Research on Chaos Synchronization of Qi System and Lü System with Different Structures

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Abstract

This paper introduces a three-dimensional chaotic synchronization method is introduced, and the advantages and disadvantages of the synchronization controller designed by this method are analyzed. Firstly, the characteristics of two chaotic systems with different structures are studied. Secondly, the mathematical model is established, and the synchronization controller is designed by direct method, so that the two chaotic systems with different structures can be synchronized at different initial values. With the help of MATLAB, the error curve of the synchronous system is drawn when the synchronous controller acts on the response system.

Keywords: Chaos synchronization, Chaotic systems with different structures, Direct method, MATLAB

1. Introduction

Chaos theory is a key subject in the study of nonlinear theory in today's academic field. In 1963, the famous American scientist Lorenz first discovered the chaotic system [1]. This equation of chaotic system plays a very important role in the history of chaos, especially in the analysis of the emergence of chaotic solutions in nonlinear equations, which has great research significance. By analyzing the formation of chaotic system, scientists design a mathematical model suitable for the system, and then analyze whether the chaotic system can carry out chaotic motion under the mathematical model, and finally verify the basic characteristics of chaotic system.

The synchronization phenomenon was first discovered by the famous physicist Huygens. Once by chance, he saw two pendulums placed side by side, and they were exactly the same when they swung. This discovery opened up a branch in the field of mathematics and physical science the theory of the disaster oscillator, which revealed the synchronization phenomenon and its mechanism in nature. In 1990, Pecora and Carroll proposed a synchronization method to drive response, which made the chaotic trajectory of the system coincide under different initial conditions, and discovered the chaotic synchronization phenomenon in the circuit for the first time [2]. Scientists have found that there are many ways to synchronize chaotic systems, such as adaptive synchronization, delay synchronization, pulse synchronization and so on. In the study of two chaotic systems, firstly, the equilibrium point of the control system is analyzed. Then, according to the stability conclusion of Lyapunov's law, the correctness of the theoretical simulation results is verified by using the function method and numerical simulation [3],[4],[5]. The adaptive synchronization controller is designed and the adaptive synchronization method of chaotic system is discussed.

The discovery of chaos synchronization greatly promotes the study of chaos. Scientists have put forward the concept of chaotic system synchronization according to different directions, and the research of chaotic synchronization has made rapid progress.

2. Mathematical models of chaotic systems

The mathematical model of the Qi chaotic system is shown below:

$$\begin{cases} \dot{x} = a(y - x) + yz \\ \dot{y} = cx - xz - y \\ \dot{z} = xy - bz \end{cases} \tag{1}$$

Unknown quantities such as x, y and z are state variables of the system. The unknown quantity is the state variable of the system is $x, y, z \in \mathrm{R}$. $a = 35$, $b = 8/3$, $c = 80$ are typical parameters of the system.

The state variables of the mathematical model are transformed on the x, y and z axes respectively $(x, y, z) \to (x, -y, -z)$, $(x, y, z) \to (-x, y, -z)$, $(x, y, z) \to (-x, -y, z)$.

It is found that only when z-axis transformation is carried out, the mathematical model of the system does not change, so it can be explained that the mathematical model of the system is symmetric about the z-axis.

The partial derivative of the mathematical model of Qi chaotic system is obtained:

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -(a + b + 1) = -\frac{116}{3} < 0 \tag{2}$$

According to the partial derivative of the above equation, the derivative value is less than zero. According to the results, it can be concluded that the mathematical model of the system is always dissipative. The change of all systems is limited and bounded. As time increases, the orbit of the system will not spread out to infinity.

2.1 Chaotic dynamic properties

Set the initial value of the Qi chaotic system to $(x_0, y_0, z_0) = (1,1,1)$. The Lyapunov index of Qi chaotic system is obtained by MATLAB operation toolbox as $\lambda_1 = 4.0517 > 0$, $\lambda_2 = -0.0027 \approx 0$. Because of the Lyapunov index is $\lambda_1 = 4.0517 > 0$, it means that Qi chaotic system can have chaotic motion at this time. By putting the three Lyapunov exponents obtained into the formula for solving, the Lyapunov dimension d_L of the system is calculated as:

$$d_L = j + \frac{\sum_{i=1}^{j} \lambda_i}{|\lambda_{j+1}|} = 2 + \frac{4.0517 - 0.0027}{42.7151} = 2.0948 \tag{3}$$

j is the largest integer satisfying the condition that $\sum_{i=1}^{j} \lambda_i > 0$. According to the above formula, d_L =2.0948 , so the dimension of the chaotic system is fractional dimension.

