Quasi-static Stability Analysis of Frictionless Planar Enveloping Grasps (Analysis of curvature effects at contact points)

Takayoshi Yamaada, Junya Sato, Kazuaki Ito, Hidehiko Yamamoto
Dept. of Mechanical Engineering, Gifu University,
Yanagido 1-1, Gifu 501-1193, Japan
E-mail: yamat@gifu-u.ac.jp

Abstract

In this paper, we discuss grasp stability of frictionless planar enveloping grasps. The grasp stability is based on potential energy of the grasp system replaced with an elastic system. A stiffness matrix of the grasp system is derived. The grasp stability is evaluated by the eigenvalues of the matrix. We show that the matrix depends on grasp positions, grasp forces, local curvatures, etc., at contact points. Moreover, we analyze curvature effects on the grasp system by differentiating the matrix by the curvatures. We show that the derivative is negative semi definite.

Keywords: Grasp stability, Enveloping grasp, Grasp stiffness matrix, Curvature effects

1. Introduction

Humans can grasp and manipulate various types of objects dexterously. In various fields like as production lines including handling tasks, picking tasks, assembly tasks, etc., manual hand works are remained. Therefore, the human-like dexterous skills are required for mechanical functions of robots. In recent years, in order to obtain appropriate grasps roughly, deep learning methods are introduced. On the other hands, in traditional ways, grasp and manipulation are analyzed from the viewpoint of mechanics, kinematics and dynamics in detail. As one of the methods, grasp stability based on potential energy of the grasp system replaced with an elastic system is investigated in order to derive grasp evaluation. In grasp forms, pinching grasp by fingertips, enveloping grasp by finger link surfaces and so on can be considered.

Ref. [1] analyzed grasp stability of pinching grasps. Local curvature effects on the stability were also investigated. Ref. [2] investigated grasp position effects on the stability, proposed an automatic generation of optimal grasp. Ref. [3] analyzed frictionless enveloping grasps by replacing joint displacement and finger surface displacement with elastic model (Figure 1).

In this paper, we discuss grasp stability of frictionless planar enveloping grasps. The stiffness matrix of the grasp system is derived. The grasp stability is evaluated by the eigenvalues of the matrix. We show that the matrix depends on grasp positions, grasp forces, local curvatures, etc., at contact points. Moreover, we analyze curvature effects on the grasp system by differentiating the matrix by the curvatures.

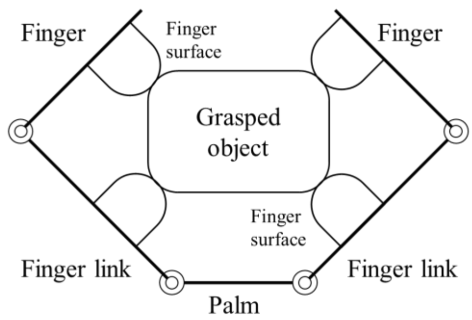

Figure 1: A planar enveloping grasp

2. Problem Formulation

2.1. *Symbols*

Symbol i is finger number, j is joint and link number. Some coordinates are defined as shown in Figure 2. Symbol Σ_b denotes a base coordinate frame on the system. Symbol Σ_o is an object coordinate frame, Σ_{bo} is its initial pose. Symbol Σ_{i0} is a i-th finger base coordinate frame, Σ_{ij} is the *j*-th link coordinate frame of the *i*-th finger, Σ_{bij} is the initial pose of Σ_{ij}. Symbols Σ_{Cfij} and Σ_{Coij} are contact coordinate frames on the finger and the object, Σ_{Lfij} and Σ_{Loij} are the initial pose of Σ_{Cfij} and Σ_{Coij}, respectively. Symbol Σ_{ijp} is a deformed coordinate frame of Σ_{Lfij}. Vectors and matrices are shown in Appendix.

