Predictive Functional Controller Design with Disturbance Observer and Its Application

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Abstract

This paper discusses a predictive functional controller (PFC) design using a disturbance observer for systems with a dead time. The PFC is a kind of model predictive controller that is an effective method for systems with a dead time. However, in the PFC, the control performance is strongly affected by the accuracy of the designed model. Therefore, this research proposes a method that the PFC maintains good control performance under the modeling error by using the disturbance observer that can estimate the disturbance even if a system has dead time. The effectiveness of the proposed control scheme is evaluated by experimental results.

Keywords: Predictive Functional Control (PFC), Disturbance Observer, Dead Time, Model Predictive Control (MPC)

1. Introduction

Model Predictive Control (MPC) [1], [2] is an effective control method for systems with a dead time. The MPC requires to solve the optimization problem online for each sampling period. So, the application of the MPC is costly. Richalet proposed predictive functional control (PFC) [3], [4] as one of the simplified MPCs**.** The PFC is effective and easy to implement for systems with a dead time. However, the accuracy of the designed model strongly affects the control performance of the PFC. It is difficult to match the designed model perfectly when the PFC is applied to a real system. Therefore, A mechanism to suppress these modeling errors is necessary. A method to suppress modeling errors using a disturbance observer (DOB) has been proposed [5]. However, the existing method using a DOB is ineffective for systems with a dead time. Therefore, this paper discusses an effective disturbance estimation method for systems with a dead time using a DOB and verifies the effectiveness of the method with experiments.

2. Predictive Functional Controller

The PFC applies only to linear time-invariant systems. Low computational cost due to methods that do not require solving optimization problems online. The basic algorithm of the PFC is explained below.

2.1. Control law derivation for PFC

Let T_s is the sampling time, and k is the arbitrary step time. The control target of a first-order system is represented by the following state-space as follows:

$$\begin{aligned}\boldsymbol{x_m}(k+1) &= \boldsymbol{A_m}x(k) + \boldsymbol{B_m}u(k)\\ \boldsymbol{y_m}(k) &= \boldsymbol{C_m}\boldsymbol{x_m}(k)\end{aligned}. \tag{1}$$

Where $\boldsymbol{x_m} \in R^n$ is the state variable vector, $u \in R$ is the control input, and $y_m \in R$ is the model output. The reference trajectory output $y_r(k+i|k)$ at time k is defined as follows:

$$y_r(k+i|k) \coloneqq r(k+i|k) - \lambda^i\{r(k) - y(k)\}. \tag{2}$$

Where $r(k)$ is the target value and $y(k)$ is the system output. where λ is the attenuation rate of the reference trajectory output and can be expressed as follows:

$$\lambda = e^{-\frac{3T_s}{t_{TRBF}}}. \tag{3}$$

t_{TRBF} is called the closed-loop response time taking the response to reach 95% of the target value, and is the main adjustment parameter of the PFC. Define an evaluation function based on the difference between the incremental reference trajectory output $\Delta y_r(k+h_j)$ and the incremental model output $\Delta y_m(k+h_j)$ at the matching point, expressed as follows:

$$J(k) := \sum_{j=1}^{n_h} \left\{ \Delta y_m(k+h_j) - \Delta y_r(k+h_j) \right\}^2. \quad (4)$$

Where $h_j(j = 1,2,\ldots,n_h)$ are the sample times of the matched points, and $n_h(n_h \geq 1)$ is the number of matched points. The increment of the reference trajectory at the coincidence point, $\Delta y(k+h_j)$, can be expressed from Eq. (2) as follows:

$$\Delta y_r(k+h_j) = [r(k) - y(k)](1 - \lambda^{h_j}) \quad (5)$$

The PFC expresses the control input $u(k)$, computed at each sampling time, as a weighted sum of polynomial basis functions (step, ramp, parabolic signal, etc.). The control input is expressed as follows:

$$u(k+i) = \sum_{l=1}^{n_B} \mu_l(k) i^{l-1} \quad (6)$$

Where $\mu_l(k)(l = 1,2,\ldots,n_b)$ are unknown weight coefficients and are optimized so that Eq. (4) is minimized. Only the step term has a value at time k among the two-term basis functions. Therefore, only u(k) is input to the control object and is expressed as follows:

$$u(k) = \sum_{l=1}^{n_B} \mu_l(k) := \boldsymbol{U_b}(0)^T \boldsymbol{\mu}(k) \quad (7)$$

