Geometry Structure Oriented Nonlinear Internal Model Based Manifold Consensus

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Abstract

This note comes with manifold consensus based on nonlinear internal model. To be special, scheme demonstrated here is not necessary to inject the nonlinear internal model with additional extraneous augmented system. And this amazing result is made possible empowered by geometry structure, to be specific, Riemannian metric is employed to modeling the internal model of the nonlinear manifold. In case of completeness, the consensus of a first order linear agent and another one second order oscillator is provided to verify the suggested program.

Keywords: geometry structure, manifold following, Riemannian metric, oscillator agent

1. Introduction

The consensus of the single simple agent and the complex dynamics of the nonlinear agent is full of meaning[1], the promising application of this research can be seen in different areas, especially in robotic fields[2-5]. Examples such as task cooperation, teams' agreement and such et al. The advantage of this heterogeneous

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agents' deployment not only saves money but also sharpens the problem.

However, it is so tough to realize consensus between nonlinear agent and linear agent, like second oscillator agent in nonlinearity and one first order linear agent in linearity. Traditional method is to inject nonlinear internal model by introducing extraneous augmented system. It has several disadvantages, the first one is that it is not easy to found the exact nonlinear internal model， especially when some uncertainties exist, and another adverse point is that the traditional method needs the augmented dimension, the augmented system increases the dimensionality of the system, and this complicates the system further.

This note takes advantage of the geometry characteristics of the nonlinear agent, especially its Riemannian metric specialty to compact the nonlinear internal model. The compacted nonlinear internal model and the linear internal model is introduced in the control function at first, then followed by the high gain linear feedback controller. The jointed action of them makes consensus possible.

The note will be arranged as follows. At start, problem formulation will be proposed, then after, the main results will be presented in the second part. followed will be some conclusions.

2. Geometry Structure Oriented Nonlinear Internal Model

This part will begin with the introduction of the geometry structure oriented nonlinear internal model.

2.1. *Problem formulation*

Assume the following agents' model is already.

$$\begin{cases} \dot{X} = f(X) + h(X)U_X, X \in R^n, \\ \dot{Y} = AY + BU_Y, Y \in R^m, \\ n > m, \end{cases} \tag{1}$$

In Eq.1, we have two kinds of agents, one is the nonlinear agent, its state demonstrated as X, and another one is linear agent, its state is demonstrated as Y here. Our aim is to design controller to realize the consensus between X and Y.

Comment 1: It is not necessary to realize the consensus of all the sub variables of the linear agent to the nonlinear agent all the times, for this requirement is too strong to be practical. The accepted result is that some of the linear agent sub variables and some of the nonlinear agent sub variables come to consensus.

2.2. *The geometry structure*

Riemannian metric can be used to describe the nonlinear degree of the dynamics of the nonlinear agents.

Let *M* be a Riemannian manifold, *TM* is the tangent vector among *M*, the metric on *TM* is often denoted by $g = (g_p)_{p \in M}$ [6-7]. Under local coordinates, Riemannian metric is often noted as:

$$g = \sum\nolimits_{ij} g_{ij} dx_i \otimes dx_j$$

or simply

$$g = \sum\nolimits_{ij} g_{ij} dx_i dx_j ,$$

where

$$g_{ij}(p) = \left\langle \left(\frac{\partial}{\partial x_i}\right)_p, \left(\frac{\partial}{\partial x_p}\right)_p \right\rangle_p \tag{2}$$

In straight and flat space, the Riemannian metric is reduced to Euclidean metric.

Comment 2: Take one circle in Euclidean space as example, its radius length is *r*, the value of *r* can demonstrate its nonlinear degree, the bigger the value of *r*, the smaller the nonlinear degree of the circle, the Riemannian metric can be characterized as its arc length.

2.3. *Nonlinear internal model*

Riemannian metric as the representation of the nonlinear degree of the dynamics of the nonlinear agents, and it can be used as the nonlinear internal model to construct the controller. Take Eq.1 as example, the controller designed under nonlinear internal model empowered by Riemannian metric can be factorized as three steps:

At the first step, the nonlinear internal model and linear internal model of the nonlinear agent and the linear agent is prepared, and at the second step the linear feedback law is added, and the third step is to pour them two together to the agent systems properly.