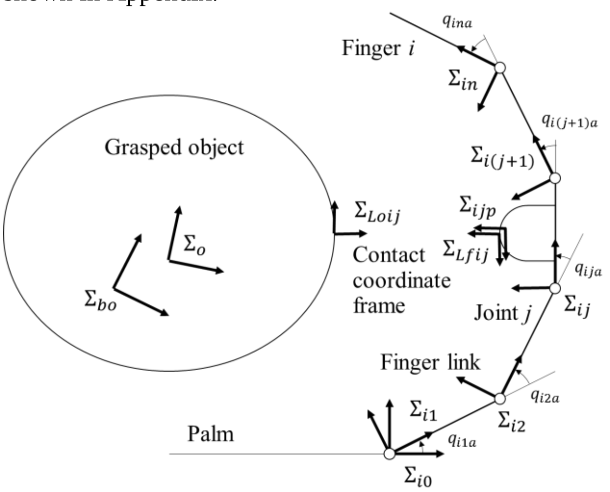

Figure 2: Coordinate frames on the object and the finger

2.2. *Joint and finger surface elastics*

We consider that the *i*-th finger is constructed of serial links with *n* joints. Position of the *j*-th joint (active joint), q_{ija}, is represented as

$$q_{ija} = q_{nija} + q_{cija} + q_{dija} \in \mathbb{R} \tag{1}$$

$$\begin{cases} q_{nija} & \text{Natural position} \\ q_{cija} & \text{Compression by the initial force} \\ q_{dija} & \text{Displacement by the object displacement} \end{cases}$$

The homogeneous transformation matrix of the *j*-th link is represented as

$$\begin{aligned} &{}^{i(j-1)}A_{ij}(q_{ija}) \\ &= \begin{cases} {}^{i(j-1)}A_{bij}(q_{nija}, q_{cija})A_r(q_{dija}) & \text{for revolute joint} \\ {}^{i(j-1)}A_{bij}(q_{nija}, q_{cija})A_t(q_{dija}\boldsymbol{u}_1) & \text{for prismatic joint} \end{cases} \end{aligned} \tag{2}$$

The link surface contacts on the object. Deformation of the link surface is represented as the pose displacement of the surface coordinate (passive joint) and represented by the following form.

$$\boldsymbol{q}_{ijp} = \boldsymbol{q}_{nijp} + \boldsymbol{q}_{cijp} + \boldsymbol{q}_{dijp} \in \mathbb{R}^3 \tag{3}$$

$$\begin{cases} \boldsymbol{q}_{nijp} & \text{Natural position} \\ \boldsymbol{q}_{cijp} & \text{Compression by the initial force} \\ \boldsymbol{q}_{dijp} & \text{Displacement by the object displacement} \end{cases}$$

The displacement is represented by the following homogeneous transformation matrix.

$$\begin{aligned} {}^{ij}A_{Lfij}(\boldsymbol{q}_{ijp}) &:= {}^{ij}A_{ijp}\,{}^{ijp}A_{Lfij}(\boldsymbol{q}_{dijp}) \\ {}^{ijp}A_{Lfij}(\boldsymbol{q}_{dijp}) &:= A_{tr}(\boldsymbol{q}_{dijp}), \quad {}^{ijp}A_{Lfij}(0_{3\times1}) = I_3 \\ \lim_{\boldsymbol{q}_{dijp}\to 0} \Sigma_{ijp} &= \Sigma_{Lfij} \end{aligned} \tag{4}$$

Displacement of the *j*-th link is represented by

$$\boldsymbol{q}_{dij} := [q_{dija}, \boldsymbol{q}_{dijp}^T]^T \in \mathbb{R}^4 \tag{5}$$

2.3. *Potential energy of the j-th link*

Elastic coefficient of the joint displacement is defined as $s_{ija} \in \mathbb{R}$. The potential energy of the *j*-th joint is obtained by the following form:

$$U_{ija}(q_{dija}) := \frac{1}{2}s_{ija}(q_{cija} + q_{dija})^2 \tag{6}$$

Initial joint torque τ_{ija} is obtained by

$$\tau_{ija} := s_{ija}q_{cija} \tag{7}$$

Symbol $S_{ijp} \in \mathbb{R}^{3\times3}$ represents stiffness coefficient of the elasticity of the link surface displacement. The potential energy of the *i*-th finger link is given by

$$U_{ijp}(\boldsymbol{q}_{dijp}) := \frac{1}{2}[\boldsymbol{q}_{cijp} + \boldsymbol{q}_{dijp}]^T S_{ijp}[\boldsymbol{q}_{cijp} + \boldsymbol{q}_{dijp}] \tag{8}$$

