An Improved Landweber Method for Electrical Impedance Tomography

Qian Wang

School of Electronic Information and Automation, Tianjin University of Science and Technology, No. 1038 Dagu Nanlu, Hexi District, Tianjin, China

Xiaoyan Chen

School of Electronic Information and Automation, Tianjin University of Science and Technology, No. 1038 Dagu Nanlu, Hexi District, Tianjin, China

E-mail: 1248265505@qq.com, cxywxr@tust.edu.cn

www.tust.edu.cn

Abstract

Electrical impedance tomography (EIT) is a non-destructive monitoring technique. The inverse problem of EIT shows serious nonlinear and ill-posed nature, which leads to the low spatial resolution of the reconstructed images. The iterative idea is an effective method to deal with the inverse problem for imaging, the existing iterative imaging methods, however, have many drawbacks, such as needing huge computational resources and unstable convergence properties. To solve the above-mentioned problems, this paper proposes an improved Landweber iterative image reconstruction framework. By carefully designing a regularization term in the optimization function of the iterative framework, this method improves the convergence speed. The tank experiment results show that the improved Landweber method is superior to the traditional Landweber method in convergence speed.

Keywords: EIT, Landweber, improved Landweber, regularization term

1. Introduction

Electrical Impedance Tomography (EIT) is a new imaging technique that has been developed in recent years. The alternating current with safe amplitude is injected into the observation domain, and then the boundary voltage responded to the surface is measured through electrodes placed on the surface of the object. The conductivity is calculated for each pixel inside the field to show the distribution of the different media. Some traditional imaging modalities, such as computed tomography (CT) and magnetic resonance imaging (MRI), have some limitations, for example, ionizing radiation, high price, and inconvenience to patient movement. These factors make it limited in practical clinical applications and prevent the implementation of long-term clinical monitoring. In contrast, EIT has the advantages of low cost, high temporal resolution, portability, and no radiation, which can effectively compensate for the shortcomings of traditional medical imaging modalities, and therefore has become an important research direction in the medical field with broad application prospects.

Currently, the core problem of solving the inverse problem of EIT, i.e., image reconstruction, is to construct a suitable image reconstruction algorithm to obtain an accurate image to describe the conductivity parameters and boundary features. In order to solve this problem, many image reconstruction methods have been studied. The image reconstruction methods researched over world are mainly divided into non-iterative and iterative categories. Among them, non-iterative methods have linear back projection (LBP)[1] and Tikhonov regularization[2], etc. The iterative methods have the conjugate gradient method[3] and Landweber method[4], etc. The LBP method is characterized by simple structure and fast imaging, but its spatial resolution is poor. It is suitable for online and fast qualitative imaging, but cannot provide accurate quantitative information. The regularization method is an effective method to overcome the ill-posed characteristic of EIT inverse problem. Nevertheless, the selection of the parameters and patterns of the penalty function is more complex, and it is usually based on an empirical setting. So, these methods have great limitations. The conjugate gradient method (CG) requires the system matrix which is a strict square matrix, and the EIT system matrix is an approximate result, so this approximation error will be magnified in the solution, and the reconstruction result contains artifacts, which cannot be applied to quantitative analysis. Landweber iterative method[5], based on the steepest descent principle, is the most commonly used iterative method for solving the problem of EIT ill-posed problems, but it requires multiple iterations and has a slow convergence speed.

Aiming at the problems of Landweber iterative algorithm requiring multiple iterations and slow convergence, this paper proposes an improved Landweber iterative algorithm. The method increases the convergence speed by adding regularization term to the objective function of EIT inverse problem. The experimental results of the tanks show that the improved Landweber iterative method outperforms the Landweber iterative method in terms of convergence speed.

2. Method

2.1. *Forward problem*

The construction of mathematical model is the basis of electrical impedance tomography reconstruction. The EIT simulation model measurement system is shown in Figure 1. This paper adopts the data acquisition method of adjacent current excitation and adjacent voltage measurement[6]. The specific acquisition method is to randomly select a pair of adjacent electrodes to inject current excitation as the initial excitation electrode, and then successively select two adjacent electrodes to measure the voltage as the measurement electrode. Using 16 pairs of adjacent electrodes as excitation electrodes in turn, 208 measurement data can be collected in total.

