

Directional Flocking of Multi-Agent system Caused by Limited Visual Field

Yongnan Jia

School of Automation and Electrical Engineering, University of Sciences and Technology Beijing, Beijing, 100083, P. R. China

Jiali Han

School of Automation and Electrical Engineering, University of Sciences and Technology Beijing, Beijing, 100083, P. R. China

Yong Xie

The System Design Institute of Mechanical-Electrical Engineering, Beijing, 100854, China.

Weicun Zhang

School of Automation and Electrical Engineering, University of Sciences and Technology Beijing, Beijing, 100083, P. R. China

E-mail: ynjia@pku.edu.cn

Abstract

Experiment evidence has proved that the visual field of each individual in biological swarms is usually non-omnidirectional. Therefore, we introduce limited visual field to the egalitarian flocking model. The directional flocking problem refers to the flocking problem that all the individuals are expected to move in a specified direction, which is decided by the leader. This paper mainly compared the limited-visual-field flocking model with the classic flocking model (that is the egalitarian one) from the point view of rate of convergence. Experimental results indicated that limited-visual-field flocking model is more efficient than the omnidirectional one for the directional flocking problem.

Keywords: Flocking model, limited visual field, egalitarian flocking model, rate of convergence.

1. Introduction

Flocking phenomena widely exist among social animals, such as bacteria, birds, bees, herds, and fishes. In the colony, each individual just relies on local interaction and simple decision-making rules to emerge a global dynamical behavior for the sake of obtaining the chance of survival. Several models, such as Boid model¹, Vicsek model², Cucker-Smale model³, have been developed for years in order to reveal the mechanism or principle behind these flocking behaviors.

During these classical flocking models, there is a common prerequisite, that is, each individual has an omnidirectional visual field. However, in nature, individuals with limited visual fields in a flock is a more universal phenomenon. For example, the visual field of

starlings is 143°, the visual field of pigeons is 158°, and the visual field of owls is 100.5°⁴⁻⁶. Therefore, limited visual field should be included into flocking models.

Compared with the flocking model with omnidirectional visual field, limited visual field not only led to limited environment information, but also brings in a polarity for the emergence of the group behavior. Thus, we define a new flocking model called directional flocking. For each pair of individuals, if they can be seen by each other, then they are neighbors and their information can be interacted bidirectional; if agent i can be seen by agent j but agent j cannot be seen by agent i , then there exists an unequal/polar relationship. This paper will quantitatively discuss the classical flocking model and the directional flocking model caused by the limited visual field. We would like to know that whether

the directional flocking is better than the classical one in the respects of rate of convergence and stability, especially for these tasks that require individuals moving from one place to another following an empirical route, such as migration of wild goose. Besides, we will try to give an explanation on the evolution of the limited visual field.

The rest of this paper is organized as follows. The flocking model is given and the directional flocking problem is formulated in section II. Section III presents the simulation results of the two flocking models. Finally, the conclusions are drawn in section VI.

2. Modeling

2.1. Flocking model

Consider N particles moving continuously (off lattice) in a free area without any boundary limitation. Without losing generality, suppose that the time interval between two updates of the directions and positions is $\Delta t = 1$.

At $t = 0$, Each particle is randomly distributed within an area of a given size and has the same absolute velocity v_d as well as randomly distributed direction θ_i, i, \dots, N . At each time step, the position of the i th particle is updated according to

$$x_i(t+1) = x_i(t) + v_i(t)\Delta t \quad (1)$$

In each time step, the velocity of a particle $v_i(t+1)$ is updated according to the following equation

$$v_i(t+1) = v_i^{align}(t+1) \quad (2)$$

where $v_i^{align}(t+1)$ is the alignment term. The alignment term was constructed to have an absolute value v_d and a direction given by the angle $\theta_i^{align}(t+1)$. The angle was obtained from the expression

$$\theta_i^{align}(t+1) = \langle \theta_i(t) \rangle + \Delta\theta(t) \quad (3)$$

where $\Delta\theta(t)$ represents noise, which is a random number chosen with a uniform probability from the interval $[-\eta/2, \eta/2]$. $\langle \theta_i(t) \rangle$ denotes the average direction of the velocities of neighbors of the given particle i . The average direction is given by the angle

$$\langle \theta_i(t) \rangle = \arctan \left(\frac{\sum_{j=1}^N l_{ij}(t) \sin(\theta_j(t))}{\sum_{j=1}^N l_{ij}(t) \cos(\theta_j(t))} \right). \quad (4)$$