The curves of the state variables of the chaotic system can be obtained by Matlab simulation as follows. Fig. 1 is the phase trajectory diagram of the chaotic system obtained by Simulink simulation, and Fig. 2 is the phase trajectory curve of Qi chaotic system in different coordinate systems.

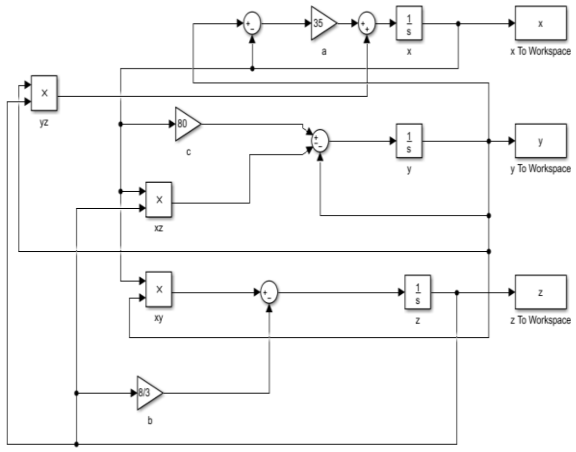
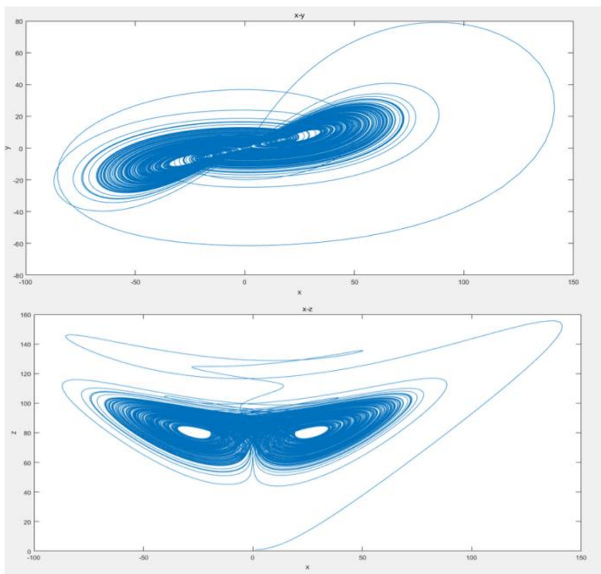


Fig. 1 Simulink simulation and construction of Qi chaotic system



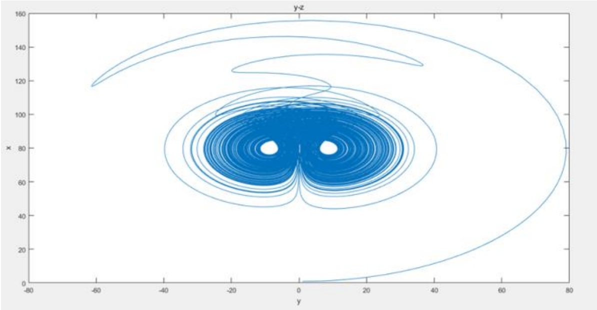


Fig. 2 Phase trajectory curve of Qi chaotic system system

Set the initial values of the system as $(x_{01}, y_0, z_0) = (1,1,1)$ and $(x_{02}, y_0, z_0) = (1.0001,1,1)$ respectively, and build a mathematical model through simulink to draw the solution curves of the corresponding three variables of the system, as shown in Fig. 3.

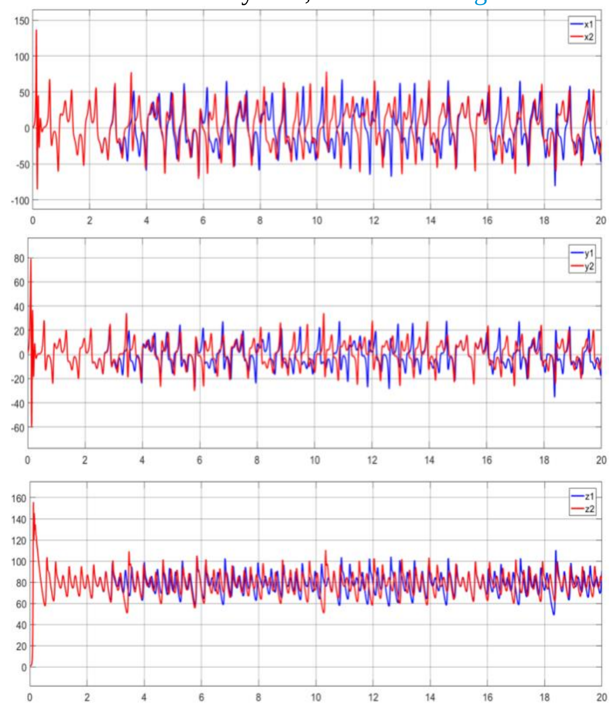


Fig. 3 Qi Solution curves of chaotic system in different states with different initial values

