Matrix Approach to Current-state Detectability of Discrete-event Systems

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Abstract

In our previous work, a matrix-based framework is proposed to tackle the problem of verifying strong detectability in the context of partially-observed nondeterministic discrete event systems (DESs). Two key concepts, namely, unobservable reach and detector, are redefined therein. Also, the dynamics of a detector, under the frameworks of the Boolean semi-tensor product of matrices, are converted equivalently into an algebraic representation. In this paper, we extend our previous work to other versions of detectability, including strong periodic detectability, weak detectability, and weak periodic detectability. Several necessary and sufficient conditions are derived for verifying aforementioned three types of detectability, respectively. Compared with the existing ones, the proposed methodology is easier to be implemented in software in the sense that it avoids the symbolic manipulations. Finally, an example is given to illustrate the theoretical results.

Keywords: Discrete event systems, state estimation, detectability, semi-tensor product of matrices.

1. Introduction

State estimation is an important and interesting topic in systems and control theory, and has many applications. For instance, in medical systems, we need to know the disease stage of a patient. Yet in remote and distributed systems, one hopes generally that a central station to be able to determine the state of a remote system with limited communications. The state estimation problem has drawn considerable attentions in the context of discrete event systems (DESs)[1-9].

The problem of state estimation of DESs has been investigated widely in terms of detectability of DESs that are based on finite automata models, which says that whether or not one can know exactly the current and subsequent states of original system after a finite delay. The notion of detectability[4] was initially proposed for the deterministic DESs, where four types of detectability, namely, strong (periodic) detectability and weak (periodic) detectability, were defined. Later on, the concepts of other types of detectability came up and have been further extended to other classes of systems by many others[5-8]. For instance, the problem of verifying detectability has been studied in the frameworks of nondeterministic DESs[5,6] and stochastic DESs[7], respectively. Recently, trajectory detectability[8] of DESs has been investigated. Complexity of determining detectability[9] for DESs has been studied. When the original system is not detectable, some approaches[10,11] have been developed to enforce provably detectability by synthesized optimal supervisor (if exists) that restricts the system's behavior in a manner.

In our previous work[6], we developed a matrix-based approach to discuss strong state detectability of

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nondeterministic DESs from a new angle. A matrix-based verification criterion with polynomial complexity in the size of system for verifying strong detectability is derived. Strong detectability says that whether or not we can know precisely the current and subsequent states of original system after a finite number of delays. However, the goal may be too rigid in practice. In this regard, extending strong detectability to strong periodic detectability and/or weak (periodic) detectability could be necessary. Additionally, developing a novel matrix-based methodology to tackle simultaneously different types of detectability is still interesting. In this paper, we will solve aforementioned these problems.

Notice that, although the study of detectability[4,5] has been considered, our verification criterions are totally different from them. First, the proposed approach is matrix-algebra-based form by using a new tool, called the semi-tensor product (STP) of matrices[12]. While the existing ones are based on design of algorithms. Second, the proposed approach is easier to be implemented in softwares in the sense that all obtained results are numerically tractable instead of graph-based symbolic manipulations.

The remainder of this paper is arranged as follows. Section 2 introduces some basic notations and concepts needed in this paper. Section 3 provides some matrix-based criterions to verify different types of detectability by means of the developed methodology. In Section 4, an example is presented to illustrate the application of the obtained results. Finally, we conclude the paper in Section 5.