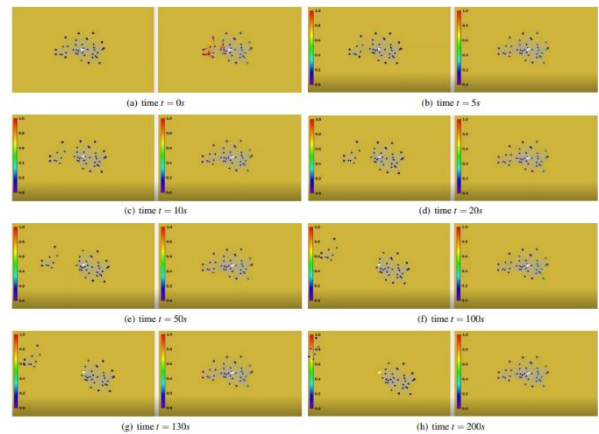
Neighbor matrix $L_N(t) = [l_{ij}(t)]_{N \times N}$ describes the neighbor relationships of particles at time t , where

$$l_{ij}(t) = c_{ij} * b_{ij} * a_{ij}(t), \forall i, j = 1, \dots, N. \quad (5)$$

Therein, $A_N(t) = [a_{ij}(t)]_{N \times N}$ is the adjacency matrix, $C_N(t) = [c_{ij}(t)]_{N \times N}$ is the contribution matrix, and $B_N(t) = [b_{ij}(t)]_{N \times N}$ is the dominance matrix.

The contribution matrix $C_N(t) = [c_{ij}]_{N \times N}$ ($c_{ij} > 0$) is defined to describe the contribution strength of each particle during the decision making process regarding the new preferred directions of the particles.

The dominance matrix $B_N = [b_{ij}]_{N \times N}$ is defined to describe the direction of information flow between each pair individuals. The direction of information flow specifies the set of particles whose behavior influences the decision of a given particle at each time step. For the classical flocking model, that is the egalitarian model, the dominance matrix is an undirected graph. For the directional flocking model, the dominance matrix is the mixed graph, since sometimes the information flow of the pairwise particles are bidirectional while sometimes the information flow of the pairwise particles is directional, which depends on whether the other one is



located in its visual field or not.

Fig. 2. The critical snapshots of the evolutionary process of the egalitarian flocking model and directional flocking model with hierarchical structure. The left one is the egalitarian flocking model and the right one is the directional flocking model.

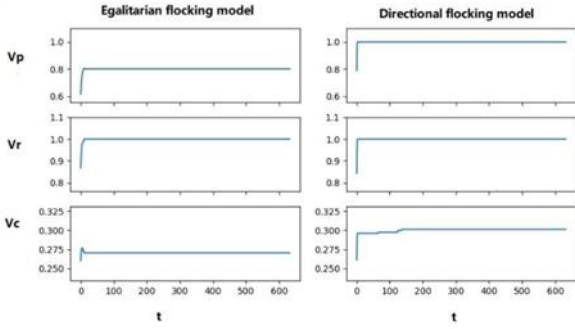


Fig. 3. Three order parameters of the two flocking models versus time.

The definition of adjacency matrix $A_N(t)$ is

$$a_{ij}(t) \begin{cases} 1, i=1, \dots, N, j \in N_i(t) \\ 0, otherwise \end{cases}, \quad (6)$$

where $N_i(t)$ is the set of j satisfying that the distance between i and j is no larger than the interaction radius r . Using the above expressions the update strategy of the velocity of each particle can be written as

$$v_i^{align}(t+1) = c^{align} v_d e_i(t) \quad (7)$$

where c^{align} is the coefficient of the alignment term. $e_i(t)$ is a unit vector with direction angle $\theta_i^{align}(t+1)$.

2.2. Order Parameters

In order to thoroughly investigate the differences between the classical flocking model with omnidirectional visual field (that is, $\theta_i = \theta_d = 180^\circ$) and the directional flocking model with limited visual field $\theta_i = \theta_d = 150^\circ$ in a quantitative way, three order parameters are given to characterize the evolutionary process of these particles governed by the above two flocking models from different point views.

The first order parameter V_p is the average value of normalized velocity of all individuals, and its formula is as follows:

$$V_p = \left| \frac{1}{N} \sum_{i=1}^N v_i \right| \quad (8)$$

of particle j . The color bar reveals the weight of the contribution of the given particle. The red particle is the strongest one, while the purple one is the weakest particle.

where $V_p \in [0, 1]$. $V_p = 1$ denotes that all the individuals in one group move with the same velocity (including value and direction), while when $V_p = 0$ equals to the initial state that the average value of the velocities of all the individuals become zero (like random distribution).

The second order parameter V_r is the cluster-dependent velocity correlation, whose definition is

$$V_r = \frac{1}{T} \frac{1}{N} \int_0^T \sum_{i=1}^N \frac{1}{N_i - 1} \sum_{j \in J_i} \frac{v_i \cdot v_j}{|v_i| |v_j|} dt \quad (9)$$

where N_i is the number of particles in the cluster that contains the i th particle, and J_i refers to the set of indices of particles that are in the same cluster as the i th particle.