Keep other conditions unchanged, only change the initial value of state variable x of Qi chaotic system by 0.01%, in a short time, the two solution curves of chaotic system will change significantly. This shows that Qi chaotic system is susceptible to small changes in initial value and has a large influence. Therefore, Qi chaotic system is very sensitive to the change of initial value, which is an important characterization of the study of chaotic system.

2.2 Characteristic of equilibrium point

Set the right side of the mathematical model equation of the system as 0, and find the equilibrium state equation is:

$$\begin{cases} a(y-x)+yz=0 \\ cx-xz-y=0 \\ xy-bz=0 \end{cases} \tag{4}$$

Solve the system of state, let:

$$\begin{cases} x_0=\sqrt{\dfrac{b\left[ac+c^2-2a+c\sqrt{(a+c)^2-4a}\right]}{2a}} \\ y_0=\dfrac{\sqrt{2ab\left[ac+c^2-2a+c\sqrt{(a+c)^2-4a}\right]}}{a+c+\sqrt{(a+c)^2-4a}} \\ z_0=\dfrac{ac+c^2-2a+c\sqrt{(a+c)^2-4a}}{a+c+\sqrt{(a+c)^2-4a}} \end{cases} \tag{5}$$

The equilibrium point of the system under the mathematical model can be obtained by substituting parameters into the formula, respectively:

$$\begin{cases} S_1=(0,0,0) \\ S_2=(x_0,y_0,z_0) \\ S_3=(-x_0,-y_0,z_0) \end{cases} \rightarrow \begin{cases} S_1=(0,0,0) \\ S_2=(26.3899,8.0531,79.6948) \\ S_3=(-26.3899,-8.0531,79.6948) \end{cases} \tag{6}$$

When the equilibrium point is the Qi chaotic system linearization, the coefficient of each variable is written into the matrix to obtain its Jacobian matrix, and then the equilibrium point into:

$$J_1=J\big|_{S_1}=\left.\begin{bmatrix} -a & a+z & y \\ c-z & -1 & -x \\ y & x & -b \end{bmatrix}\right|_{S_1}=\begin{bmatrix} -a & a & 0 \\ c & -1 & 0 \\ 0 & 0 & -b \end{bmatrix} \tag{7}$$

Transform the above matrix into a determinant, make the determinant equal to zero, and then expand the determinant to obtain the characteristic equation as follows:

$$f(s)=(s+b)\left[s^2+(a+1)s-a(c-a)\right]=0 \tag{8}$$

Through calculation, the characteristic root of the matrix can be obtained as:

$$s_1 = -b = -2.6667 \tag{9}$$

$$s_2 = \frac{-\left(a+1\right)+\sqrt{\left(a+1\right)^2+4a\left(c-1\right)}}{2} \approx 37.5788 \tag{10}$$

$$s_3 = \frac{-\left(a+1\right)-\sqrt{\left(a+1\right)^2+4a\left(c-1\right)}}{2} \approx -73.5788 \tag{11}$$

The equilibrium $S_1 = \left(0,0,0\right)$ is a saddle node.

The analysis method of the remaining two equilibrium points is the same as that of S_1, and S_2 and S_3 are symmetric with respect to the z-axis. So we can just analyze one of the two equilibria, and then we will analyze the equilibrium S_2.

By linearizing Qi chaotic system at the equilibrium point $S_2 = \left(26.3899, 8.0531, 79.6948\right)$ (same as S_1), its J matrix is

$$J_2 = \begin{bmatrix} -a & a+79.6948 & 8.0531 \\ c-79.6948 & -1 & -26.3899 \\ 8.0531 & 26.3899 & -b \end{bmatrix} \tag{12}$$

According to the above method, the matrix is transformed into the determinant, and the determinant is set to zero. Then, the determinant is expanded to get the characteristic equation. Through the operation, the characteristic root of J_2 are:

$$s_1 = -45.8958 \tag{13}$$

$$s_{2,3} = 3.6146 \pm 32.3465\mathrm{j} \tag{14}$$

$s_1<0$ and the values of the other two equilibrium points are a pair of conjugate complex roots, and the real parts of the two values are greater than zero. According to Rouse stability criterion, it can be concluded that the properties of the equilibrium points S_2 and S_3 are the same, both of which are unstable focal points of the system.