2. Preliminaries

2.1. Notations

$|X|$ denotes the cardinality of set X. $\mathbb{M}_{m\times n}$ denotes the set of $m\times n$ real matrices. $M_{(i,j)}$ denotes the *(i,j)* element of matrix M. $Col_j(M)$ denotes the *j-th* column of matrix M. $Col(M)$ denotes the set of all columns of matrix M. A matrix $B\in\mathbb{M}_{m\times n}$ is a Boolean matrix if $B_{(i,j)}\in\mathcal{D}=\{0,1\},\ \forall\ 1\le i\le m,\ 1\le j\le n$. We use $B_{m\times n}$ to denote the set of $m\times n$ Boolean matrices. $\delta_n^0:=[0,0,\cdots,0]^T$. $\delta_n^k:=Col_k(I_n)$, where I_n is the identity matrix of dimension n, $1\le k\le n$. $\Delta_n:=\{\delta_n^1,\delta_n^2,\cdots,\delta_n^n\}$; $\tilde{\Delta}_n:=\{\delta_n^1,\delta_n^2,\cdots,\delta_n^n\}$. $L\in\mathbb{M}_{m\times n}$ is a generalized logical matrix if $Col(L)\subseteq\tilde{\Delta}_m$.We denote the set of $m\times n$ generalized logical matrix by $\mathcal{L}_{m\times n}$. For brevity, $L\in\mathcal{L}_{m\times n}$ is denoted as $L=\delta_m[i_1,\ i_2,\cdots,i_n]$, $i_k\in\{0,1,\cdots,m\},1\le k\le n$.

2.2. Semi-tensor product (STP) of matrices

Definition 2.1[12] *Let* $A\in\mathbb{M}_{m\times n}$ *and* $B\in\mathbb{M}_{p\times q}$. *The STP of A and B is defined as*

$$A\ltimes B=\left(A\otimes I_{t/n}\right)\left(B\otimes I_{t/p}\right),\tag{1}$$

where t denotes the least common multiple of n and p, i.e., t = lcm(n, p); $\otimes$ *is the Kronecker product.*

Remark 2.1 *When* $n=p$, $A\ltimes B=AB$. *Hence, the STP is a generalization of the standard matrix product. We mostly omit the symbol "*$\ltimes$*" hereinafter.*

Lemma 2.1[12] *Let* $X\in\mathbb{R}^m$ *and* $Y\in\mathbb{R}^n$ *be two column vectors. Then*

$$W_{[m,n]}XY=YX,\quad W_{[m,n]}YX=XY.\tag{2}$$

where $W_{[m,n]}=[\delta_n^1\delta_m^1,\cdots,\delta_n^n\delta_m^1,\cdots,\delta_n^1\delta_m^m,\cdots,\delta_n^n\delta_m^m]$.

Lemma 2.2 [12] *Let* $\delta_{n_1}^{i_1}\delta_{n_2}^{i_2}=\delta_{n_1n_2}^{i_{12}}$, *then we have* $i_{12}=(i_1-1)n_2+i_2$.

2.3. Boolean algebra

Here we introduce some notations from the binary algebra of binary matrices (or called Boolean matrices) that will be used later on.

Definition 2.2[13] *Assume that* $A=(a_{ij})_{m\times n}\in\mathcal{B}_{m\times n}, B=(b_{ij})_{m\times n}\in\mathcal{B}_{m\times n}$. *The Boolean addition of A and B is defined as*

$$A\times_{\mathcal{B}}B:=(a_{ij}\vee b_{ij})_{m\times n}\in\mathcal{B}_{m\times n},\tag{3}$$

where the symbol "$\vee$*" is the logical operators OR.*

Definition 2.3[13] *Assume that* $A=(a_{ij})_{m\times n}\in\mathcal{B}_{m\times n}$, $B=(b_{ij})_{n\times s}\in\mathcal{B}_{n\times s}$. *The Boolean product of A and B is defined as*

$$A\times_{\mathcal{B}}B:=C=(c_{ij})_{m\times s}\in\mathcal{B}_{m\times s},\tag{4}$$

where $c_{ij}=\vee_{k=1}^n\left(a_{ik}\wedge b_{kj}\right)$; *the symbols "*$\vee$*" and "*$\wedge$*" denote the logical operators OR and AND, respectively.*

Definition 2.4 *Assume that* $A=(a_{ij})_{m\times n}\in\mathcal{B}_{m\times n}$, $B=(b_{ij})_{p\times q}\in\mathcal{B}_{p\times q}$. *The Boolean semi-tensor product of A and B is defined as*

$$A\ltimes_{\mathcal{B}}B:=\left(A\otimes I_{t/n}\right)_{\mathcal{B}}\left(B\otimes I_{t/p}\right),\tag{5}$$

where t = lcm(n, p).