The third one V_c is also a velocity correlation but not depends on cluster, whose definition is

$$V_c = \frac{1}{T} \frac{1}{N(N-1)} \int_0^T \sum_{i=1}^N \sum_{j \in J_i} \frac{v_i \cdot v_j}{|v_i| |v_j|} dt \quad (10)$$

3. Simulation Analysis

The simulations were carried out in a free two-dimensional area. We considered groups of $N=40$ particles. We aimed at comparing the rate of convergence and stability of egalitarian flocking model versus directional flocking model with hierarchical structure. In order to keep the comparability of these simulation results, both models have the same initial positions and the same initial velocities. For the directional flocking model, the visual field of each particle is $\theta_d = 150^\circ$, while that of each particle of egalitarian flocking model is $\theta_d = 180^\circ$. Besides, the element c_{ij} of the contribution matrix of directional flocking model satisfies log-normal distribution with mean value 0 and standard deviation 1, while that of egalitarian flocking model is the average value of these c_{ij} of directional flocking model. For egalitarian flocking model, pairwise particles are equal, that is, if particle i is the neighbor of particle j , then j is also the neighbor of particle i . However, for directional flocking model, pairwise particles sometimes are equal, sometimes are non-equal. That is to say, we cannot obtain the conclusion that j is the neighbor of particle i according to the condition that particle i is the neighbor

Furthermore, we record the three order parameters during the evolutionary process of 40 particles versus time in Fig. 3. It is clearly to see that the directional flocking model has a better consensus properties and a

faster convergent rate according to V_p and V_c . V_p denotes that these particles in each cluster converge to consensus finally.

4. Conclusion

We have investigated the directional flocking problem of multiagent system governed by hierarchical structure. The limited visual field of each particle produces a polar for the interaction network of these particles. The well ordered initial states and limited visual field force the flocking model has better convergent rate and consensus property than the classical flocking model (the egalitarian one), especially for these navigation tasks of multi-agent system. It is amazing that the interaction network of these particles in directional flocking model caused by limited visual field gradually from a mixed state (including bidirectional interaction and directional interaction) to a pure directional one. We cannot explain why they evolve like this. However, the conclusion conform to the collective behaviors of these real social animals in nature. Displayed equations should be numbered consecutively in each section, with the number set flush right and enclosed in parentheses.

References

1. Reynolds, C.W., "Flocks, herds, and schools: a distributed behavioral model", *ACM SIGGRAPH Computer Graphics*, 21(4), 25–34,1987.
2. Vicsek, T., Czirok, A., Ben-Jacob, E., Cohen, I., and Shochet, O. "Novel type of phase transition in a system of self-driven particles", *Physical Review Letters*, 75(6), 1226,1995.
3. Cucker, F. and Smale, S., "Emergent behavior in flocks. *IEEE Transactions on Automatic Control*", 52(5), 852–862,2007.
4. Couzin, I., Krause, J., and James, R., "Collective memory and spatial sorting in animal groups", *Journal of Theoretical Biology*, 218(1), 1–11,2002.
5. Martin, G.R., " Visual fields in woodcocks scolopax rusticola (scolopacidae; charadriiformes) ", *Journal of Comparative Physiology A*, 174(6), 787-793,1994.
6. Andreas, H. and Christian, W., " The simulation of the movement of fish schools " , *Journal of Theoretical Biology*, 156(3), 365-385, 1992.
7. Jia, Y.N. and T., V. "Modelling hierarchical flocking", *New Journal of Physics*, 21, 093048, 2019.

Authors Introduction

Dr. Yongnan Jia



She is currently an associate professor of University of Science and Technology Beijing. She received her Ph.D degree in Intelligent Control Laboratory, College of Engineering, Peking University in 2014. She is currently a post-doctoral lecturer in the School of Automation and Electrical Engineering, University of Sciences and Technology Beijing. Her research interests include intelligent robots, biomimetic robots, and collective behaviors of multi-robot system.

Ms. Jiali Han



She received her B.S. degree in automation in 2021 from the School of Engineering, Beijing Forestry University in China. He is acquiring the M.E. in University of Science and Technology Beijing. Her research interests include collective behavior analysis, and multi-agent system.

Mr. Yong Xie



He is currently an engineer in the System Design Institute of Mechanical-Electrical Engineering. He received his Master's degree in 2015 from the School of Aeronautic Science and Engineering, BUAA University in China. His research interests include intelligent control theory, positioning, and navigation of unmanned aerial vehicles.

Dr. Weicun Zhang



He is currently an associate professor of University of Science and Technology Beijing. He received his Doctor's degree in Control Theory from Tsinghua University (Beijing) in 1993. He has been a visiting research fellow at the University of Michigan from 1997 to 1998. His research interests include adaptive control, multi-agent system, positioning and navigation of unmanned vehicles.
