A New Hyperchaotic Financial System

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Abstract

In this paper, a new hyperchaotic financial system is obtained based on a financial system. It is first transformed into Kolmogorov model, which is composed of conservative torque, dissipative torque and external torque, to study the reason why the new system can generate chaos. Then, by studying energy exchange and combining different torques, dynamics of the new system is analyzed, the external torque is found to be the main reason the new system generate chaos. The paper provides a new method of analyzing chaotic dynamics in financial system, and further promotes new strategies are found to control chaos in financial market.

Keywords: hyperchaos, Kolmogorov model, Hamiltonian energy, financial market

1. Introduction

Nowadays, the economic society is becoming more and more complex and full of uncertainty. In 1980, Stutzer, an American economist, for the first time revealed the chaotic phenomenon of macroeconomic system in the economic growth equation in reference [1]. This made people realize the limitations of the economic model based on the traditional economic theory, and applied the chaotic model to economics. In 2007, the U.S. subprime mortgage crisis triggered the global economic crisis, which once again showed the existence of butterfly effect and chaos in the financial system. This has attracted extensive attention of researchers [2-8].

As we all know, most physical models are open systems [9-11], which can dissipate energy, generate energy, store energy and exchange energy with the external environment. Financial system is a complex nonlinear system, which is composed of many elements. It is open and far from equilibrium. In this nonlinear system, there is energy exchange. Therefore, chaos is a very common phenomenon. Since the discovery of chaos in economics, it has had a great impact on the famous economics. So far, there are few research results on the mechanics and energy analysis of hyperchaotic financial system.

In this paper, a new dynamic model of hyperchaotic financial system is constructed and studied. The Hamiltonian energy of the system under different torque combinations is studied by numerical method and MATLAB. It is found that the external torque is an important reason for the chaotic behavior of the system. The result analysis is helpful to predict and solve some crisis problems in the economic market, and has important theoretical and practical significance in the financial field.

2. A New Hyperchaotic Financial System Model

References [12] reported that a hyperchaotic financial system model consists of four state variables: the interest rate x, the investment demand y, the price exponent z, and the average profit margin u. The hyperchaotic finance system is given as follows:

$$\begin{cases}\dot{x}=z+(y-a)x+u\\ \dot{y}=1-by-x^2\\ \dot{z}=-x-cz\\ \dot{u}=-dxy-ku\end{cases}\tag{1}$$

Based on system (1), the change of interest rate is not only affected by the average profit margin, but also easily affected by the interaction between investment cost and profit margin. A new hyperchaotic financial system is found, which more accurately reflects the actual financial market. The model is as follows:

$$\begin{cases}\dot{x}=z+(y-a)x+duy\\ \dot{y}=1-by-x^2\\ \dot{z}=-x-cz\\ \dot{u}=-dxy-ku\end{cases}\tag{2}$$

Where a is the saving, b is the per investment cost, c is the elasticity of demands of commercials, and they are positive constants.

3. Transformation of Hyperchaotic Financial System

Every parameter and state variable of financial system has practical significance. Studying and analyzing their impact on chaotic behavior is of great value to the prevention of financial risk. Convert system (2) into Kolmogorov form [13-15]:

$$\dot{X}=\{X,H\}-\Lambda X+f$$
$$=\begin{pmatrix}0 & x & 1 & dy\\ -x & 0 & 0 & 0\\ -1 & 0 & 0 & 0\\ -dy & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}x\\ y\\ z\\ u\end{pmatrix}-\begin{pmatrix}ax\\ by\\ cz\\ ku\end{pmatrix}+\begin{pmatrix}0\\ 1\\ 0\\ 0\end{pmatrix}\tag{3}$$

Generally, $\{X,H\}$ refers to the conserved parts of (3), it includes the inertial torque and the internal torque. ΛX and f represents the dissipative part and the external torque of (3), respectively. Furthermore, there are three torques in (2): the conservative torque, the dissipative torque and the external torque.