Remark 2.2 *From now on, the following all matrix product (resp., matrix addition) is assumed to be the*

Boolean semi-tensor product (resp., Boolean addition) and the symbol "$\ltimes_{\mathcal{B}}$" (resp., $+_{\mathcal{B}}$) will also be omitted hereinafter when there is no danger of confusion.

2.4. System model

The discrete event system (DES) of interest is modeled as a nondeterministic finite automaton (NFA) that is a five-tuple $G=(X,\Sigma,\delta,X_0,X_m)$, where X is a finite set of states, Σ is a finite set of events, $X_0\subseteq X$ is the set of initial states, $X_m\subseteq X$ is the set of marked states (or accepted states), $\delta: X\times\Sigma\rightarrow 2^X$ is the partial transition function, which describes the system dynamics: given states $x,y\in X$ and an event $\sigma\in\Sigma$, $y\in\delta(x,\sigma)$ means that there is a transition labeled by event σ from state x to state y. Note that $\delta(x,\sigma)$ is undefined when the event σ cannot be executed at state x. We use $\delta(x,\sigma)!$ to denote that $\delta(x,\sigma)$ is well-defined. Obviously, the transition function can be extended to $\delta: X\times\Sigma^*\rightarrow 2^X$ in the usual manner, where Σ^* denotes the set of finite strings on the alphabet Σ, including the empty string ϵ.

For brevity, we assume that $G=(X,\Sigma,\delta,X_0,X_m)$ is deadlock free (also called alive), i.e., for each state $x\in X$, there is at least a corresponding event $\sigma\in\Sigma$ such that $\delta(x,\sigma)!$. It should be pointed out that this assumption is without essential loss of generality, since it can be relaxed by adding observable self-loops at terminal states.

When system G is partially observed, its event set is partitioned into two disjoint parts: the observable part Σ_o and the unobservable part Σ_{uo}, i.e., $\Sigma_o\cup\Sigma_{uo}=\Sigma$ and $\Sigma_o\cap\Sigma_{uo}=\emptyset$. The natural projection $P\colon \Sigma^*\rightarrow\Sigma_o^*$ is defined by

$$P(\epsilon)=\epsilon \text{ and } P(s\sigma)=\begin{cases}P(s)\sigma, & if \sigma\in\Sigma_o;\\ P(s), & if \sigma\in\Sigma_{uo}.\end{cases} \quad (6)$$

3. Detectability of partially-observed DESs

3.1. Problem statement

In the paper, so-called state estimation is based on observations of some events and states. More explicitly, the event observation is described by the projection $P\colon\Sigma^*\rightarrow\Sigma_o^*$, while the state observation is described by the output map $h: X\rightarrow Y$ where Y denotes a finite output set. In this regard, a partially-observed nondeterministic DES with the event and state observations can be described as follows.

$$G_o=(G,P,h,\Sigma_o,Y), \quad (7)$$

where $G=(X,\Sigma,\delta,X_0,X_m)$.

Consequently, the problem of state estimation of partially-observed DESs, under the framework of detectability, can be formalized as follows. Given a partially-observed nondeterministic DES (7), we do not know the set of initial states of system G_o, while we have partial event observations (i.e., $\Sigma_o\subset\Sigma$) and some state observations (i.e., $Y\neq\emptyset$). Whether we can know exactly the current and subsequent states of system (7) after a finite number of observations. Formally, we give the concepts of three types of detectability of system (7) below.

Definition 3.1 *A partially-observed nondeterministic DES (7) is said to be weakly detectable, if its current and subsequent states can be precisely determined by some admissible input-output strings after a finite number of observations.*

Definition 3.2 *A partially-observed nondeterministic DES (7) is called strongly (resp., weakly) periodically detectable, if its current state can be periodically determined by all (resp., some) admissible input-output strings after a finite number of observations.*

3.2. Algebraic expression of detector

To investigate aforesaid three types of detectability, our previous work[6] defines the unobservable reach of a state $x\in X$ for partially-observed nondeterministic DES (7).

