Formation control for rectangular agents with communication maintenance and collision avoidance

Yaxin Li, Yingmin Jia*, Jintao Hu

The Seventh Research Division and the Center for Information and Control, School of Automation Science and Electrical Engineering, Beihang University (BUAA), 37 Xueyuan Road, Haidian District, Beijing, 100191, China

E-mail: lyx2018@buaa.edu.cn, ymjia@buaa.edu.cn

www.buaa.edu.cn

Abstract

This paper mainly discusses the rectangular agents, which is not often considered in other papers. Coordinate transformation is used to describe the location relationship between two agents. Obstacle potential function between agents, obstacle function between agents and obstacles are designed to ensure no collisions. The distributed control protocol is designed to achieve desired formation with no collisions and communication maintenance between agents. Stability analysis proves the effectiveness of the algorithm.

Keywords: rectangular agents, formation control, communication maintenance, collision avoidance

1. Introduction

With the progress of science and technology, especially the unprecedented development of robots, sensors, artificial intelligence, the coordinated control system of multi-agent system has also become a research hotspot in the field of automatic control[1-3].

In the application scenario of coordinated control of multi-agent system, due to the limited communication and measurement range of a single agent, it is necessary to use multi-agent system to form a specific formation to achieve the maximum coverage and detection, such as seabed exploration and disaster search and rescue or in some military exercises and operations. Due to the rich application prospect and the challenge of group coordinated control, formation control has become one of the multi-agents coordinated control[4-6].

In the process of formation or other tasks, designing obstacle avoidance strategies between agents and between agents and environmental obstacles is the prerequisite for the safe and reliable operation of multi-agent system. Artificial potential function is often used to solve formation problems[7]. Formation and obstacle avoidance of first-order rectangular multi-agent systems have been studied[8]. Formation Maneuvering with collision avoidance and communication maintenance is also studied[9].

This article mainly solves two problems: For second-order rectangular multi-agents system

(i). How to design the control law to make the system reach the desired formation and avoid collision?

(ii). How to design the control law to make the system reach the desired formation, avoid collision and maintain communication?

Notation. *The length of the rectangle agent i is l_i, the width of the rectangle agent i is w_i. L_s denotes the Laplacian matrix of sensing graph. L_c denotes the Laplacian matrix of communication graph.*

2. Preliminaries and Problem Statement

2.1. *Graph theory*

A directed graph $G(v,\varepsilon)$ consists of a node set $v=\{1,2,\cdots n\}$ and a set of edges $\varepsilon \subset v\times v$. If the edge (j,i) is in edge ε, *j* is the in-neighbor of agent *i* and *i* is the out-neighbor of agent *j*.
For a directed graph, if there exists a path from node q_2 to node q_1, we say that node q_1 is reachable from node q_2. If there exists a node q_1, which is reachable from any other node q_i, we say the directed graph is rooted. Supposed existing a non-singleton subset of nodes Q:{Q_1, Q_2, …, Q_n}, if the exists a path from nodes {Q_1, Q_2, …, Q_n} to q_1 after removing any node in Q, we say q_1 is 2-reachable from Q. If there exists a subset of two nodes, which is 2-reachable from any other node, we say the directed graph is 2-rooted.

2.2. *Coordinate transformation*

Consider two rectangle agents 1 and 2 and one obstacle (obs) as shown in Fig. 1, where the coordinate of center of rectangle 1 (O_1) is denoted by (x_1, y_1) and vertex P_{11} (x_{11}, y_{11}) is denoted by P_{11} in the frame OXY, the coordinate of the center of rectangle 1 (O^1_1) is denoted by (x^1_1, y^1_1) and vertex P_{11} (x^1_{11}, y^1_{11}) is denoted by P^1_{11} in the frame $O_1X_1Y_1$.

