# Cartesian Space Coordinated Impedance Control of Redundant Dual-Arm Robots

#### Yang Zhang

School of Energy and Power Engineering, Beihang University (BUAA), Beijing 100191, China.

### Yingmin Jia\*

The Seventh Research Division and the Center for Information and Control, School of Automation Science and Electrical Engineering, Beihang University (BUAA),

Beijing, 100191, China

E-mail: zhyang19@buaa.edu.cn, ymjia@buaa.edu.cn,

www.buaa.edu.cn

#### Abstract

This paper presents a cartesian space coordinated impedance control method to achieve coordination when a dual-arm robot operates an object. First, the relative positional and force errors when the two arms operate the object are defined. Then, these relative errors are introduced into the general impedance controller to achieve coordinated impedance control. Compared to the conventional impedance control, this scheme ensures the coordination between the two arms and reduces the contact force error between the end-effectors and the object.

Keywords: Impedance control, Dual-arm robot, Coordinated control, Relative error

## 1. Introduction

There has been an increasing interest in robotics in recent decades. For additional research and application needs, researchers have developed and studied various types of robots. Among these robots, redundant dual-arm robots are playing an increasingly important role in numerous engineering applications.

When a dual-arm robot operates an object, it can be broadly classified into uncoordinated and coordinated operation according to the constraint relationship between the two arms.[1] In an uncoordinated operation, two robotic arms perform different operation tasks in the same task space, while a coordinated operation means that both robotic arms perform the same or multiple related operation tasks in the same operation space. Compared with the uncoordinated operation, there are stricter motion constraints and force constraints between

the robotic manipulators in coordinated operation tasks. The main methods to solve the flexible control of robotic arm by symmetric coordinated control scheme are hybrid force/position control method,[2] impedance control method [3] and intelligent control method.

The impedance control method does not directly control the contact force or the position/velocity but instead satisfies the desired motion characteristics by equating the robot end-operating force-position interactions to a spring-mass-damping model.[4] When a dual-arm robot performs a coordinated operation task, two robotic arms grip the same object and they form a closed-chain motion mechanism.[5] The impedance control of the closed-chain system takes into consideration the distribution of internal forces based on the single-arm impedance control. When controlling a dual-arm robot, the relative error between the arms and the absolute error of the individual arms should be considered. Otherwise, the

<sup>\*</sup>Corresponding author.

superimposed error of the two arms will accumulate and then seriously affect the performance of the operation.[6] The problem of error compensation during the flexible operation of a dual-arm robot has yet to be addressed in the existing literature. However, this issue has an essential role in reducing contact force error and ensuring the safety of the operated object.

This paper proposes a coordinated impedance control method in cartesian space, aiming at the problem of the high requirements for the coordination of the position and force of the arms when a dual-arm robot operates an object. Compared to the traditional impedance control strategy, the proposed method reduces the contact force error between the robotic manipulator and the object, improving the robot's performance when manipulating the object with the dual arm.

#### 2. System Model

The dual-arm robot platform used in this research is shown in Fig. 1.

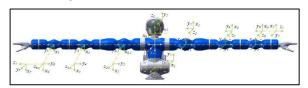


Fig. 1. Redundant dual-arm robot platform

The robot is designed based on humanoid ideas, and each robotics manipulator contains seven rotating joints in a spherical-roll-spherical configuration.

### 2.1. Redundant dual-arm robot model

The dynamics model of the redundant dual-arm robot is shown below:

$$M_D(\theta)\ddot{\theta} + C_D(\theta,\dot{\theta})\dot{\theta} + G_D(\theta) = \tau_a - J_D^T F_D$$
 (1)

where,  $\tau_a \in R^{2n}$  is the joint torque;  $F_D \in R^{2m}$  is the contact force from the external environment;  $\theta \in R^{2n}$  is the joint angle;  $J_D \in R^{2m \times 2n}$  is the Jacobi matrix.

The kinematic model of the redundant dual-arm robot is shown below:

$$\begin{cases} \dot{\chi}_D = J_D(\theta)\dot{\theta} \\ \ddot{\chi}_D = J_D(\theta)\ddot{\theta} + \dot{J}_D(\theta,\dot{\theta})\dot{\theta} \end{cases}$$
(2)

where,  $\chi_D \in R^{2m}$  is the position and orientation of the dual-arm and satisfies  $\chi_D = [\chi_R^T \quad \chi_L^T]^T$ .

### 2.2. Dual-arm manipulation model

The schematic diagram of a robot manipulating an object with dual-arm is shown in Fig. 2.