Where $\boldsymbol{U_b}(i)$ and $\boldsymbol{\mu}(k)$ are expressed as follows:

$$\begin{cases} \boldsymbol{U_b}(i) = [1 \quad i \quad \cdots \quad i^{n_B-1}]^T \\ \boldsymbol{\mu}(k) = [\mu_1(k) \quad \mu_2(k) \quad \cdots \quad \mu_{n_B}(k)]^T \end{cases} \quad (8)$$

In the PFC, the mod output $y_m(k+h_j)$ at the coincident point is expressed sum of the free response $y_l(k+h_j)$ the forced response $y_f(k+h_j)$. The free-response $y_l(k+h_j)$ is expressed as follows from the state-space representation of Eq. (1).

$$y_l(k+h_j) = \boldsymbol{C_m A_m^{h_j} x_m}(k) \quad (9)$$

Also, the forced response $y_f(k+h_j)$ is expressed as follows:

$$y_f(k+h_j) = \sum_{l=1}^{n_B} \mu_l(k) y_{b_l}(i) \quad (10)$$

Where $y_{b_l}(i)$ is the response of the system to the base input $u_{b_l}(i) := i^{l-1}$. From Eqs. (1), (9), and (10), the incremental model output $\Delta y_m(k+h_j)$at the matching point is expressed as follows:

$$\Delta y_m(k+h_j) = \boldsymbol{C_m}\left(\boldsymbol{A_m^{h_j}} - I\right)\boldsymbol{x_m}(k) + \sum_{l=1}^{n_B} \mu_l(k) y_{b_l}(i) \quad (11)$$

Where $\boldsymbol{y_b}(h_j)$ is expressed as follows:

$$\boldsymbol{y_b}(h_j) = [y_{b_1}(h_j) \quad y_{b_2}(h_j) \quad \cdots \quad y_{b_{nB}}(h_j)]^T \quad (12)$$

The evaluation function in Eq. (4) is expressed using Eqs. (5), (11), and (12) as follows.

$$J(k) = \sum_{j=1}^{n_h} \left\{ \begin{matrix} \boldsymbol{y_b}(h_j)^{\boldsymbol{T}} \boldsymbol{\mu}(k) + \boldsymbol{C_m}\left(\boldsymbol{A_m^{h_j}} - I\right)\boldsymbol{x_m}(k) \\ +(\lambda^{h_j} - 1)(r(k) - y(k)) \end{matrix} \right\} \quad (13)$$

The unknown weight coefficient vector $\boldsymbol{\mu}(k)$ that minimizes the evaluation function in Eq. (13) is obtained from $\partial J(k)/\partial \mu(k) = 0$ as follows:

$$\boldsymbol{\mu}(k) = -\left\{ \sum_{j=1}^{n_h} \boldsymbol{y_b}(h_j) \boldsymbol{y_b}(h_j)^{\boldsymbol{T}} \right\}^{-1}$$

$$\sum_{j=1}^{n_h} \left\{ C_m\left(A_m^{h_j} - I\right) x_m(k) + (\lambda^{h_j} - 1)(r(k) - y(k)) \right\} \quad (14)$$

where the vector v is defined as follows:

$$\boldsymbol{v} := \boldsymbol{y_b}(h_j) \left\{ \sum_{j=1}^{n_h} \boldsymbol{y_b}(h_j) \boldsymbol{y_b}(h_j)^{\boldsymbol{T}} \right\}^{-1} \boldsymbol{U_b}(0) \quad (15)$$

Also, using Eq. (15), the constants $k_0$and the constant vector $\boldsymbol{v_x}$ are defined as follows:

$$\begin{cases} k_0 = \boldsymbol{v}^T [1 - \lambda^{h_1} \quad \cdots \quad 1 - \lambda^{h_{n_h}}]^T \\ \boldsymbol{v_x} = -\left[C_m(A_m^{h_1} - I) \quad \cdots \quad C_m\left(A_m^{h_j} - I\right)\right]^T \boldsymbol{v} \end{cases} \quad (16)$$

Using Eqs. (7), (14)-(16), the optimal control input is expressed as follows:

$$u(k) = k_0\{r(k) - y(k)\} + \boldsymbol{v_x^T x_m}(k) \quad (17)$$

Where in Eq. (17), the constants k_0 and the constant vector can $\boldsymbol{v_x}$can be computed offline. Therefore, the online calculations at each sampling time can be performed by taking the difference between the target value and the control output and the state variables of the model and multiplying them by a constant or a vector of constants.