Denote $P_1=\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}, P_{1k}=\begin{bmatrix} x_{1k} \\ y_{1k} \end{bmatrix}$ in the frame OXY, and $P^1{}_{1k}=\begin{bmatrix} x^1_{1k} \\ y^1_{1k} \end{bmatrix}$ in the frame $O_1X_1Y_1$ for k=1,2,3,4,

$$P^1_{1k}=R(\varphi_1)(P_{1k}-P_1) \qquad (1)$$

where $R(\varphi_1)=\begin{bmatrix} \cos(\varphi_1) & \sin(\varphi_1) \\ -\sin(\varphi_1) & \cos(\varphi_1) \end{bmatrix}$

The distance between two agents is defined as follows.

$$d_{x1}(p_{2k})=\begin{cases} \left|x^1_{2k}\right|-\dfrac{l_1}{2}, & if\ \left|x^1_{2k}\right|>\dfrac{l_1}{2} \\ 0, & otherwise \end{cases}$$

$$d_{y1}(p_{2k})=\begin{cases} \left|y^1_{2k}\right|-\dfrac{w_1}{2}, & if\ \left|y^1_{2k}\right|>\dfrac{w_1}{2} \\ 0, & otherwise \end{cases}$$

$$d_1(p_{2k})=\sqrt{d_{x1}(p_{2k})^2+d_{y1}(p_{2k})^2}$$

We define the distance between of agent 1 and agent 2 is $d_{12}=\min\left(\min_k\left(d_1(p_{2k})\right),\min_k\left(d_2(p_{1k})\right)\right)$.

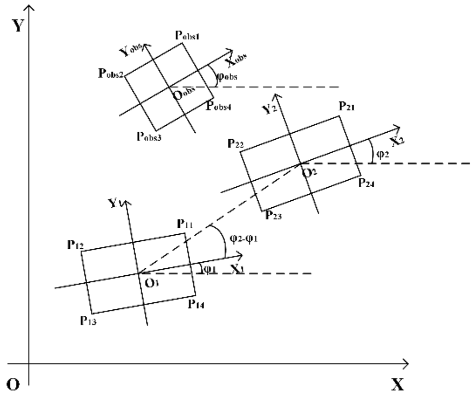

Fig. 1. Coordinate transformation between agents and obstacles

2.3. *Collision and connection region*

We define two different areas for obstacle avoidance (Ψ_i^a) and maintaining communication state (Ψ_i^c).

$\Psi_i^a=\left\{j\in\mathbb{C}: r_a\le\left\|d_{ij}\right\|\le R_a\right\}$ is safe range of obstacle avoidance. If $\left\|d_{ij}\right\|\le r_a$, obstacle avoidance may occur.

$\Psi_i^a=\left\{j\in\mathbb{C}: r_m\le\left\|d_{ij}\right\|\le R_m\right\}$ is range of maintaining communication state. If $\left\|d_{ij}\right\|\ge R_m$, agents will lose contact.

2.4. *Desired formation*

Given a desired relative position formation β: $\{0,\beta_2,\cdots,\beta_n\},\beta_i\in\mathbb{C}$, if the final position can be expressed as $\lim_{t\to\infty}p(t)=\varpi_1\beta+\varpi_2\mathbf{1_n}+v_nt1_n,\ \varpi_1,\varpi_2\in\mathbb{C}$, We say the target formation is formed.

3. Control Design

We consider a system with n agents, and the dynamic model is as follows:

$$\begin{cases} \dot{p}_i=v_i \\ \dot{v}_i=a_i \end{cases} \qquad (2)$$

where $p\in\mathbb{C}$ donates the position, $v\in\mathbb{C}$ donates the velocity, $a\in\mathbb{C}$ denotes the acceleration.
Now we consider the problem (i).
The control protocol is designed as follows:

$$u_i=u_{if}+u_{ic}, i=1,\dots,n \qquad (3)$$

where u_{if} is for formation maneuvering and u_{ic} is for collision avoidance.
u_{if} is obtained from the following equation:

$$\begin{cases} \dot{\tilde{v}}_i = \sum_{j\in M_i} a_{ij}\left(\tilde{v}_j - \tilde{v}_i\right) \\ \dot{\tilde{p}}_i = -\sum_{j\in \mathcal{N}_i^+} s_{ij}\left(p_j - p_i\right) - \alpha \tilde{p}_i \\ u_{if} = -\sum_{j\in \mathcal{N}_i^+} s_{ij}^* \tilde{p}_i + \sum_{j\in \mathcal{N}_i^-} s_{ji}^* \tilde{p}_j + \tilde{v}_i - v_i \end{cases} \tag{4}$$

where $\tilde{p}_i$ is an auxiliary variable which is used to achieve the desired formation, $\tilde{v}_i$ is an auxiliary variable which is used to make the velocity synchronize.
M_i is the neighbor of agent i in communication graph, N_i^+ is the in-neighbor of agent i in sensing graph, N_i^- is the out-neighbor of agent i in communication graph.
a_{ij} can be any positive real number, $s_{ij} \in \mathbb{C}$ satisfies $\sum_{j\in \mathcal{N}_i^+} s_{ij}(\zeta_j - \zeta_i) = 0,\ for\ i = 1,\dots,n$.
The value of variable α satisfies

$$\alpha > \frac{\sqrt{1+4\lambda_{\max}\left(L_s^* L_s\right)} - 1}{2}$$

Proof.
Donate

$$p = \left[p_1, p_2, \cdots p_n\right]^T, v = \left[v_1, v_2, \cdots v_n\right]^T, \tilde{p} = \left[\tilde{p}_1, \tilde{p}_2, \cdots \tilde{p}_n\right]^T,$$

$$v = \left[\tilde{v}_1, \tilde{v}_2, \cdots \tilde{v}_n\right]^T, K_1 = \begin{bmatrix} 0 & I_n & 0 \\ 0 & -I_n & -L_s^* \\ L_s & 0 & -\alpha I_n \end{bmatrix}, K_2 = \begin{bmatrix} 0 \\ I_n \\ 0 \end{bmatrix},$$

$$\sigma_1 = p - \bar{v}t\mathbf{1}_n, \sigma_2 = v - \bar{v}\mathbf{1}_n, \sigma_3 = \tilde{v} - \bar{v}\mathbf{1}_n, \bar{v} = \frac{c^T \tilde{v}(0)}{c^T \mathbf{1}_n}$$

Then the system can be described as:

$$\begin{bmatrix} \dot{p} \\ \dot{v} \\ \dot{\tilde{p}} \\ \dot{\tilde{v}} \end{bmatrix} = \begin{bmatrix} 0 & I_n & 0 & 0 \\ 0 & -I_n & -L_s^* & I_n \\ L_s & 0 & -\alpha I_n & 0 \\ 0 & 0 & 0 & L_c \end{bmatrix} \begin{bmatrix} p \\ v \\ \tilde{p} \\ \tilde{v} \end{bmatrix}$$

We transform this system into a cascade system

$$\boxed{\dot{\tilde{v}} = -L_c \tilde{v}} \xrightarrow{+} \otimes \xrightarrow{-} \boxed{\begin{bmatrix} \dot{p} \\ \dot{v} \\ \dot{\tilde{p}} \end{bmatrix} = K_1 \begin{bmatrix} p \\ v \\ \tilde{p} \end{bmatrix} + K_2 \tilde{v}}$$

By analyzing the asymptotic convergence of the zero input system, we can solve the range of α .
We design a potential function Θ_{ij} :

$$\Theta_{ij} = \begin{cases} \left(\dfrac{\left|p_i - p_j\right|^2}{\sqrt{\tan \mu_1\left(\left|p_i - p_j\right|^2 - r_a\right)}} \right)^{1.4}, & \text{if } p_j \in \Psi_i^a \\ 0, & \text{otherwise} \end{cases}$$

where μ_1 is a constant.