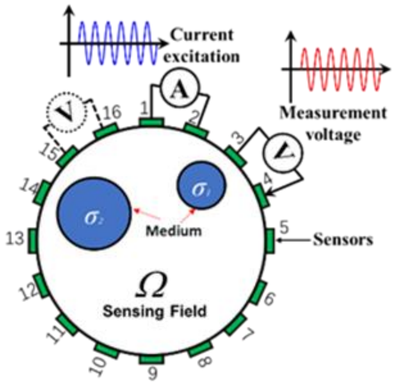

Fig. 1 EIT simulation model measurement system

In this paper, according to the complete electrode model (CEM)[7], the mathematical expression of the forward problem of CEM is as follows Eq. (1)

$$\begin{cases} \nabla\cdot(\sigma(x)\nabla\varphi(x)) = 0, x\in\Omega, \\ \varphi(x) = u(x), x\in\partial\Omega\setminus\bigcup_{L=1}^{16}e_L, \\ \sigma(x)\dfrac{\partial\varphi(x)}{\partial n} = 0, x\in\partial\Omega\setminus\bigcup_{L=1}^{16}e_L, \\ \int_{e_L}\sigma(x)\cdot\dfrac{\partial\varphi(x)}{\partial n}dS = I_L, x\in e_L, L=1,2,\cdots,16, \\ \varphi(x)+\rho_L\sigma(x)\cdot\dfrac{\partial\varphi(x)}{\partial n} = U_L, x\in e_L, L=1,2,\cdots,16. \end{cases} \tag{1}$$

where, $x\in\Omega$ is the spatial position, $\sigma(x)$ is the conductivity distribution in the measurement field, $\varphi(x)$ represents the potential distribution in the measurement field, Ω represents the measurement field, n represents the external normal vector on the boundary, $\partial\Omega$ represents the boundary information of the measurement area, and ρ_L represents the contact impedance between the electrode and the contact surface, U_L is the response voltage on the L-th electrode.

In order to solve the Laplace equation numerically with CEM boundary conditions, numerical techniques are needed. Generally, finite element method[8] is used to solve this problem. Assuming that the measurement noise is additive Gaussian noise, the observation model of EIT can be written as follows Eq. (2)

$$V = U(\sigma) + e \tag{2}$$

where, V is the vector of the measured voltage, $U(\sigma)$ is a forward solution based on the finite element method, and e is Gaussian noise.

2.2. *Inverse problem*

The mathematical model of EIT inverse problem can be expressed as Eq. (3)

$$y = Ax + b \tag{3}$$

where, y (matrix $y \in R^{m*1}$) represents the difference in boundary voltage values, and x (matrix $x \in R^{n*1}$) represents the difference in conductivity distribution. A is the sensitivity matrix, representing the mapping between conductivity change and boundary voltage. b represents additive noise.

Because the mathematical model of the EIT inverse problem is ill-conditioned, Eq. (2) cannot be solved uniquely and is usually calculated with the least square error. Namely Eq. (4)

$$\hat{x} = \arg\min_x \frac{1}{2}\|Ax - y\|_2^2 \tag{4}$$

2.3. *Landweber method*

Now we analyze the principle of Landweber method from a mathematical perspective[9]. Assume that matrix A_0 is the approximate matrix of matrix A^{-1}, and matrix P is the residual matrix of matrix A^{-1} and matrix A_0. Then there is formula Eq. (5)

$$P = I - A_0 A \tag{5}$$

Eq. (6) can be obtained from Eq. (5)

$$A^{-1} = (I - P)^{-1} A_0 \tag{6}$$

If the radius $\rho(P) < 1$ of the residual matrix P, it will be expanded. Step k is expanded as Eq. (7)

$$A_k = (I + P + P^2 + \cdots + P^{k-1}) A_0 \tag{7}$$

Because Eq. (8)

