Generation and Analysis of a Multi-scroll Conservative Chaotic System

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Abstract

Multi scroll conservative chaotic system is a new kind of chaotic system, which has attracted extensive attention due to its complex dynamic characteristics. In this paper, we study a conservative chaotic system and introduce a sine function without multiple angles to make the conservative chaotic system generate multiple scrolls, so as to construct a multiple scroll conservative chaotic system. The system generates one-dimensional linear multi roll and two-dimensional grid like multi roll distributions by adjusting nonlinear functions.

Keywords: Multi-Scroll, Conservative Chaotic System, Balance Point Analysis, Lyapunov Index

1. Introduction

In recent years, the research on multi-scroll chaotic attractors [1] has become an important branch of chaos research. The dynamic behavior of multi-scroll chaotic system is more complex than that of single-scroll and double-scroll chaotic system. At present, due to the shortcomings of the analysis method, the characteristics of the multi-scroll chaotic system is only partially analyzed. But in practice, the multi-scroll chaotic system has its special value, for example, it plays a great role in the fields of secure communication and fuzzy recognition. At the same time, because the chaos attractors generated by the multi-scroll chaotic system are usually obvious, the study of the multi-scroll chaotic system can deepen the understanding of the generation mechanism of chaos attractors in the multi-scroll chaotic system, and is also conducive to the development of new chaotic systems.

2. Generation and Analysis of a Multi-scroll Conservative Chaotic System

The multi-scroll chaotic system with more complex dynamic characteristics is more suitable for chaos security and chaos compression and other applications. Based on the conservative chaotic system, the non-double Angle sine function is used to generate multiple scrolls in the conservative chaotic system, and the dynamic characteristics of the multi-scroll attractor are analyzed by means of Lyapunov exponential diagram, bifurcation diagram and attractor phase diagram.

2.1. The emergence of multi-scroll chaotic systems

The multi-scroll conservative chaotic system is constructed, and the non-double Angle sine function is introduced into the equilibrium equation (1).

$$\begin{cases} 0 = ay + byw \\ 0 = -ax \\ 0 = cw \\ 0 = -bxy - cz \end{cases} \tag{1}$$

Set $\nabla H(x) = (\sin(x), y, z, w)$ to get the new system equation:

$$\begin{cases} \dot{x} = ay + byw \\ \dot{y} = -a\sin(x) \\ \dot{z} = cw \\ \dot{w} = -b\sin(x)y - cz \end{cases} \tag{2}$$

The divergence of system equation (2) is $\nabla \cdot f = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} = 0$, The divergence tells us that the system is still conservative.

Let $\dot{x} = 0, \dot{y} = 0, \dot{z} = 0, \dot{w} = 0$ to get equilibrium equation (3).

$$\begin{cases} 0 = ay + byw \\ 0 = -a\sin(x) \\ 0 = cw \\ 0 = -b\sin(x)y - cz \end{cases} \tag{3}$$

Its equilibrium points should be classified into two categories. The first category is $(2k\pi, 0,0,0)$ $(k\epsilon 1,2,3,\cdots)$, the second type is $((2k+1)\pi, 0,0,0)$ $(k\epsilon 0,1,2,3,\cdots)$, such equilibrium points are common in multi roll chaotic systems. Equation (4) is its Jacobian matrix [2].

$$J(x) = \begin{bmatrix} 0 & a+bw & 0 & by \\ -a\,cos(x) & 0 & 0 & 0 \\ 0 & 0 & 0 & c \\ -bcos(x)y & -bsin(x) & -c & 0 \end{bmatrix} \tag{4}$$

Order $J - \lambda E = 0$, when the balance point is the first type of balance point, $\lambda_1 = -ai, \lambda_2 = ai, \lambda_3 = -ci, \lambda_4 = +ci, a > 0, b > 0, c > 0$, such balance point is the center point; When the balance point is the second type of balance point, $\lambda_1 = a, \lambda_2 = -a, \lambda_3 = -ci, \lambda_4 = +ci, a > 0, b > 0, c > 0$, such equilibrium points are unstable saddle points [3].

2.2. Fundamental dynamic analysis of a multi-scroll chaotic system

Take an initial value of change as$[\pi/2 + 2k\pi, 1,1,1], k = (0,1,2)$. When k=0, the initial value is [π/2,1,1,1], and the parameters a=9, c=14, the Lyapunov exponent of system equation (2) is shown in Fig.1.