$$\vartheta_i = -\sum_{p_j \in \Psi_i^a} \frac{\partial \Theta_{ij}}{\partial p_i},\ i = 1,\dots,n \quad u_{ic} = \left(\left|u_{if}\right| + \mu_3\right)\operatorname{sgn}\left(\vartheta_i\right) \tag{5}$$

where μ_3 is a constant. □

Theorem 1. *Using control protocol in Eq. (3)(4)(5), the system can achieve the desired formation and avoid collision.*

Proof[7]**.** The proof is similar to Theorem 3.1. □
Now we consider the problem (ii).
The control protocol is designed as follows:

$$u_i = u_{if} + u_{ic} + u_{ia}, i = 1,\dots,n \tag{6}$$

We design a potential function Λ_{ij} :

$$\Lambda_{ij} = \begin{cases} \left(\sqrt{\tan \mu_2\left(\left|p_i - p_j\right|^2 - r_m^2\right)} \right)^2, & \text{if } p_j \in \Psi_i^c \\ 0, & \text{otherwise} \end{cases}$$

where μ_2 is a constant.

$$\zeta_i = -\sum_{p_j \in \Psi_i^a} \frac{\partial \Lambda_{ij}}{\partial p_i},\ i = 1,\dots,n \quad u_{ia} = \left(\left|u_{if}\right| + \mu_4\right)\operatorname{sgn}\left(\zeta_i\right) \tag{7}$$

where μ_4 is a constant.

Theorem 2. *Using control protocol in Eq. (4)(6)(7), the system can achieve the desired formation, avoid collision and maintain.*

Proof. The proof is similar to Theorem 3.1. □

4. Conclusion

This paper solves two problems for second-order rectangle agents: communication maintenance and collision avoidance. We use the coordinate transformation of different coordinate systems to obtain the distance information between agents. We design the potential function to keep the agents at a corresponding distance from each other. The corresponding control law is designed to achieve the desired goal.

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Authors Introduction

Mr. Yaxin Li

He received the B.S. degree in automation from Shandong University at 2019. He is currently pursuing the Ph.D. degree with the Seventh Research Division and the Center for Information and Control, School of Automation Science and Electrical Engineering, Beihang University.

Prof. Yingmin Jia

He received the B.S. degree in control theory from Shandong University, China, in 1982, and the M.S. and Ph.D. degrees both in control theory and applications from Beihang University, China, in 1990 and 1993, respectively. Then, he joined the Seventh Research Division at Beihang University where he is currently Professor of automatic control. From February 1995 until February 1996 he was a Visiting Professor with the Institute of Robotics and Mechatronics of the German Aerospace Center (DLR), Oberpfaffenhofen, Germany. He held an Alexander von Humboldt (AvH) research fellowship with the Institute of Control Engineering at the Technical University Hamburg-Harburg, Hamburg, Germany, from December 1996 until March 1998, and a JSPS research fellowship with the Department of Electrical and Electronic Systems at the Osaka Prefecture University, Osaka, Japan, from March 2000 until March 2002. He was a Visiting Professor with the Department of Statistics at the University of California Berkeley from December 2006 until March 2007. Dr. Jia was the recipient of the National Science Fund for Distinguished Young Scholars in 1996, and was appointed as Chang Jiang Scholar of the Ministry of Education of China in 2004. He has been Chief Scientist of the National Basic Research Program of China (973 Program) since 2011, and in particular he won the Second Prize of National Technology Invention Award in 2015. His current research interests include robust control, adaptive control and intelligent control, and their applications in robots systems and distributed parameter systems.

Mr. Jintao Hu

He received the B.S. degree in automation in 2019 from Chongqing University in China. He is currently pursuing the Ph.D. degree with the Seventh Research Division and the Center for Information and Control, School of Automation Science and Electrical Engineering, Beihang University.