$$(I - P)(I + P + P^2 + \cdots + P^{k-1}) A_0 \tag{8}$$

According to Eq. (5), Eq. (6) and Eq. (8), we can get Eq. (9)

$$P^k A_0 = A_0 (I - A A_k) \tag{9}$$

If A_{k+1} is very close to A^{-1} in step k+1, we can get Eq. (10)

$$x_{k+1} = x_k + P^k A_0 y \tag{10}$$

Then by Eq. (9) and Eq. (10), we can get the iterative equation Eq. (11) of Landweber method

$$x_{k+1} = x_k + A_0 A^T (y - Ax_k) \tag{11}$$

In practical applications, the gain factor A_0 selection in the iterative method is very important because both the measurement voltage error and the bias brought by linearizing the nonlinear problem in the positive and inverse problems affect the final calculation results. In 2019, Han Guanghui demonstrated that the convergence of the function is faster and the imaging of the image is better when the gain factor is $A_0 = 2/(\lambda_{\max} + \lambda_{\min})$ (λ_{max} and λ_{min} are respectively the maximum and minimum eigenvalues of the matrix A^TA)[10].

2.4. *Improved Landweber method*

In order to improve the convergence speed of Landweber, a regularization term is added to the iteration Eq. (10) of Landweber method. The improved function is expressed as Eq. (12)

$$\hat{x} = \arg\min_x \frac{1}{2}\{\alpha\|Ax - y\|_2^2 + \lambda\|x_k - x_{k-1}\|^2\} \tag{12}$$

where, first item is the difference between the calculated voltage value and the measured voltage value. The second item is the difference between the conductivity distribution at the current time and the previous time. α is a gain factor. λ is a regularization term coefficient.
Combined with Eq. (11), the iterative formula of the improved Landweber iterative method proposed in this paper is as follows Eq. (13)

$$x_{k+1} = x_k + \alpha A^T (y - Ax_k) + \lambda\|x_k - x_{k-1}\|^2 \tag{13}$$

Step length in equation (11) α and λ It can be selected as a fixed value according to experience. Based on human experience $\alpha = 2/(\lambda_{\max} + \lambda_{\min})$ (λ_{max} and λ_{min} are the maximum and minimum eigenvalue of matrix A^TA respectively), λ= 0.6.

3. Experiment

3.1. *Evaluation metrics*

To quantitatively describe the imaging quality of the Landweber method and the improved Landweber method, the Relative Error (RE), Correlation Coefficient (CC) and Structure Similarity Index Measure (SSIM) of the reconstructed image are used as objective evaluation indicators. The calculation formula is as follows Eq. (14), Eq. (15) and Eq. (16).

$$RE = \frac{\left|x - \hat{x}\right|}{|x|} \tag{14}$$

$$CC = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(\hat{x}_i - \bar{\hat{x}})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(\hat{x}_i - \bar{\hat{x}})^2}} \tag{15}$$

$$SSIM = \frac{2 * \frac{1}{n}\sum_{i=1}^{n} x_i * \frac{1}{n}\sum_{i=1}^{n} \hat{x}_i * \mathrm{cov}(x, \hat{x})}{\left[\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2 + \left(\frac{1}{n}\sum_{i=1}^{n} \hat{x}_i\right)^2\right] * \mathrm{cov}^2(x, \hat{x})} \tag{16}$$

where, $\hat{x}$ represents the conductivity distribution of the reconstructed image, x represents the conductivity distribution of the real image, $\bar{\hat{x}}$ represents the mean value of the conductivity distribution of the reconstructed image, and $\bar{x}$ represents the mean value of the conductivity distribution of the real image.

3.2. *Experimental device*

The boundary voltage signal acquisition equipment used in this paper is composed of a high-precision and fast EIT data acquisition system (EIT-DAS), an image reconstruction system and a circular tank model. The schematic diagram[11] of the device is shown in Figure 2.

