenomenon is solved by designing a SMC, which optimizes the rotational path. Meanwhile, the strong robustness and high precision control can be achieved.

## Acknowledgements

This work was supported in part by the NSFC (62133001, 61520106010) and the National Basic Research Program of China (973 Program: 2012CB821200, 2012CB821201).

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