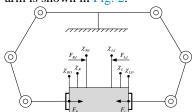


Fig. 2. The analysis of a dual-arm robot manipulating an object

Assume that the two robotic manipulators clamp the object separately and that the end-effectors of the two arms have no relative moving with the object. The mapping between the velocity of the object and the velocity of the end-effector is as follows:

$$\dot{\chi}_D = J_{DO} \dot{y} \tag{3}$$

where,  $J_{DO} \in R^{2m \times p}$  is the grasping matrix for the dualarm.  $y \in R^p$  is the position and orientation of the object. The complete kinematic relationship is as follows:

$$\dot{y} = J_{DO}^{\dagger} J_{D} \dot{\theta} \tag{4}$$

The dynamic equation of the manipulated object is

$$M_{Y}(y)\ddot{y} + C_{Y}(y,\dot{y})\dot{y} + G_{Y}(y)y = F_{Y}$$
 (5)

where,  $F_{\gamma}$  is the combined force that the two robotics manipulators acting on the centroid of the object.

The mapping between the combined force on the centroid of the object and the force at the end of the robotics manipulator is as follows:

$$F_{Y} = J_{DO}^{T} F_{D} \tag{6}$$

## 3. Coordinated Impedance Controller Design

## 3.1. Cartesian space impedance controller

The cartesian space impedance control is mainly to solve the problem that when the robotic arm is disturbed by the environment, the impedance relationship can still be achieved to ensure the flexibility of the system. The expression of the general impedance model is

$$M\,\tilde{\ddot{\chi}} + B\,\tilde{\dot{\chi}} + K\,\tilde{\chi} = F\tag{7}$$

where, M is the expected inertia matrix; B is the expected damping matrix; K is the expected stiffness matrix; F is the contact force.  $\tilde{\chi} \square \chi - \chi_d$  is the position error of end effector.

When grasping an object with dual arms, assume that the outputs of the Cartesian space impedance controller are  $F_{RI}$  and  $F_{LI}$ , and they satisfy:

$$\begin{cases} F_{RI} = F_R - F_{Rd} \\ F_{LI} = F_L - F_{Ld} \end{cases}$$
 (8)

Define the output of impedance control as  $\chi_{RI}$  and  $\chi_{LI}$ , respectively, and the following relationship can be obtained.

$$\begin{cases}
M_R \tilde{\ddot{\chi}}_{RI} + B_R \tilde{\dot{\chi}}_{RI} + K_R \tilde{\chi}_{RI} = F_{RI} \\
M_L \tilde{\ddot{\chi}}_{LI} + B_L \tilde{\dot{\chi}}_{LI} + K_L \tilde{\chi}_{LI} = F_{LI}
\end{cases} \tag{9}$$

The tracking trajectory of the general cartesian space impedance control can be obtained as

$$\begin{cases}
\chi_R = \chi_{RI} + \chi_{Rd} \\
\chi_L = \chi_{LI} + \chi_{Ld}
\end{cases}$$
(10)

When the impedance-based control method is used to achieve dual-arm flexible control, the position accuracy of the end-effector is difficult to guarantee, making it more challenging to achieve dual-arm coordination.

### 3.2. Definition of relative error

The difference between each arm's actual and desired pose is defined as the absolute error.

$$\begin{cases}
e_R = \chi_{Rd} - \chi_R \\
e_L = \chi_{Ld} - \chi_L
\end{cases}$$
(11)

Define the product of each arm's absolute error and the corresponding ratio factor as the relative error.

$$\begin{cases} e_{Rk} \square k_{Rk} (e_R - e_L) \\ e_{Lk} \square k_{Lk} (e_R - e_L) \end{cases}$$
 (12)

After coordinated control, the end pose of the robotics manipulators satisfies the following relationship.

$$\begin{cases}
\chi_{Rk} = \chi_R + e_R + e_{Rk} \\
\chi_{Lk} = \chi_L + e_L + e_{Lk}
\end{cases}$$
(13)

The controller must compensate for the relative errors for the robot to achieve dual-arm coordination.

## 3.3. Coordinated impedance controller

First, based on the desired trajectory of the object and the external disturbance of the robotics manipulator, we can obtain the impedance control acceleration as

$$\begin{cases}
\ddot{\chi}_{RI} = M_R^{-1} \left( F_{RI} - B_R \tilde{\chi}_{RI} - K_R \tilde{\chi}_{RI} \right) + \ddot{\chi}_{Rd} \\
\ddot{\chi}_{LI} = M_L^{-1} \left( F_{LI} - B_L \tilde{\chi}_{LI} - K_L \tilde{\chi}_{LI} \right) + \ddot{\chi}_{Ld}
\end{cases} (14)$$

where,  $M_R, B_R, K_R \in R^{m \times m}$  and  $M_L, B_L, K_L \in R^{m \times m}$  are the inertia, damping and stiffness of the Cartesian space desired by the right arm and left arm, respectively.