By observing this table, it can be seen that two of the three equilibrium points obtained by the Qi chaotic system show instability, so the orbits near the two equilibrium points will disperse out. When the time increases, the tracks at these two points will disperse out, indicating that the system has the butterfly effect.

This chaotic system has the property of dissipation, chaotic motion can occur, so the whole system is stable. Because the dissipative property has the effect of stabilizing the system, it makes the outer orbitals of the system attractor gather into the attractor. But the adjacent orbitals repel, and they have to be separated exponentially. Therefore, the system is stable overall but unstable locally, and the system will exhibit a very cumbersome structure.

Similarly, the numerical simulation shows that Lü chaotic system has the same chaotic behavior as Qi chaotic system.

3. Design of synchronous controller

3.1 Driving system

In this paper, Qi chaotic system is taken as the driving system, and the mathematical model of Qi chaotic system is given as:

$$\begin{cases} \dot{x}_1 = a_1\left(y_1 - x_1\right) + y_1 z_1 \\ \dot{y}_1 = c_1 x_1 - x_1 z_1 - y_1 \\ \dot{z}_1 = x_1 y_1 - b_1 z_1 \end{cases} \tag{15}$$

Among them, the typical parameters of Qi chaotic system are: $a = 35$, $b = 8/3$, $c = 80$.

3.2 Response system

In this paper, the Lü chaotic system is taken as the response system. The equation with the synchronization controller is as follows:

$$\begin{cases} \dot{x}_2 = a_2\left(y_2 - x_2\right) + u_{c1} \\ \dot{y}_2 = -x_2 z_2 + c_2 y_2 + u_{c2} \\ \dot{z}_2 = x_2 y_2 - b_2 z_2 + u_{c3} \end{cases} \tag{16}$$

Typical parameters of Lü chaotic system are: $a = 36$, $b = 3$, $c = 20$. The synchronization controller is $u_c = \left[u_{c1} \quad u_{c2} \quad u_{c3}\right]^T$.

By using MATLAB software to simulate the error system and state synchronization curve of the controller, judge whether these two chaotic systems with different structures can show complete synchronization.

3.3 Direct method

When the equilibrium state of the system is very stable and the output of the system has reached the equilibrium state. With the increase of time, the energy stored in the system will decrease until it reaches the minimum stable value of the equilibrium state. Lyapunov direct method is from the point of view of energy. The movement of the system will consume energy, but the capacity of the system will not be consumed to zero.

So Lyapunov by introducing a fictitious function, this function is called Lyapunov function, denoted as $v(x,t)$ or $v(x)$. Let $v(x)$ be any scalar function, where x is the unknown variable of the system, if $v(x)$ satisfies the following properties:

(1) $\dot{v}(x)=\frac{d_{v(x)}}{dt}$ is continuous and can reflect the trend of energy change;

(2) $v(x)$ is positive definite and can reflect the magnitude of energy;

(3) When $\|x\|\to\infty$, $v(x)\to\infty$ reflects the distribution of energy, function $v(x)$ is called Lyapunov function.

Given two systems: $A=(x_1,y_1,z_1)$, $Y=(x_2,y_2,z_2)$, u_c indicates the control quantity.

$$\begin{cases} A=f(A) \\ B=g(B)+u_c \end{cases} \tag{17}$$

The way to synchronize two chaotic systems with different structures is to find a suitable u_c that make $\lim_{t\to\infty}\|y(t)-x(t)\|=0$.

For any initial values $x(0)$ and $y(0)$, the control synchronization problem of the system can be transformed into the problem of system error.

Set the error $e=\begin{bmatrix}e_1 & e_2 & e_3\end{bmatrix}^T=\begin{bmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\end{bmatrix}^T$, choose the Lyapunov function $v(x)=\frac{1}{2}\sum_{i=1}^{n}e_i^2$, obviously $v(x)$ is positive definite, if $v(x)$ is negative definite, then the error equation of state is asymptotically stable at the origin, that is, $\dot{v}(x)=\dot{e}_1e_1+\dot{e}_2e_2+\dot{e}_3e_3=0$ at $t\to\infty$ so you have to find a suitable u_c to make $v(x)$ negative definite.