Definition 3.3 *Given a partially-observed nondeterministic DES (7), the unobservable reach of state $x\in X$, denoted by $UR(x)$, is defined as*

$$UR(x)=\{x\}\cup\left\{\begin{matrix}\tilde{x}\in X\mid \exists e\in\Sigma_{uo}^* \; s.t.\, \tilde{x}\in\delta(x,e)\\ and\; h(x)=h(\tilde{x})\end{matrix}\right\} \quad (8)$$

Intuitively, $UR(x)$ represents the set of all states that are reachable from x through unobservable strings and they have the same output as state x.

A detector[6] that is a deterministic finite automaton, denoted by G_{det}, is constructed for partially-observed nondeterministic DES (7). Formally,

$$G_{det}=(\tilde{X},\Sigma_o\cup\{\emptyset\}\times Y,\delta_{det},\tilde{x}_0), \quad (9)$$

where the state set is $\tilde{X}\subseteq 2^X$, $\Sigma_o\cup\{\emptyset\}\times Y$ denotes the input-output set, δ_{det} is partial transition function, $\tilde{x}_0=X$ is the initial state.

To obtain the dynamics of detector (9), let us consider the partially-observed nondeterministic DES (7), where $X=\{x_1,x_2,\cdots,x_n\}$; the set of events, without loss of generality, is $\Sigma=\Sigma_o\cup\Sigma_{uo}$ with $\Sigma_o=\{e_1,e_2,\cdots,e_{s-1}\}$ and $\Sigma_{uo}=\{e_s,e_{s+1},\cdots,e_m\}$; the output set is $Y=$

$\{y_1, y_2, \cdots, y_q\}$. Identify x_i (resp., e_{j_1}, e_{j_2}, y_k) with δ_n^i (resp., $\delta_s^{j_1}$, δ_s^s, δ_q^k) for simplicity, $(1 \le i \le n)$ (resp., $1 \le j_1 \le s-1$, $s \le j_2 \le m$, $1 \le k \le q$). we call δ_n^i, $\delta_s^j (j = j_1, s)$ and δ_q^k are the vector forms of x_i, e_j $(j = j_1, j_2)$ and y_k, respectively. Thus $X \sim \Delta_n$, $\Sigma \sim \Delta_s$ and $Y \sim \Delta_q$. To this end, an admissible input-output pair (δ_s^j, δ_q^k) can be identified with δ_{sq}^p by means of the formula $\delta_{sq}^p = \delta_s^j \ltimes \delta_q^k$ given in Lemma 2.2.

Next, we construct a matrix $F_p \in \mathcal{B}_{n\times n}$, called input-output transition structure matrix associated with the input-output pair (δ_s^j, δ_q^k), as follows.

$$F_{p(i,t)} = \begin{cases} 1, \delta_n^a \in \delta(\delta_n^t, \delta_s^j), \delta_n^i \in UR(\delta_n^a), h(\delta_n^a) = \delta_q^k; \\ 0, otherwise. \end{cases} \quad (10)$$

Further, the input-output transition structure matrix (abbreviated as IOTSM) associated with all admissible input-output pairs is defined as

$$F = [F_1, F_2, \cdots, F_{sq}] \in \mathcal{B}_{n\times nsq}. \quad (11)$$

Proposition 3.1 *Given a partially-observed nondeterministic DES (7), then the dynamics of detector (9) can be equivalently described by the following algebraic equation*

$$\tilde{x}(t+1) = Fu(t)y(t)\,\tilde{x}(t), \quad (12)$$

where the matrix F *defined in (11) is called the IOTSM of detector (9),* $\tilde{x}(1) = [1,1,\cdots,1]^{\mathrm{T}}$ *is the initial state of detector (9),* $\tilde{x}(t) = (x_1(\mathrm{t}), x_2(\mathrm{t}), \cdots, x_n(\mathrm{t}))^{\mathrm{T}}$ *is the vector form of state of detector (9) at step t, whose* i_τ*-th* $(\tau = 1,2,\cdots,k)$ *entry equals to 1 means that the state of detector (9) at step t is* $\{x_{i_1}, x_{i_2}, \cdots, x_{i_k}\}$, $u(t)y(t) \in \Delta_{sq}$ *is the vector form of input-output pair at step t.*

Using Lemma 2.1, we have

$$\tilde{x}(t+1) = FW_{[n,sq]}\tilde{x}(t)u(t)y(t), \quad (13)$$

Define matrix $\tilde{F}^{[t]} =: (FW_{[n,sq]})^{[t]}\tilde{x}(1)$, *then Eq.(13) becomes*

$$\tilde{x}(t+1) = \tilde{F}^{[t]} \ltimes_{j=1}^{t} u(j)y(j). \quad (14)$$

3.3. Verification of detectability

Now, we further develop a matrix-based methodology in terms of Eq. (13) to verify three types of detectability for partially-observed nondeterministic DES (7). To this end, we need the following some preliminaries.