Initial contact force $\boldsymbol{\tau}_{ijp}$ is obtained by

$$\boldsymbol{\tau}_{ijp} := S_{ijp}\boldsymbol{q}_{cijp} \tag{9}$$

The potential energy of the *j*-th joint is given as

$$U_{ij}(\boldsymbol{q}_{dij}) := U_{ija}(q_{dija}) + U_{ijp}(\boldsymbol{q}_{dijp}) \tag{10}$$

The potential energy of the *i*-th finger is obtained by

$$U_i(\boldsymbol{q}_{di}) := \sum_{j=1}^{n} U_{ij}(\boldsymbol{q}_{dij}) \tag{21}$$

where

$$\begin{aligned} \boldsymbol{q}_{di} &:= [\boldsymbol{q}_{di1}^T, \cdots, \boldsymbol{q}_{din}^T]^T \\ &= [q_{di1a}, \boldsymbol{q}_{di1p}^T, \cdots, q_{dina}, \boldsymbol{q}_{dinp}^T]^T \in \mathbb{R}^{4n} \end{aligned} \tag{32}$$

In the case that the number of fingers is m, the potential energy of the grasp system is obtained by

$$U(\boldsymbol{q}_d) := \sum_{i=1}^{n} U_i(\boldsymbol{q}_{di}), \qquad \boldsymbol{q}_d := [\boldsymbol{q}_{d1}^T, \cdots, \boldsymbol{q}_{dm}^T]^T \tag{13}$$

2.4. *Joint and finger surface elastics*

In the case that the *j*-th link surface contacts on the object, we obtain the following constraint:

$$\begin{aligned} &{}^bA_{bo}\,{}^{bo}A_o(\boldsymbol{\varepsilon}_o)\,{}^oA_{Loij}\,{}^{Loij}A_{Coij}(\alpha_{oij}) \\ &= {}^bA_{i0}\,{}^{i0}A_{i1}(q_{i1a}) \times \cdots \times {}^{i(j-1)}A_{ij}(q_{ija}) \\ &\times {}^{ij}A_{Lfij}(\boldsymbol{q}_{ijp})\,{}^{Lfij}A_{Cfij}(\alpha_{fij})\,{}^{Cfij}A_{Coij} \\ &(j = 1,2,\cdots,n) \end{aligned} \tag{14}$$

where parameter $\boldsymbol{\varepsilon}_o$ represents position and orientation of the object. Parameters α_{oij} and α_{fij} represent surface displacement of the contact points on the object and the link.

$$
\begin{aligned}
{}^{Loij}A_{Coij}(\alpha_{oij}) &:= {}^{Loij}A_{\kappa oij}A_r(\kappa_{oij}\alpha_{oij}){}^{Loij}A_{\kappa oij}^{-1} \\
{}^{Lfij}A_{Cfij}(\alpha_{fij}) &:= {}^{Lfij}A_{\kappa fij}A_r(\kappa_{fij}\alpha_{fij}){}^{Lfij}A_{\kappa fij}^{-1} \\
{}^{Loij}A_{\kappa oij} &:= A_t(-\kappa_{oij}^{-1}\boldsymbol{u}_1), \quad {}^{Lfij}A_{\kappa fij} := A_t(-\kappa_{fij}^{-1}\boldsymbol{u}_1)
\end{aligned} \tag{15}
$$

Curvature κ is given as the following characteristics:

$$
\begin{cases} \kappa > 0 & \text{convex surface} \\ \kappa = 0 & \text{flat surface} \\ \kappa < 0 & \text{concave surface} \end{cases} \tag{16}
$$

Relation of the contact coordinate frame between the link and the object surfaces is set as the following condition:

$$
{}^{Cfij}A_{Coij} = A_r(\pi) \tag{17}
$$

From (15), we have

$$
\begin{aligned}
A_{tr}(\boldsymbol{q}_{dijp}) = {}&{}^{ijp}A_{ij}{}^{i(j-1)}A_{ij}^{-1}(q_{ija}) \times \cdots \\
&\times {}^{i0}A_{i1}^{-1}(q_{i1a}){}^{i0}A_{bo}{}^{bo}A_o(\boldsymbol{\varepsilon}_o){}^{o}A_{Loij} \\
&\times {}^{Loij}A_{Coij}(\alpha_{oij}){}^{Cfij}A_{Coij}^{-1}{}^{Lfij}A_{Cfij}^{-1}(\alpha_{fij})
\end{aligned} \tag{18}
$$