Set the controller to:

$$u_C=\begin{bmatrix}u_{c1} & u_{c2} & u_{c3}\end{bmatrix}^T \tag{18}$$

By comparing the mathematical model of the drive system with that of the response system, the error system can be written as follows:

$$\begin{aligned} \dot{e}_1 &= a_2(e_2-e_1)+(a_2-a_1)(y_1-x_1)-y_1z_1+u_{c1} \\ \dot{e}_2 &= -x_2z_2+c_2e_2+(c_2+1)y_1+x_1z_1-c_1x_1+u_{c2} \\ \dot{e}_3 &= x_2y_2-b_2e_3-x_1y_1+(b_1-b_2)z_1+u_{c3} \end{aligned} \tag{19}$$

Choose the Lyapunov function $v(x)=\frac{1}{2}\sum_{i=1}^{n}e_i^2$ and calculate $\dot{v}(x)=\dot{e}_1e_1+\dot{e}_2e_2+\dot{e}_3e_3$.

$$u_c=\begin{bmatrix}(a_2-1)e_1-a_2e_2+(a_2-a_1)(x_1-y_1)+y_1z_1 \\ x_2z_2-(1+c_2)e_2-(c_2+1)y_1-x_1z_1+c_1x_1 \\ x_1y_1+(b_2-1)e_3-x_2y_2+(b_2-b_1)z_1\end{bmatrix} \tag{20}$$

The error formula and the typical parameters of the two systems are obtained one generation after another.

$$\begin{cases} \dot{x}=35(y_1-x_1)+(x_1-x_2)+y_1z_1 \\ \dot{y}=-y_2-x_1z_1+80x_1 \\ \dot{z}=x_1y_1-z_2-\frac{5}{3}z_1 \end{cases} \tag{21}$$

Finally, according to the equation of the system and the mathematical model of the Qi chaotic system, the model of the two systems is built, and the error curve and state synchronization curve of the Qi system and the Lü system are drawn to verify whether this method can synchronize the two systems with different structures.

Here, the initial values of the two systems are: $(x_1,y_1,z_1)=(1,1,1)$, $(x_2,y_2,z_2)=(30,30,30)$. Simulation software was used to draw the error curves and state curves of the two systems, as shown in the Fig. 4 and Fig. 5.

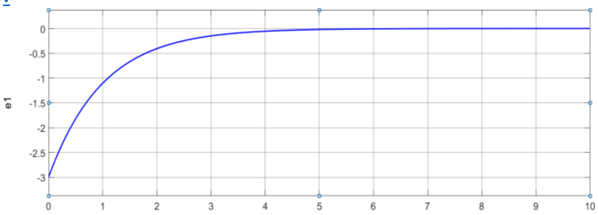
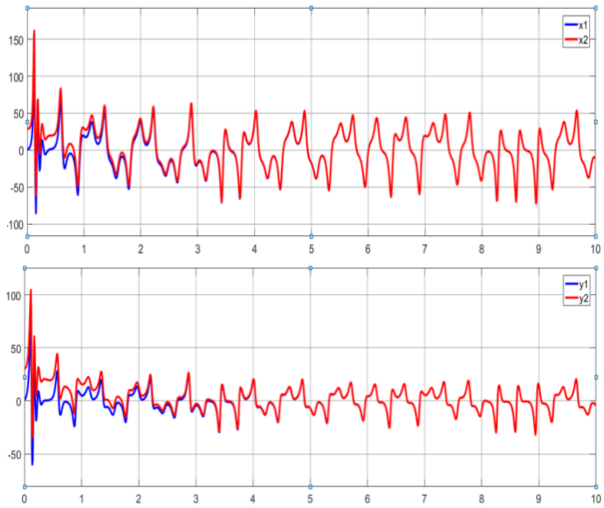


Fig. 4 Simulation error curve of synchronous controller designed by center translation method



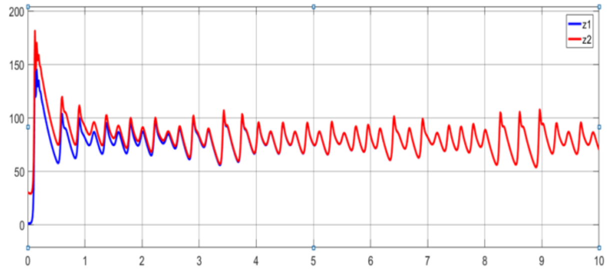


Fig. 5 Simulated state synchronization curve of synchronization controller designed with direct method

From the simulated image, it can be seen that the system can reach equilibrium in about 4.5 seconds.

4. Conclusion

By using direct method to design synchronous controller is simple in theory and easy to implement. In practical application, it effectively saves manpower, time and energy, and can effectively realize the synchronization problem of two different structures.

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