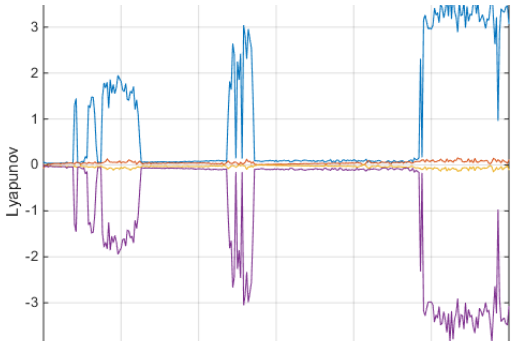

Fig.1. Laplace exponent diagram when a=9, c=14

Bifurcation Diagram Corresponding to Lyapunov Exponential Graph when Parameters a=9, c=14 is shown in Fig.2.

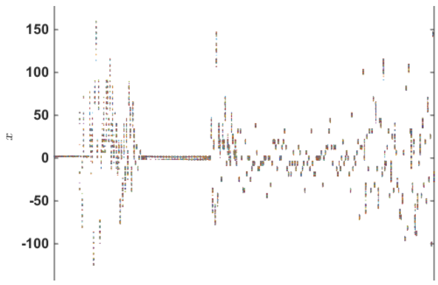

Fig.2. Bifurcation Diagram

When k=0, the initial value is [π/2,1,1,1], and the parameters a=4, c=4, the Lyapunov exponent of system equation (2) is shown in Fig.3.

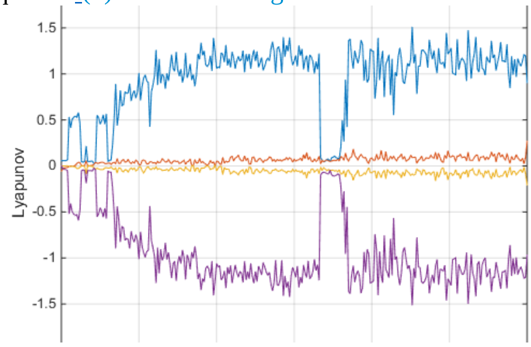

Fig.3. Laplace exponent diagram when a=4, c=4

Bifurcation Diagram Corresponding to Lyapunov Exponential Graph when Parameters a=4, c=4 is shown in Fig.4.

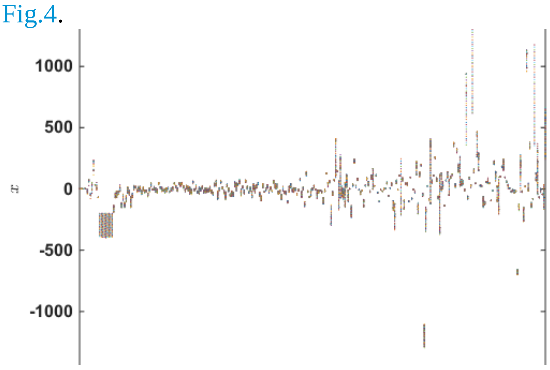

Fig.4. Bifurcation Diagram

By observing Lyapunov exponential graph and bifurcation diagram, it can be seen from system (2) that when parameters change, the distribution of Lyapunov exponential graph changes significantly. System (2) can generate chaos in significantly different parameter ranges.

2.3. A multi-scroll attractor and its stability analysis

The change of parameters obviously changes the interval in which the system can generate chaos. Here, the phase diagram of the system is observed by changing the initial value of the system, and the attractor characteristics of the system are analyzed by changing the phase diagram.

Take the system parameter $a = b = c = 4$, then only the initial value of the system is changed.

Set initial value $y_0 = z_0 = w_0 = 1$, and the Lyapunov exponent with initial value x is shown in Fig.5.

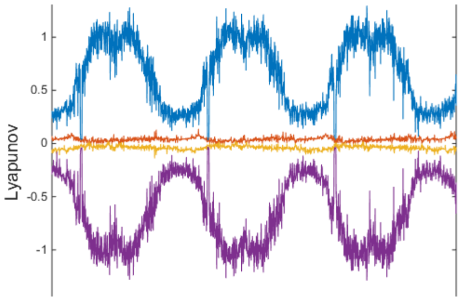

Fig.5. Laplace exponent diagram when a=4, c=4

Bifurcation Diagram Corresponding to Lyapunov Exponential Graph when Parameters a=4, c=4 is shown in Fig.6.

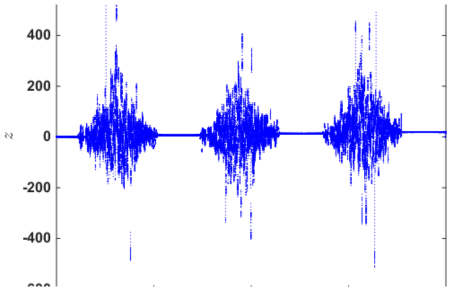

Fig.6. Bifurcation Diagram

By comparing the Lyapunov exponent diagram in Fig.5 and the bifurcation diagram in Fig.6, it can be seen that the range of chaos generated by the system is periodic. Within the range of $[0, 2\pi]$, the change of Lyapunov exponent within this range is similar to the image of a sine function. Then, taking 2π as the period, it can be seen that its image is the same as that within the range of $[0, 2\pi]$. That is, when the system changes with the initial value x, if $x = p, p \in (0,2\pi)$ is taken, then x can be taken as $p + 6k, k \in R^N$. The results obtained by the system can be the same.