The displacement on the link surface, $\boldsymbol{q}_{dijp}$, is given by a function of the object displacement $\boldsymbol{\varepsilon}_o$, joint position displacement $q_{di1a}, \cdots, q_{dija}$, contact position displacement $\boldsymbol{\alpha}_{ij}$.

$$
\boldsymbol{q}_{dijp}(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_{ij}, q_{di1a}, \cdots, q_{dija}), \quad (j = 1,2,\cdots,n) \tag{19}
$$

where

$$
\boldsymbol{\alpha}_{ij} := \begin{bmatrix} \alpha_{oij} \\ \alpha_{fij} \end{bmatrix} \in \mathbb{R}^2 \tag{20}
$$

Consequently, the potential energy of the *i*-th finger is represented as the following form:

$$
\begin{aligned}
U_{iq}(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i) &:= U_i(q_{di1a}, \boldsymbol{q}_{di1p}^T(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_{i1}, q_{di1a}), \cdots, \\
&\qquad q_{dina}, \boldsymbol{q}_{dinp}^T(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_{in}, q_{di1a}, \cdots, q_{dina})) \\
&= \sum_{j=1}^{n} \{U_{ija}(q_{dija}) \\
&\quad + U_{ijp}(\boldsymbol{q}_{dijp}(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_{ij}, q_{di1a}, \cdots, q_{dija}))\}
\end{aligned} \tag{21}
$$

where

$$
\boldsymbol{\alpha}_i := \begin{bmatrix} \boldsymbol{\alpha}_{i1} \\ \vdots \\ \boldsymbol{\alpha}_{in} \end{bmatrix} \in \mathbb{R}^{2n}, \quad \boldsymbol{\beta}_i := \begin{bmatrix} q_{di1a} \\ \vdots \\ q_{dina} \end{bmatrix} \in \mathbb{R}^n \tag{22}
$$

3. Partial derivative of potential energy

3.1. *The first derivative of the potential energy*

The first partial derivative of the potential energy is derived and its initial condition is considered, then we have the followings:

$$
\begin{aligned}
\left.\frac{\partial U_{iq}(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i)}{\partial \boldsymbol{\varepsilon}_o}\right|_0 &= \sum_{l=1}^{n} {}^{Lfil}B_o^T \boldsymbol{\tau}_{ilp} \\
\left.\frac{\partial U_{iq}(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i)}{\partial \boldsymbol{\alpha}_{ij}}\right|_0 &= K_{ij}^T \boldsymbol{\tau}_{ijp}, \quad K_{ij} := \begin{bmatrix} -\boldsymbol{u}_2 & -\boldsymbol{u}_2 \\ \kappa_{oij} & -\kappa_{fij} \end{bmatrix} \\
\left.\frac{\partial U_{iq}(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i)}{\partial q_{dija}}\right|_0 &= \tau_{ija} - \boldsymbol{v}_\zeta^T \sum_{l=j}^{n} {}^{Lfil}B_{ij}^T \boldsymbol{\tau}_{ilp}
\end{aligned} \tag{23}
$$

3.2. *The second derivative of the potential energy*

The second partial derivative of the potential energy is derived and its initial condition is considered, then we have the followings:

$$
\begin{aligned}
&\frac{\partial^2 U_{iq}(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i)}{\partial \boldsymbol{\varepsilon}_o \partial \boldsymbol{\varepsilon}_o^T} \\
&= \sum_{l=1}^{n} \{{}^{Lfil}B_o^T S_{ilp}{}^{Lfil}B_o + \boldsymbol{v}_\zeta[\boldsymbol{\tau}_{ilp}^T I_{23}^T{}^{Lfil}\boldsymbol{p}_o]\boldsymbol{v}_\zeta^T\} \\
&\left.\frac{\partial^2 U_{iq}(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i)}{\partial \boldsymbol{\alpha}_{ij} \partial \boldsymbol{\varepsilon}_o^T}\right|_0 = K_{ij}^T S_{ijp}{}^{Lfij}B_o + \begin{bmatrix} 1 \\ 1 \end{bmatrix}[\boldsymbol{\tau}_{ijp}^T \boldsymbol{v}_x]\boldsymbol{v}_\zeta^T \\
&\left.\frac{\partial^2 U_{iq}(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i)}{\partial q_{dija} \partial \boldsymbol{\varepsilon}_o^T}\right|_0 \\
&\quad = -\sum_{l=j}^{n} [\boldsymbol{v}_\zeta^T{}^{Lfil}B_{ij}^T S_{ilp}{}^{Lfil}B_o + \boldsymbol{\tau}_{ilp}^T I_{23}^T \Omega I_{23}{}^{Lfil}B_o] \\
&\left.\frac{\partial^2 U_{iq}(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i)}{\partial \boldsymbol{\alpha}_{ik} \partial \boldsymbol{\alpha}_{ij}^T}\right|_0 \\
&= \begin{cases} K_{ik}^T S_{ijp} K_{ij} + [\boldsymbol{\tau}_{ijp}^T \boldsymbol{v}_x]\begin{bmatrix} \kappa_{oij} & \kappa_{oij} \\ \kappa_{oij} & -\kappa_{fij} \end{bmatrix} & (1 \le k = j \le n) \\ 0_{2\times 2} & (\text{otherwise}) \end{cases} \\
&\left.\frac{\partial^2 U_{iq}(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i)}{\partial q_{dika} \partial \boldsymbol{\alpha}_{ij}^T}\right|_0 \\
&= \begin{cases} -\boldsymbol{v}_\zeta^T{}^{Lfij}B_{ik}^T S_{ijp} K_{ij} - \boldsymbol{\tau}_{ijp}^T \boldsymbol{v}_x[1 \quad 1] & (1 \le k \le j \le n) \\ 0_{1\times 2} & (\text{otherwise}) \end{cases} \\
&\left.\frac{\partial^2 U_{iq}(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i)}{\partial q_{dika} \partial q_{dija}}\right|_0 \\
&= \begin{cases} s_{ija} + \sum_{l=j}^{n} [\boldsymbol{v}_\zeta^T{}^{Lfil}B_{ik}^T S_{ilp}{}^{Lfil}B_{ij}\boldsymbol{v}_\zeta + \boldsymbol{\tau}_{ilp}^T I_{23}^T{}^{Lfil}\boldsymbol{p}_{ij}] \\ \qquad (1 \le l \le k = j \le n) \\ \sum_{l=j}^{n} [\boldsymbol{v}_\zeta^T{}^{Lfil}B_{ik}^T S_{ilp}{}^{Lfil}B_{ij}\boldsymbol{v}_\zeta + \boldsymbol{\tau}_{ilp}^T I_{23}^T{}^{Lfil}\boldsymbol{p}_{ij}] \\ \qquad (1 \le l \le k < j \le n) \end{cases}
\end{aligned} \tag{24}
$$

4. Frictionless enveloping grasp

4.1. *Constraints of frictionless contact*

In the case that the i-th finger surface contacts on the objects with frictionless condition, contact point displacement $\boldsymbol{\alpha}_i$ and joint position displacement $\boldsymbol{\beta}_i$ shift to the position locally minimizing the energy. Consequently, we have the following constraint:

$$\begin{bmatrix} \dfrac{\partial U_{iq}(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i)}{\partial \boldsymbol{\alpha}_i} \\ \dfrac{\partial U_{iq}(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i)}{\partial \boldsymbol{\beta}_i} \end{bmatrix} = \begin{bmatrix} 0_{2n} \\ 0_n \end{bmatrix} \tag{25}$$

From this condition, we have $3n$ constraints, then the freedom of the system is given only as the parameter $\boldsymbol{\varepsilon}_o$.

$$U_{iq}^{fs}(\boldsymbol{\varepsilon}_o) := U_{iq}\big(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i(\boldsymbol{\varepsilon}_o), \boldsymbol{\beta}_i(\boldsymbol{\varepsilon}_o)\big) \tag{26}$$

The first partial derivative is derived, its initial condition is considered. Considering the constraint of Eq. (25), we have the following gradient:

$$G_i^{fs} := \left.\frac{\partial U_{iq}^{fs}(\boldsymbol{\varepsilon}_o)}{\partial \boldsymbol{\varepsilon}_o}\right|_0 = \sum_{l=1}^{n} {}^{Lfil}B_o^T \boldsymbol{\tau}_{ilp} = \sum_{l=1}^{n} {}^{o}W_{Lfil} {}^{Lfil}\boldsymbol{f} \tag{27}$$

where the symbol $\boldsymbol{\tau}_{ilp}$ is given by

$$\boldsymbol{\tau}_{ilp} = I_{23}^T {}^{Lfil}\boldsymbol{f}, \quad {}^{Lfil}\boldsymbol{f} = \begin{bmatrix} {}^{Lfil}f_x \\ {}^{Lfil}f_y \end{bmatrix}, \quad {}^{Lfil}f_x < 0, \quad {}^{Lfil}f_y = 0 \tag{28}$$