2.2. A dead time compensation for the PFC

Let L is the dead time of the control target. The internal model expressed in Eq. (1) has a form that does not include a dead time. Therefore, the control output $y(k+L)$ after a dead time can be expressed using the model output as follows:

$$y(k+L) = y(k) + y_m(k) + y_m(k-L) \quad (18)$$

Let $\hat{y}(k+L|k) := y(k+L)$ be the predicted value, the control input when the control target contains wasted time is as follows:

$$u(k) = k_0\{r(k) - \hat{y}(k+L|k)\} + \boldsymbol{v_x^T x_m}(k) \quad (19)$$

3. Modeling error compensation using DOB

In the PFC, the control performance is strongly affected by the accuracy of the designed model. However, it is difficult to prepare a complete model of an actual control target in advance. Therefore, this paper considers a method to reduce the effects of modeling errors by using a DOB.

3.1. Basic structure of DOB

A compensator with a DOB reduces the effect of modeling error by taking the effect of modeling error as a disturbance and canceling it out. The basic DOB structure is shown in Fig.1. Where $G(s)$ is the transfer function of a controlled object and $G^{-1}(s)$ is the inverse function of the transfer function of the controlled object. To make the observer proper, a low-pass filter $F(s)$ must be used in combination. In this paper, $F(s)$ is designed as a multistage coupling of a first-order system expressed as follows:

$$F(s) = \left(\frac{\omega_c}{s + \omega_c}\right)^{n_s} \tag{20}$$

When the control target includes the wasted time as shown in Fig.1, the estimated input $\hat{U}(s)$ obtained by applying the system output $Y(s)$ to $G^{-1}(s)$ can be expressed as follows:

$$\hat{U}(s) = (U(s) + d)e^{-Ls} \tag{21}$$

When the control input $U(s)$ is subtracted from Eq. (21), the estimated disturbance $\hat{d}$ is expressed as follows:

$$\hat{d} = U(s)(1 - e^{-Ls}) + de^{-Ls} \tag{22}$$

Eq. (22) indicates that the disturbance cannot be estimated because of the dead time.

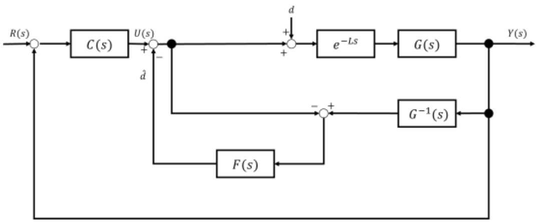

Fig. 1. Basic structure of DOB

3.2. DOB considering a dead time

The structure of the proposed DOB considering the dead time is shown in Fig.2. For the structure shown in Fig. 2, the estimated disturbance $\hat{d}$ is expressed as follows:

$$\hat{d} = U(s)\left(e^{-\hat{L}s} - e^{-Ls}\right) + de^{-Ls} \tag{23}$$

Where $\hat{L}$ represents the estimated dead time of the system. Since $\hat{L} = L$ when a dead time is known, Eq. (23) is expressed as follows:

$$\hat{d} = de^{-Ls} \tag{24}$$

Eq. (24) shows that the proposed DOB configuration is capable of suppressing disturbances in the situation where the system does not have a large amount of dead time.

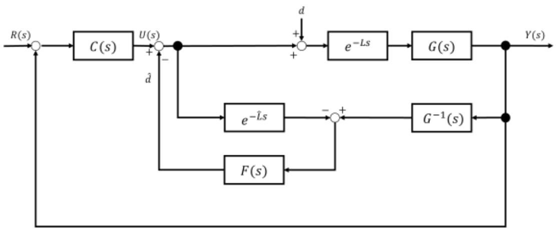

Fig. 2. DOB considering a dead time

4. Numerical example and experimental

The effectiveness of the proposed method is verified by numerical examples and actual experiments. The temperature experiment apparatus used is shown in Fig.3. Approximating the target as a first order + dead time system, the system was identified and $K = 4.27$, $T = 37.6$, $L = 2$ were obtained.

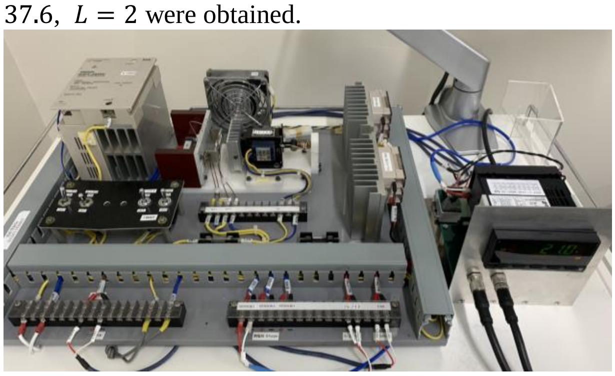

Fig. 3. Temperature experimental device

4.1. Simulation

A numerical example is performed using the obtained model. The system is a first order delay + dead time system with $K \in [2,5], T \in [30,50], L = 2$. Gain and time constants were prepared using random numbers within a range. Conditions for numerical examples are shown in Table 1. The results of applying only the PFC are shown in Fig.4, applying the DOB for the basic structure is shown in Fig.5, and the DOB for the proposed structure is shown in Fig.6. Ten trials were used for the display. When only the PFC is used, the control performance varies due to modeling errors. The response of the basic structure is oscillating when a dead time is included. However, the proposed structure of the DOB appropriately reduces the influence of modeling error and improves controllability.