When k=0, the initial value is [π/2,1,1,1], and the phase diagram generated by system equation (2) is shown in Fig.7.

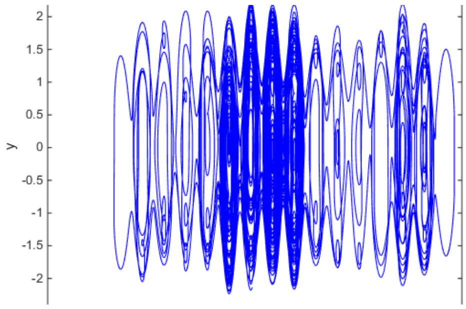

Fig.7. 16-scroll chaotic attractors

When k=1, the initial value is [5π/2,1,1,1], and the phase diagram generated by system equation (2) is shown in Fig.8.

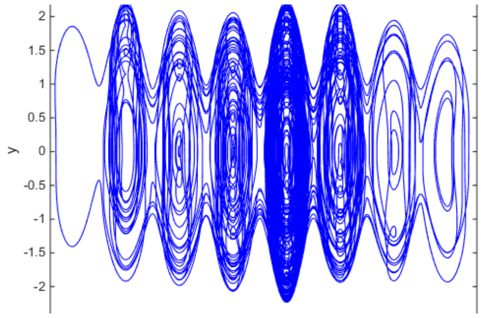

Fig.8. 8-scroll chaotic attractors

When k=2, the initial value is [9π/2,1,1,1], and the phase diagram generated by system equation (2) is shown in Fig.9.

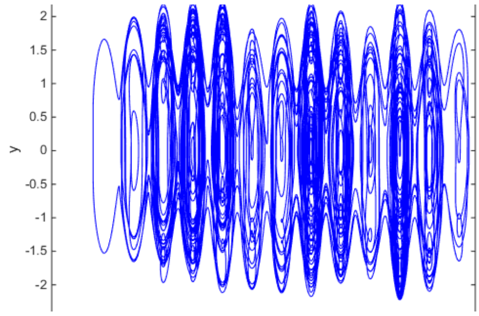

Fig.9. 13-scroll chaotic attractors

From Fig.7, Fig.8 and Fig.9, it can be seen that the number of scrolls in the x-y phase diagram varies significantly with the initial value, that is, the system has multiple stability, and the system is still a conservative system from the appearance. It can be seen from the multi roll phase diagram that the introduction of $\sin(x)$ leads to a one-dimensional multi roll chaotic attractor, which verifies the multi stability. Similarly, the introduction of angle free sine functions to y, z, w makes the system generate multi roll.

In Jacobian matrix (4), a two-dimensional nonlinear function $\nabla H(x) = (\sin(x), \sin(y), z, w)$ is introduced to generate multi scroll chaotic attractors

Take $a = b = c = 4$ to calculate the eigenvalues of different equilibrium points and analyze their characteristics. Characteristic value and type of balance point is shown in Table1.

Take $N = 2n\pi$ to limit the width of nonlinear function [4]. Equation (5) is the formula of nonlinear function.

Table 1. Characteristic value and type of balance point

balance point (x_0, y_0, z_0, w_0)	characteristic value $(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$	type
$(2k\pi, 2k\pi, 0,0)$	$(-4i, 4i, -4i, +4i)$	Center point
$(2k\pi, (2k+1)\pi, 0,0)$	$(4, -4, -4i, +4i)$	Unstable saddle point
$((2k+1)\pi, 2k\pi, 0,0)$	$(4, -4, -4i, +4i)$	Unstable saddle point
$((2k+1)\pi, (2k+1)\pi, 0,0)$	$(-4i, 4i, -4i, +4i)$	Center point

$$f(u) = \begin{cases} u + N, u < -N \\ \sin(u), -N \le u \le N \\ u - N, u > N \end{cases} \quad (5)$$

Lyapunov Exponential Graphs Varying with Initial Values is shown in Fig.10.