The second partial derivative is given as

$$H_i^{fs} := \left.\frac{\partial^2 U_{iq}^{fs}(\boldsymbol{\varepsilon}_o)}{\partial \boldsymbol{\varepsilon}_o \partial \boldsymbol{\varepsilon}_o^T}\right|_0 = U_{iq,\varepsilon\varepsilon} + Q_i^{fs} \begin{bmatrix} U_{iq,\varepsilon\alpha} \\ U_{iq,\varepsilon\beta} \end{bmatrix} \tag{29}$$

where the symbol Q_i^{fs} express

$$Q_i^{fs} := \left[\left.\frac{\partial \boldsymbol{\alpha}_i^T}{\partial \boldsymbol{\varepsilon}_o}\right|_0 \quad \left.\frac{\partial \boldsymbol{\beta}_i^T}{\partial \boldsymbol{\varepsilon}_o}\right|_0 \right] = -\left[U_{iq,\alpha\varepsilon} \quad U_{iq,\beta\varepsilon}\right] \begin{bmatrix} U_{iq,\alpha\alpha} & U_{iq,\beta\alpha} \\ U_{iq,\alpha\beta} & U_{iq,\beta\beta} \end{bmatrix}^{-1} \tag{30}$$

For example, the symbol $U_{iq,\alpha\varepsilon}$ means the second partial derivative of $U_{iq}(\boldsymbol{\varepsilon}_o, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i)$ by $\boldsymbol{\alpha}_i$ and $\boldsymbol{\varepsilon}_o$. The potential energy of the grasp system is given by the following the formula:

$$U^{fs}(\boldsymbol{\varepsilon}_o) := \sum\nolimits_{i=1}^{m} U_{iq}^{fs}(\boldsymbol{\varepsilon}_o) \tag{31}$$

Total grasp wrench (force and moment) of the grasp system and the grasp stiffness matrix are obtained by

$$G^{fs} := \left.\frac{\partial U^{fs}(\boldsymbol{\varepsilon}_o)}{\partial \boldsymbol{\varepsilon}_o}\right|_0 = \sum\nolimits_{i=1}^{m} G_i^{fs}, \quad H^{fs} := \left.\frac{\partial^2 U^{fs}(\boldsymbol{\varepsilon}_o)}{\partial \boldsymbol{\varepsilon}_o \partial \boldsymbol{\varepsilon}_o^T}\right|_0 = \sum\nolimits_{i=1}^{m} H_i^{fs} \tag{32}$$

The matrix H^{fs} depends on contact position, contact direction, contact force, local curvature at contact points.

5. Effect of local curvatures at contact points

5.1. *Partial derivative of the local curvatures*

The grasp stiffness matrix is partially differentiated by the local curvature, then we have the following formula:

$$\begin{aligned} \frac{\partial H_i^{fs}}{\partial \kappa_{oij}} &= \frac{\partial U_{iq,\varepsilon\varepsilon}}{\partial \kappa_{oij}} + \left\{ \frac{\partial}{\partial \kappa_{oij}} \begin{bmatrix} U_{iq,\varepsilon\alpha} \\ U_{iq,\varepsilon\beta} \end{bmatrix} \right\}^T \left[Q_i^{fs}\right]^T \\ &\quad + Q_i^{fs} \left\{ \frac{\partial}{\partial \kappa_{oij}} \begin{bmatrix} U_{iq,\varepsilon\alpha} \\ U_{iq,\varepsilon\beta} \end{bmatrix} \right\} \\ &\quad + Q_i^{fs} \left\{ \frac{\partial}{\partial \kappa_{oij}} \begin{bmatrix} U_{iq,\alpha\alpha} & U_{iq,\beta\alpha} \\ U_{iq,\alpha\beta} & U_{iq,\beta\beta} \end{bmatrix} \right\} \left[Q_i^{fs}\right]^T \\ &= \boldsymbol{a}\boldsymbol{b}_1{}^T + \boldsymbol{b}_1\boldsymbol{a}^T + \left({}^{Lfij}\boldsymbol{f}^T\boldsymbol{u}_1\right)\boldsymbol{b}_1\boldsymbol{b}_1{}^T \\ &= \left({}^{Lfij}\boldsymbol{f}^T\boldsymbol{u}_1\right)\boldsymbol{b}_1\boldsymbol{b}_1{}^T \preccurlyeq 0_{3\times3} \\ \frac{\partial H_i^{fs}}{\partial \kappa_{fij}} &= -\boldsymbol{a}\boldsymbol{b}_2{}^T - \boldsymbol{b}_2\boldsymbol{a}^T + \left({}^{Lfij}\boldsymbol{f}^T\boldsymbol{u}_1\right)\boldsymbol{b}_2\boldsymbol{b}_2{}^T \\ &= \left({}^{Lfij}\boldsymbol{f}^T\boldsymbol{u}_1\right)\boldsymbol{b}_2\boldsymbol{b}_2{}^T \preccurlyeq 0_{3\times3} \end{aligned} \tag{33}$$