Table 1. Numerical example conditions

t_{TRBF}	h_1	h_2	h_3	ω_c	u_{max}	u_{min}
30	10	15	30	5	100	0

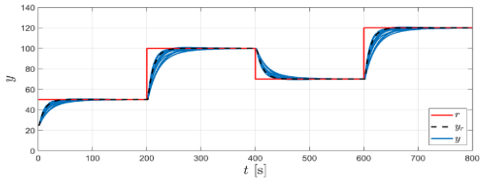

Fig. 4. Simulation result for the PFC

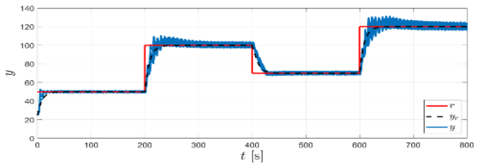

Fig. 5. Simulation result for Basic DOB

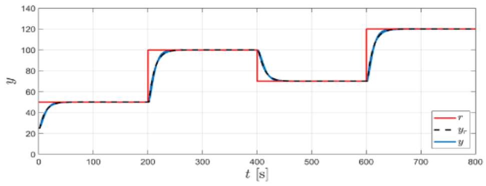

Fig. 6 Simulation result for proposed DOB

4.2. Experimental

The effectiveness of the proposed method is verified by the experiments shown in Fig.3. Experimental conditions are the same as in Table 1. The results of applying only the PFC are shown in Fig.7, and the results of applying the DOB for the proposed structure are shown in Fig.8. The results confirm the effectiveness of the proposed method through experiments.

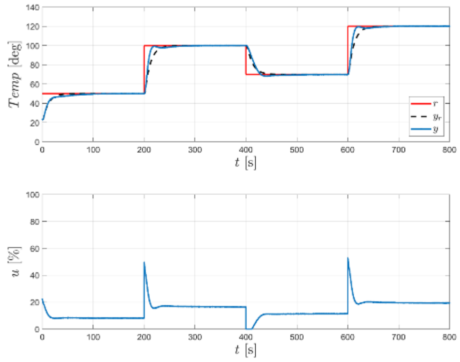

Fig. 7. Experimental result for the PFC

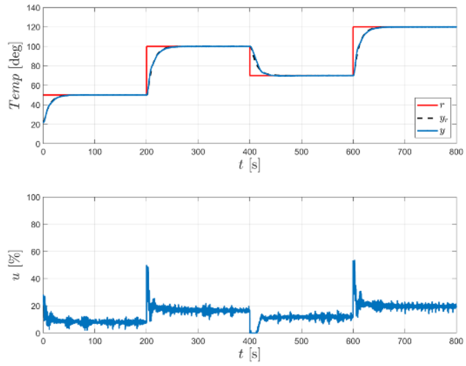

Fig. 8. Experimental result for proposed DOB

5. Conclusion

In this paper, the Eq using the DOB considering the dead time was proposed. The effectiveness of the proposed method was demonstrated through numerical examples and experiments for a system with a dead time. It was confirmed that the proposed structure does not cause oscillations in the response of the system even for systems with a dead time and reduces the influence of modeling errors. In the future, consider designing a data-driven observers that can be applied to larger environmental changes.

References

1. J. M. Maciejowski: *Predictive Control with Constraints*, Prentice-Hall（2002）
2. E. F. Camacho and C. Bordons: *Model Predictive Control (Second Edition)*, Springer-Verlag (2007)

3. J. Richalet, S. Abu el Ata-Doss, C. Arber, H.B. Kuntze, A. Jacubasch and W. Schill: Predictive functional control: application to fast and accurate robot, *Proc. of IFAC 10th World Congress*, 251/258 (1987)
4. J. Richalet, D. O' Donovan: *Predictive Functional Control*, Springer, (2009)
5. T. Sato, K. Kaneko, N. Saito: Improvement of Tracking Performance in Predictive Functional Control Using Disturbance Observer, *Transactions of the Society of Instrument and Control Engineers*, vol. 10, no. 15, pp. 132-139, (2011) (in Japanese)

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