To be formalized, the controller for Eq.1 can be described as follows:

$$\begin{cases} U_X = [h(X)]^{-1} \left\lfloor g_{ij}(Y) - g_{ij}(X) + K(Y - X) \right\rfloor, \\ U_Y = B^{-1} \left[g_{ij}(X) - g_{ij}(Y) + K(X - Y) \right], \end{cases} \tag{3}$$

where that capital K is the coefficient matrix of the linear feedback controller, it reflects the feedback gain.

Comment 3: Here the nonlinear internal model is embedded in geometry metric, and the geometric metric is related to its local coordinate, that is geometric is localized quantity.

Comment 4: The regulation law of Equ.3 is composed of two different parts, one is from the nonlinear internal model representation, and another one is from the linear feedback controller.

Comment 5: The nonlinear internal model is responsible for driving the controlled system into the flat and straight space, and the linear feedback controller is in charge of driving the error of the consensus into the satisfied region.

3. Case Study

In this section, the case study of geometry metric oriented nonlinear internal model based consensus will be realized for an oscillator agent of the second order and a first order linear agent.

3.1. *The mathematical model*

The mathematical model of the targeted system is described as follows:

$$\begin{cases} \dot{X} = f(X) + h(X)U_X, X \in R^n, \\ \dot{Y} = AY + BU_Y, Y \in R^m, \\ f(X) = \begin{bmatrix} -x_2 \\ x_1 \end{bmatrix}, \\ h(X) = 1, \\ X \in R^2, \\ A = 0, \\ B = 1, \\ Y \in R^1, \end{cases} \tag{4}$$

For oscillator agent without controller coming in, its locus is a circle and the radius of the circle is determined by the initial value of the agent. And its geometry metric is listed as follows:

$$\begin{cases} g_{ij}(X) = \frac{1}{2}\left|x_i x_j\right|, \\ g_{ij}(Y) = y_i y_j, \end{cases} \tag{5}$$

Under this assumption, we can write down the regulation law of the controller to Equ.4 as follows:

$$\begin{cases} U_X = U_{Xa} + U_{Xb}, \\ U_{Xa} = y^2 - \frac{1}{2}\left|x_1 x_2\right|, \\ U_{Xb} = k(y - x_1), \\ U_Y = U_{Ya} + U_{Yb}, \\ U_{Ya} = \frac{1}{2}\left|x_1 x_2\right| - y^2, \\ U_{Yb} = k(x_1 - y), \end{cases} \tag{6}$$

Comment 6: Notice that abiding to the match principle, regulation action to oscillator is added on its first sub variable, for we want to realize the consensus among the first sub variable of the oscillator to the first order linear agent.

3.2. *Simulation results*

To oscillator agent and linear agent described in Eq.4, we select feedback gain *k=400*, and the initial values of the oscillator as -8, 2, and the initial value of the linear agent as 0, the simulation result is described in Fig.1 as follows:

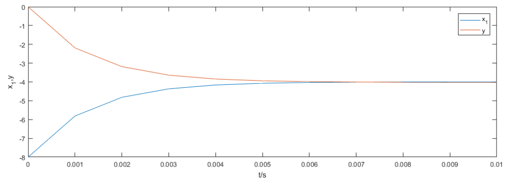

Fig.1 Case study example results

It is not so difficult for us to get that the consensus of the linear agent and the oscillator agent is made possible here.

3.3. *Further comments*

Inspiration initialized here can be generalized to the system of chaos, synchronization as well as robotics cooperation, it is just start of the suggested scheme, promising results are expected in the near future.

System complexity, could not be tractable if we are kept apart away from the concrete systems. Studies like performance improvement of vehicle driving[8-9], can be the main carrier. Time varying and distributive delays, can also be the negative factors. Adaptive strategy in Jia[10] gave a good example.

4. Conclusion

Nonlinear internal model is badly reputed in construction for its complexity, this note provides an alternative way to construct nonlinear internal model to realized consensus among oscillator agent and the linear agent, some promising results will be expected of this research in the near future.

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