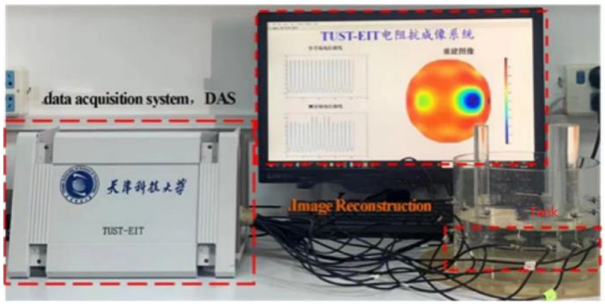

Fig. 2 TUST-EIT electrical impedance imaging System

The DAS data acquisition system consists of a host computer for data processing and image reconstruction, a data acquisition and signal processing module, and a tank with 16 titanium electrode sensors. A tank with a radius of 0.095m is used in the experiment, and 16 electrodes are distributed at the same interval and height on the tank wall. Uniform inclusions (glass rods) are placed in the tank, and the experimental data is obtained by changing the size and position of the inclusions. In the experiment, appropriate NaCl is added to tap water to obtain the background conductivity of the experiment (the conductivity is set to be about 1S/m), and the conductivity of the glass rod model can be approximately 0S/m. During the experiment, the current is excited through the hardware circuit. The amplitude of the exciting current is 4.5mA, and the frequency of the exciting current is 100kHz. Then the voltage between adjacent electrodes is measured to obtain the boundary voltage of the field.

3.3. *Tank Experiments*

In order to verify the performance of the improved Landweber method proposed in this paper, the Landweber method and the improved Landweber method will be used for image reconstruction. Four groups of tank experiments are set up in this paper. Tank 1 is inclusions with a radius of 0.02m and symmetrical position, tank 2 is inclusions with a radius of 0.015m and symmetrical position, tank 3 is inclusions with a radius of 0.02m and asymmetrical position, and tank 4 is inclusions with a radius of 0.015m and asymmetrical position. The image reconstruction results are shown in Figure 3。

As can be seen from Figure 3 and Table 1, when the boundary clarity, visualization effect and evaluation index values of reconstructed images are similar, iterations of the improved Landweber method decreased by more than 50% compared with the Landweber method. Therefore, the improved Landweber method proposed in this paper can greatly reduce iterations of image reconstruction and improve the speed of image

Table 1 Evaluation metrics and iterations of tank experiment

		Tank 1	Tank 2	Tank 3	Tank 4
Landweber	RE	0.4119	0.4331	0.3979	0.4115
	CC	0.6922	0.6137	0.7141	0.5849
	SSIM	0.6139	0.5379	0.6370	0.5229
	Iterations	**2500**	**3500**	**2400**	**2550**
Improved Landweber	RE	0.4107	0.4309	0.3965	0.4013
	CC	0.6944	0.6149	0.7173	0.7085
	SSIM	0.6162	0.5400	0.6399	0.6311
	Iterations	**1010**	**1420**	**1020**	**1020**

reconstruction.

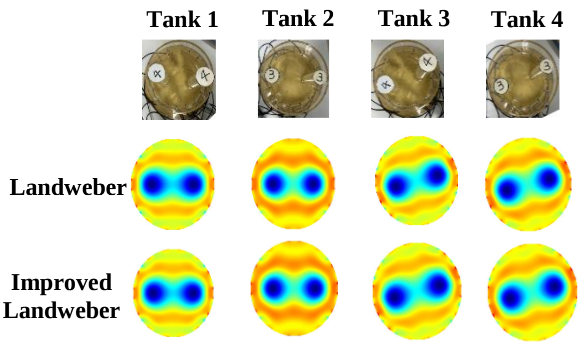

Fig. 3 Image reconstruction results of tank experiments

4. Conclusion

To solve the problem of the slow convergence speed of Landweber iterative method, an improved Landweber iterative reconstruction method is proposed. The method improves the convergence speed by adding regularization terms to the objective function of the inverse problem of EIT. From the results of the tank experiment, it can be seen that iterations of the Landweber method is more than double that of the improved Landweber method when the evaluation metrics of the reconstructed images are similar. In summary, the improved Landweber iterative method converges faster than the Landweber iterative method.