The vectors $\boldsymbol{a}$, $\boldsymbol{b}_1$, $\boldsymbol{b}_2 \in \mathbb{R}^3$ are given by

$$\begin{aligned} &\boldsymbol{a} := {}^{Lfij}B_o^T S_{ijp}\boldsymbol{v}_\zeta \\ &+Q_i^{fs} \begin{bmatrix} \begin{bmatrix} 0_{2(j-1)\times1} \\ K_{ij}^T S_{ijp}\boldsymbol{v}_\zeta + \boldsymbol{u}_2\left[{}^{Lfij}\boldsymbol{f}^T\boldsymbol{u}_1\right] \\ 0_{2(n-j)\times1} \end{bmatrix} \\ -\begin{bmatrix} \boldsymbol{v}_\zeta^T {}^{Lfij}B_{i1}^T S_{ijp}\boldsymbol{v}_\zeta \\ \vdots \\ \boldsymbol{v}_\zeta^T {}^{Lfij}B_{ij}^T S_{ijp}\boldsymbol{v}_\zeta \\ 0_{(n-j)\times1} \end{bmatrix} \end{bmatrix} = \boldsymbol{0}, \\ &\boldsymbol{b}_1 := Q_i^{fs} \begin{bmatrix} 0_{2(j-1)\times1} \\ \boldsymbol{u}_1 \\ 0_{2(n-j)\times1} \\ 0_{n\times1} \end{bmatrix}, \quad \boldsymbol{b}_2 := Q_i^{fs} \begin{bmatrix} 0_{2(j-1)\times1} \\ \boldsymbol{u}_2 \\ 0_{2(n-j)\times1} \\ 0_{n\times1} \end{bmatrix} \end{aligned} \tag{34}$$

Because we have ${}^{Lfij}\boldsymbol{f}^T\boldsymbol{u}_1 < 0$, the derivatives $\frac{\partial H_i^{fs}}{\partial \kappa_{oij}}$ and $\frac{\partial H_i^{fs}}{\partial \kappa_{fij}}$ are negative semi definite. The value of the local curvature is smaller, the grasp stability is higher.

6. Conclusions

This paper analyzed frictionless planar enveloping grasps. Not only joint displacement but also link surface displacement is replaced with elastic feature. Contact constraints between the finger links and the grasped object were derived, and independent parameters were clarified. By using independent parameters, the potential energy of the grasp system was derived. In the case of frictionless contact, the wrench vectors were obtained by the first partial derivative of the energy, the grasp stiffness matrix was obtained by the second partial derivative. By using partial derivative of the stiffness matrix by local curvature at contact points, the effect of the curvature on the grasp stability was clarified.

References

[1] T. Yamada, R. Johansson, A. Robertsson, and H. Yamamoto, Static Stability Analysis of a Planar Object Grasped by Multifingers with Three Joints, Robotics, 4, 464-491, doi:10.3390/robotics4040464, 2015.

[2] T. Yamada, K. Niwa, H. Yamamoto, H. Kawasaki, and T. Mouri, Automatic generation of grasp positions using the partial differentiation of the grasp stiffness matrix, J. of Robotic and Mechatronic Systems, Vol. 2, No. 2, pp. 1-15, 2017.