Then, integrating the above equation yields the output pose of the impedance controller. The desired output of the coordinated impedance controller is as follows.

$$\begin{cases} \chi_{RId} = \chi_{RI} + {}_{L}^{R}T\chi_{LI} \\ \chi_{LId} = \chi_{LI} + {}_{L}^{R}T\chi_{RI} \end{cases}$$
 (15)

After considering the relative error, the desired output of the coordinated impedance controller is as follows.

$$\begin{cases} \chi_{Rk} = e_{Rk} + k_{RI} \chi_{RId} \\ \chi_{Ik} = e_{Ik} + k_{IJ} \chi_{IId} \end{cases}$$
 (16)

By combining the kinematic equations of the two arms, the desired acceleration can be obtained as

$$\ddot{\theta}_{d} = J_{D}^{-1} \left[ \ddot{\chi}_{Dk} - \dot{J}_{D}^{-1} \dot{\theta} \right] + N_{D}^{T} \lambda \tag{17}$$

where,  $\ddot{\mathcal{Z}}_{Dk} = [\ddot{\mathcal{Z}}_{Rk}^T \quad \ddot{\mathcal{Z}}_{Lk}^T]^T$ ;  $N_D$  is the null space matrix for  $J_D$ .  $\lambda$  represents an optional vector.

Finally, combining the above equation with the dynamics model yields the desired output torque as

$$\tau_D = M_D J_D^{-1} \left[ \ddot{\chi}_{Dk} - \dot{J}_D^{-1} \dot{\theta} \right] + M_D N_D^T \lambda - J_D^T F_D \quad (18)$$

## 4. Simulation Analysis

This section verified the proposed algorithm using Adams and Matlab simulation platforms. Meanwhile, to demonstrate the superiority of the proposed method, general and coordinated impedance control methods were used in the same task, respectively. The simulation process is that the dual-arm robot holds an object for motion, and the simulation model is shown in Fig. 3.

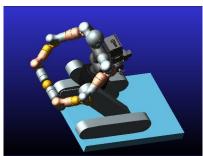


Fig. 3. Coordinated operation of the dual-arm robot

The output force of the robotics manipulators and the total force acting on the object under different controllers are shown in Fig. 4 and Fig. 5.

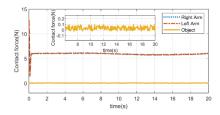


Fig. 4. Variation of contact force based on Cartesian space impedance control method

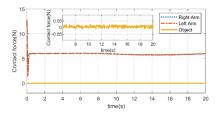


Fig. 5. Variation of contact force based on Cartesian space coordinated impedance control method

Comparing Fig. 4 and Fig. 5, it can be found that the coordination between the arms is significantly improved by using coordinated impedance control, and the fluctuation of the combined force on the object is significantly reduced, which achieves the purpose of regulating the contact force of the object.

### 5. Conclusion

Aiming at the coordination problem during manipulation of objects by a dual-arm robot, this article proposes a coordinated impedance control method. The contact force error between the dual-arm is compensated by introducing the relative error, which reduces the internal force on the object and improves the system's safety.

### Acknowledgements

This work was supported in part by the NSFC under Grant 62133001, and Grant 61520106010, and in part by the National Basic Research Program of China (973 Program) under Grant 2012CB821200 and Grant 2012CB821201.

# References

- C. Smith, et al., "Dual arm manipulation—A survey," Robot. Auton. Syst., vol. 60, no. 10, pp. 1340-1353, 2012.
- C. Chen, Z. Liu, Y. Zhang and S. Xie, "Coordinated Motion/Force Control of Multiarm Robot with Unknown Sensor Nonlinearity and Manipulated Object's Uncertaint-

- y," IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 47, no. 7, pp. 1123-1134, July, 2017.
- 3. Z. Li, C. Xu, Q. Wei, C. Shi and C. Su, "Human-Inspired Control of Dual-Arm Exoskeleton Robots with Force and Impedance Adaptation," IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 50, no. 12, pp. 5296-5305, December, 2020.
- G. Xiong, Y. Zhou and J. Yao, "Null-space impedance control of 7-degree-of-freedom redundant manipulators based on the arm angles," Int. J. Adv. Robot. Syst., vol. 17, no. 3, pp. 255688463, May, 2020.
- J. Lee, P.H. Chang and R.S. Jamisola, "Relative Impedance Control for Dual-Arm Robots Performing Asymmetric Bimanual Tasks," IEEE Trans. Ind. Electron., vol. 61, no. 7, pp. 3786-3796, July, 2014.
- D. Jinjun, G. Yahui, C. Ming and D. Xianzhong, "Symmetrical adaptive variable admittance control for position/force tracking of dual-arm cooperative manipulators with unknown trajectory deviations," Robot. Comput.-Integr. Manuf., vol. 57, pp. 357-369, 2019.

### **Authors Introduction**

Mr. Yang Zhang



He received the B.S degree in aircraft design and engineering from Northwestern Polytechnical University, Xian, China, in 2016, and the M.S degree in Aerospace Engineering from National University of Defense Technology, Changsha, China, in 2019. He is currently working toward a Ph.D. with the School of Energy and Power Engineering,

Beihang University. His main research focuses on motion planning and control of collaborative robot, robust and nonlinear control.

Prof. Yingmin Jia



He received the B.S. degree in control theory from Shandong University, China, in 1982, and the M.S. and PhD degrees both in control theory and applications from Beihang University, China, in 1990 and 1993, respectively. Then, he joined the Seventh Research Division at Beihang University where he

is currently Professor of automatic control. His current research interests include robust control, adaptive control and intelligent control, and their applicat-ions in robot systems and distributed parameter systems.