References

1. J. Solomon, D. Marinet, K. Roy Choudhury, et al., "Effect of radiation dose reduction and reconstruction method on image noise, contrast, resolution, and detectability of subtle hypoattenuating liver lesions at multidetector CT: filtered back projection versus a commercial model-based iterative reconstruction method," Radiology, vol. 283, no. 3, pp. 777–787, 2017.
2. Y. Xu, Y. Pei, and F. Dong, "An adaptive tikhonov regularization parameter choice method for electrical resistance tomography," Flow Measurement and Instrumentation, vol. 50, pp. 1–12, 2016.
3. L. Miao, Y. Ma and J. Wang, "ROI-Based Image Reconstruction of Electrical Impedance Tomography Used to Detect Regional Conductivity Variation," in IEEE Transactions on Instrumentation and Measurement, vol. 63, no. 12, pp. 2903-2910, Dec. 2014, doi: 10.1109/TIM.2014.2326765.
4. S. Liu, L. Fu and W. Yang. "Optimization of an iterative image reconstruction method for electrical capacitance tomography[J]," Measurement Science and Technology, vol. 10, no. 7, pp.37-39, 1999.
5. W. Yang, Spink D M, York T A, et al., "An image-reconstruction method based on Landweber's iteration method for electrical-capacitance tomography [J],"Measurement Science and Technology, vol. 10, no. 11, pp.1065-1069, 1999.
6. D. C. Barber and B. H. Brown, "Applied potential tomography," J. Physics E. Sci. instrum, Vo1.17, pp.723-733, 1984.
7. M. Cheney, D. Isaacson, E. J. Somersalo, et al., "Layer-stripping reconstruction method for impedance imaging," 1992 14th Annual International Conference of the IEEE Engineering in Medicine and Biology Society, pp.1694-1695, 1992, doi: 10.1109/IEMBS.1992.5761994.
8. P. V auhkonen, M. Vauhkonen, T. Savolainen, et al., "Three-dimensional electrical impedance tomography based on the complete electrode model," IEEE Trans. Biomed. Eng, vol. 46, no. 9, pp. 1150–1160, Sep. 1999.
9. D. Liu, D. Gu, D. Smyl, et al., "B-Spline Level Set Method for Shape Reconstruction in Electrical Impedance Tomography," in IEEE Transactions on Medical Imaging, vol. 39, no. 6, pp. 1917-1929, June 2020, doi: 10.1109/TMI.2019.2961938.
10. B. Grychtol and A. Adler, "FEM electrode refinement for electrical impedance tomography," 2013 35th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC), pp. 6429-6432, 2013,doi: 10.1109/EMBC.2013.6611026.
11. R. Fu, Z. Wang, X. Zhang, D. Wang, X. Chen and H. Wang, "A Regularization-Guided Deep Imaging Method for Electrical Impedance Tomography," in IEEE Sensors Journal, vol. 22, no. 9, pp. 8760-8771, 1 May1, 2022, doi: 10.1109/JSEN.2022.3161025.

Authors Introduction

Qian Wang

She (1248265505@qq.com) graduated from Tangshan University with a bachelor's degree in 2018. She is currently pursuing a master's degree in School of Electronic Information and Automation, Tianjin University of Science and Technology.

Her research interests include electrical tomography (especially clinical applications of electrical impedance tomography), numerical solutions of inverse problems, and finite/boundary element methods.

Xiaoyan Chen

She, professor of Tianjin University of Science and Technology, graduated from Tianjin University with PH.D (2009), worked as a Post-doctor at Tianjin University (2009.5-2015.5). She had been in RPI, USA with Dr. Johnathon from Sep.2009 to Feb.2010 and in Kent, UK with Yong Yan from Sep-Dec.2012. She has researched electrical impedance tomography technology in monitoring lung ventilation for many years. She is in charge of the TUST-EIT lab and guides young researchers and graduate students to improve the electrical data acquisition hardware platform, to study the traditional and novel reconstruction algorithms with the prior structural information. Recently, her research team is focus on the novel methods through deep learning network models.