4. Torque and Energy Analysis

4.1. The conservative torque

When there is only conservative moment, the system equation is:

$$\begin{cases}\dot{x}=xy+z+duy\\ \dot{y}=-x^2\\ \dot{z}=-x\\ \dot{u}=-dxy\end{cases}\tag{4}$$

Eq. (4) is a conservative system only affected by the inertial torque and the internal torque, and its corresponding Hamiltonian energy is a constant and $\dot{H}=0$. Which means the Hamiltonian energy is not exchanged with the dissipative energy and external supplied energy. When the initial value is $(1,2,0.5,0.5)$, the $H=2.75$. The system state variables and Hamiltonian energy are shown in Fig.1. All these results from numerical analysis further verify system under the conservative torque is conservative. Therefore, it can be obtained that the system can't generate chaos only under the conservative torque, and the corresponding Hamiltonian energy is invariable.

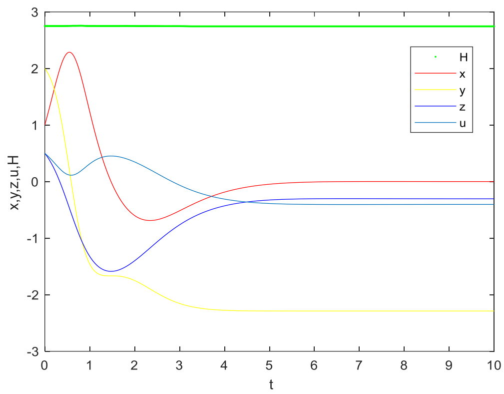

Fig.1 State variables and Hamiltonian energy of system (4)

In this case, chaos is not found to exist in the system. The interest rate, price index and average profit margin tend to zero, indicating that the economic market has lost its vitality.

4.2. The conservative torque and the dissipative torque

Under both the conservative torque and the dissipative torque, the system equation is:

$$\begin{cases}\dot{x}=z+(y-a)x+duy\\ \dot{y}=-by-x^2\\ \dot{z}=-x-cz\\ \dot{u}=-dxy-ku\end{cases}\tag{5}$$

At this time, the Hamiltonian energy function of the system (5) is selected as the Lyapunov function. Hamiltonian derivative is

$\dot{H}=-(ax^2+by^2+cz^2+ku^2)\leq 0$. According to Lyapunov stability theory, the system (5) is asymptotically stable. The state variable converges to zero. Because there is no external input, the system energy dissipates, and the phase space volume shrinks to zero over time. The state variables and Hamiltonian energy of the system (5) are shown in Fig.2. It can be seen that the interest rate, investment cost, price index, average profit margin and Hamiltonian energy all tend to zero. The economic market is paralyzed and completely lost its vitality.

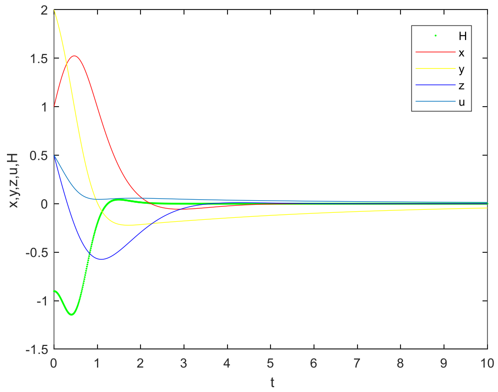

Fig.2 State variables and Hamiltonian energy of system (5)

4.3. The conservative torque and the external torque

Under both the conservative torque and the external torque, the system equation is:

$$\begin{cases}\dot{x}=xy+z+duy\\ \dot{y}=1-x^2\\ \dot{z}=-x\\ \dot{u}=-dxy\end{cases} \quad (6)$$

At this time, the Hamiltonian energy derivative is $\dot{H}=y$, Hamiltonian energy is $H=\frac{1}{2}y^2$, Hamiltonian energy and its derivatives are shown in Fig.3. Investment demand is also affected by interest rate, price index and average profit margin. Hamiltonian energy will change and alternate with the irregular movement of investment cost. Therefore, it is chaotic in a limited area, as shown in Fig.4.