[3] T. Yamada, Y. Kishi, J. Sato, K. Ito, H. Yamamoto, Quasistatic Stability of Frictionless Planar Enveloping Grasps, Proc. of JSME Robomech2019 (CD-ROM), 2P1-H05, 2019.

Appendix A

In two dimensions, a homogeneous transformation matrix of a frame Σ_b with respect to a frame Σ_a is represented by the following form:

$$ {}^{a}A_b := \begin{bmatrix} {}^{a}R_b & {}^{a}\boldsymbol{p}_b \\ 0_{1\times 2} & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3} \tag{35} $$

where ${}^{a}\boldsymbol{p}_b \in \mathbb{R}^2$ is a position vector, ${}^{a}R_b \in \mathbb{R}^{2\times 2}$ is a rotation matrix expressing relative orientation. The following symbols are also defined in this paper.

$$ \begin{gathered} \boldsymbol{u}_1 := \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{u}_2 := \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ \boldsymbol{v}_x := \begin{bmatrix} \boldsymbol{u}_1 \\ 0 \end{bmatrix}, \quad \boldsymbol{v}_y := \begin{bmatrix} \boldsymbol{u}_2 \\ 0 \end{bmatrix}, \quad \boldsymbol{v}_\zeta := \begin{bmatrix} 0_{2\times 1} \\ 1 \end{bmatrix} \\ \mathrm{Rot}(\zeta) := \begin{bmatrix} \cos\zeta & -\sin\zeta \\ \sin\zeta & \cos\zeta \end{bmatrix}, \\ \Omega := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \mathrm{Rot}\left(\frac{\pi}{2}\right) \\ A_t(\boldsymbol{x}) := \begin{bmatrix} I_2 & \boldsymbol{x} \\ 0_{1\times 2} & 1 \end{bmatrix}, \qquad A_r(\zeta) := \begin{bmatrix} \mathrm{Rot}(\zeta) & 0_{2\times 1} \\ 0_{1\times 2} & 1 \end{bmatrix} \\ A_{tr}(\boldsymbol{\varepsilon}) := A_t(\boldsymbol{x})A_r(\zeta), \qquad \boldsymbol{\varepsilon} := \begin{bmatrix} \boldsymbol{x} \\ \zeta \end{bmatrix}, \qquad \boldsymbol{x} := \begin{bmatrix} x \\ y \end{bmatrix} \\ {}^{a}B_b := \begin{bmatrix} {}^{a}R_b & -\Omega\, {}^{a}\boldsymbol{p}_b \\ 0_{1\times 2} & 1 \end{bmatrix}, \quad I_{23} = \begin{bmatrix} I_2 & 0_{2\times 1} \end{bmatrix} \in \mathbb{R}^{2\times 3} \\ {}^{a}W_b := \begin{bmatrix} {}^{a}R_b \\ {}^{a}\boldsymbol{p}_b \times {}^{a}R_b \end{bmatrix} = \begin{bmatrix} I_{23}\, {}^{b}B_a \end{bmatrix}^T \end{gathered} \tag{36} $$

Authors Introduction

Dr. Takayoshi Yamada

He received the B.E., M.E, Ph.D. degrees in Mechanical Engineering from Nagoya Institute of Technology, Japan, in 1991, 1993, 1995, respectively. He is a Professor with the Dept. of Mechanical Engineering, Gifu University. His research interests include grasping, manipulation, sensing, and automation systems.

Dr. Junya Sato

He received the B.E., M.E., and Ph.D. degrees in Eng. from Iwate University in Japan, in 2012, 2014, and 2017, respectively. He is an Assistant Professor with the Dept. of Mech. Eng., Faculty of Eng., Gifu Univ., Japan. His research interests include applications of evolutionary computations to computer vision.

Dr. Kazuaki Ito

He received the B.S., M.S. and Ph.D. degrees in electrical and computer engineering from Nagoya Institute of Technology, Japan, in 1998, 2000, and 2003, respectively. He is a Professor with the Dept. of Mech. Eng., Gifu University. His current research interests include applications of motion control theory and soft computing techniques for mechatronic systems and real-world haptics.

Dr. Hidehiko Ymamoto

He received the M.E. and Ph. D. degrees in Mech. Eng. from Nagoya Institute of Technology, in 1980, 1991. He was a professor with the Dept. of Mech. Eng., Gifu University. He is currently a specially appointed professor, Gifu University.