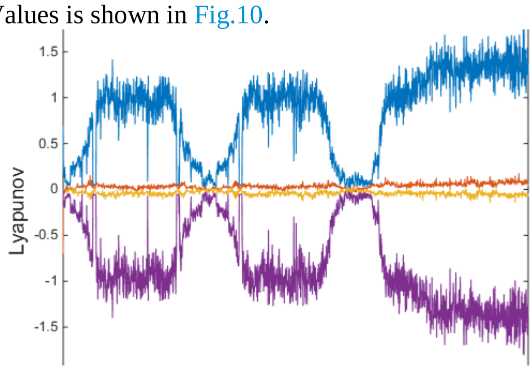

Fig.10. Laplace exponent diagram

In Fig.10, before $x = 4\pi$, the change rule of the Lyapunov exponent of the system that is greater than zero is still an approximate periodic change. After $x = 4\pi$, the change rule of the system has changed due to the use of two sine functions without multiple angles, breaking the previous change rule. It can be seen from the image that the Lyapunov exponent has a trend of gradual expansion after 4π.

The initial value is $[\pi/2, \pi/2, 1, 1]$, and the parameter is $a = b = c = 4$. We can get the two-dimensional multi roll attractor phase diagram, and the number of attractors is related to the value of n. If the number of scroll attractors generated by the system is M, then there is a relationship (6).

$$M = (2n)^2 + (2n+1)^2 \quad (6)$$

If the order of the generated two-dimensional multi scroll is R, the order relationship (7) can be obtained.

$$R = 4n + 1 \quad (7)$$

Taking $n = 1$, the phase diagram of the scroll attractor in Fig.11 is obtained.

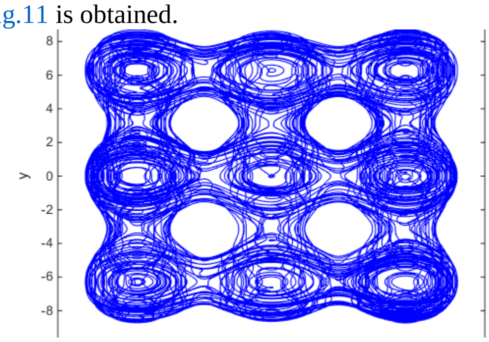

Fig.11. 5×5 multi-scroll chaotic attractor phase diagram

Fig.11 shows that the number of attractors is $M = (2)^2 + (2+1)^2 = 13$.

Taking $n = 2$, the phase diagram of the scroll attractor in Fig.12 is obtained.

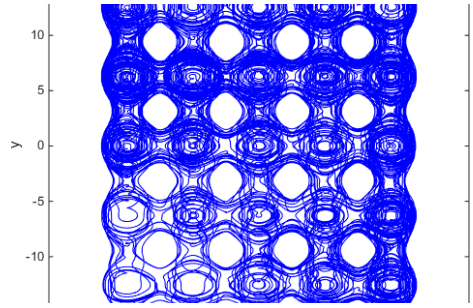

Fig.12. 9×9 multi-scroll chaotic attractor phase diagram 1

On the basis of $n = 2$, change the initial value, let $[\pi/2, 5\pi/2, 1, 1]$, and draw the phase diagram, as shown in Fig.13.

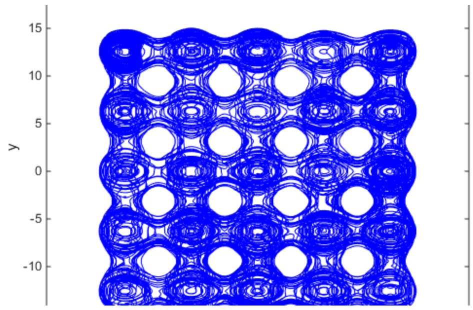

Fig.13. 9×9 multi-scroll chaotic attractor phase diagram 2

By changing the initial value and comparing Fig.12 and Fig.13, it can be seen that changing the initial value can change the internal motion track of the phase diagram and ensure that the number of scrolls does not change.

3. Conclusion

In this paper, nonlinear functions are introduced into a four-dimensional conservative chaotic system to generate multiple scrolls. After the introduction of nonlinear function, the equilibrium point of the system changes from a fixed point to a set of equilibrium points. The system carries out basic characteristic analysis, and discusses its divergence, equilibrium point and whether the energy is conservative. The equilibrium points obtained by introducing one-dimensional nonlinear function are divided into two categories. For its Lyapunov exponent analysis, after introducing the sine function without multiple angles, The Lyapunov exponents of the system equations show similar periodic characteristics to the sine function, and the Lyapunov exponents obtained by changing the initial values are very different. With the change of initial value, the phase diagrams obtained are also different, and the number of vortex attractors formed is also different, which verifies the multi stability.

Then, the nonlinear function is extended, two nonlinear functions are introduced, and the system with two nonlinear functions is further analyzed. The obtained phase diagram changes from one-dimensional to two-dimensional scroll attractor, and the relationship between the number and arrangement of scroll and the threshold width is obtained. By changing the initial value, the phase diagram with different internal distribution but the same number of scroll is obtained.

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