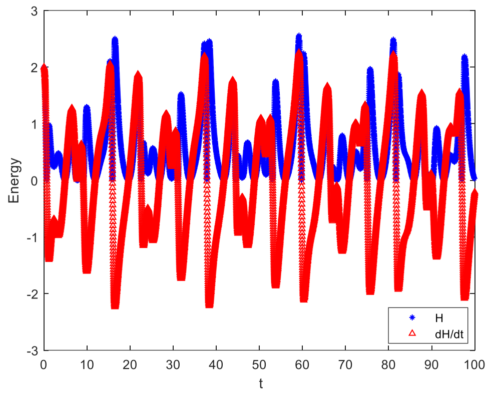

Fig.3 Hamiltonian energy and its derivative of system (6)

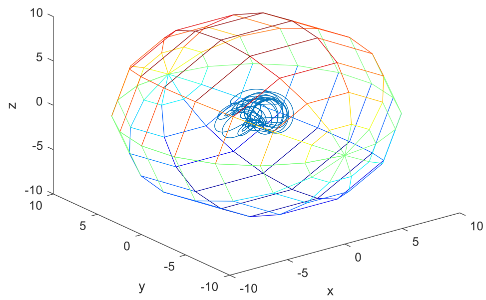

Fig.4 Phase diagram in finite region of system (6)

The external torque f is equivalent to an external excitation, which adds the investment rate 1 to the system, increases the dynamic behavior of the system. This makes the state variable no longer tend to zero, and the system appears chaos, which means that it adds vitality to the economic market. It can be seen that external incentive is essential to market economy, and it is also the main reason for the chaos of financial market.

4.4.The full torque

Under the full torque, the equation is system (3). At this time, the Hamiltonian energy derivative $\dot{H}=-(ax^2+by^2+cz^2+ku^2)+y$ is shown in Fig.5. When the system parameter is $a=0.9$, $b=0.2$, $c=1.5$, $d=0.33$, $k=0.17$, and the initial value point is selected $(1,2,0.5,0.5)$, the Hamiltonian energy distribution of the system (3) is shown in Fig.6. Different colors represent different Hamiltonian energies, and the color from yellow to dark blue represents the energy from high to low, indicating the energy change of the attractor. Dissipative torque dissipates energy, and external torque promotes energy absorption. The two interact, and their energy changes are disordered, and the Hamiltonian energy derivative is high and low.

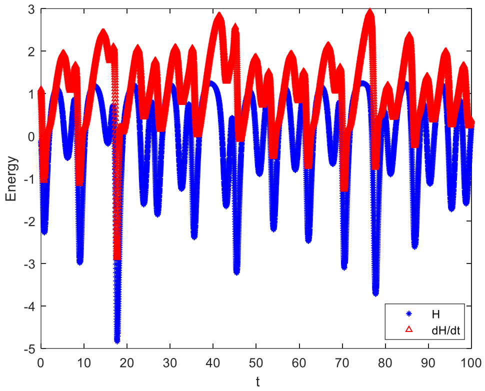

Fig.5 Hamiltonian energy and its derivative of system (3)

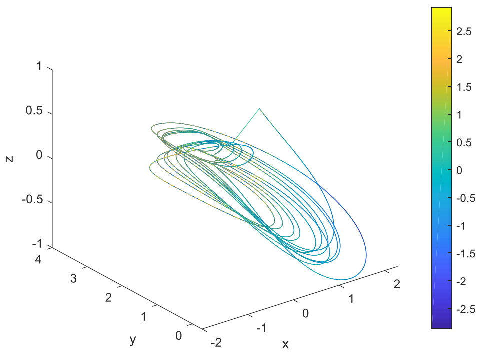

Fig.6 Chaos attractor with Hamiltonian energy

By further analyzing, we find that the interaction between the external torque and other torques is main reason why hyperchaotic financial system can generate chaos and its dynamics characteristics is related to the variation of Hamiltonian energy.

5. Conclusion

Based on the original financial system and combined with the actual situation, a new hyperchaotic financial system is established. In addition, by transforming the system into Kolmogorov type system, the dynamic characteristics of the system are analyzed. It is found that the interaction between external torque and other torques is the main reason for the chaos of the system. At the same time, the influence of energy change on dynamic characteristics is studied by energy analysis. Finally, combined with the practical significance of various parameters and variables, this paper makes a reasonable explanation for various phenomena, which is of great value to the regulation of financial market.

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Authors Introduction

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