Using Theorem 2 existed in other paper[14] and Eq. (13) and/or (14), we can obtain easily the state set $\tilde{X}$ of detector (9), and denote by $\tilde{X} = \{z_1, z_2, \cdots, z_n\}$. Identifying $z_i \sim \eta_i$, where η_i is the vector form of state z_i, $1 \le i \le N$. Further, we construct that the vector $A_p \in \mathcal{B}_{N\times 1}(1 \le p \le N)$ is as follows.

$$A_{p(i,j)} = \begin{cases} 1, \eta_i \in Col(FW_{[n,sq]})\eta_j, 1 \le i,j \le N; \\ 0, otherwise. \end{cases}$$
(15)

Intuitively, the matrix $A = [A_1, A_2, \cdots, A_N] \in \mathcal{B}_{N\times N}$ is the adjacency matrix of the state transition diagram of the detector (9).

On the other hand, we call a state $z \in \tilde{X}$ a single state if $|z| = 1$ and denote by $X_{single} = \{z \in \tilde{X} \mid |z| = 1\}$. Let $X_{single} = \{z_{i_1}, z_{i_2}, \cdots, z_{i_w}\}$. $\nu = \{i_1, i_2, \cdots, i_w\}$ is said to be the subscript set of X_{single}.

The following results provide the matrix-based criteria of verifying aforesaid four types of detectability for partially-observed nondeterministic DES (7).

Theorem 3.1 *A partially-observed nondeterministic DES (7) is weakly detectable if and only if*

$$\{\sigma \mid (\textstyle\sum_{k=1}^{\omega}(P^TAP)^K)_{(\sigma,\sigma)} = 1\} \ne \emptyset \quad (16)$$

where $P = \delta_N[i_1, i_2, \cdots, i_w]$.

Proof. Since $\nu = \{i_1, i_2, \cdots, i_w\}$ is the subscript set of the set of single states X_{single}, then (16) holds if and only if there exists at least a loop in G_{det} in which all states belongs to X_{single}. This means that there exists at least an admissible infinite input-output string by which the current and subsequent states of system (7) can be know exactly after a finite delay. Therefore, by Definition 3.1, system (7) is weakly detectable if and only if (16) holds.

Theorem 3.2 *A partially-observed nondeterministic DES (7) is strongly periodically detectable if and only if*

$$\{\sigma \mid (\textstyle\sum_{k=1}^{N-\omega}(Q^TAQ)^K)_{(\sigma,\sigma)} = 1\} = \emptyset \quad (17)$$

where $Q = \delta_N[1, \cdots, i_1 - 1, i_1 + 1, \cdots i_\omega - 1, i_\omega + 1, \cdots, N]$.

Proof. We know that (17) holds if and only if there is no loop of all states belong to $\tilde{X}/X_{single}$ in G_{det}. This implies that we determine periodically the current state of system (7) for all admissible input-output strings after a finite number of delays. Consequently, by Definition 3.2, (17) holds if and only if system (7) is strongly periodically detectable.

Theorem 3.3 *A partially-observed nondeterministic DES (7) is weakly periodically detectable if and only if*

$$\{\sigma \mid (\textstyle\sum_{k=1}^{N}A^K)_{(\sigma,\sigma)} = 1\} \cap \nu \ne \emptyset \quad (18)$$

Proof. The proof of Theorem 3.3 follow directly from Definition 3.2 and Theorem 3.1, here we omit it.

Remark 3.1 Consider a partially-observed nondeterministic DES (8) with n state nodes, $s-1$ observable event nodes and q output nodes. In other paper[9], author proved that there exists no polynomial algorithm for verifying weak (periodic) detectability unless $P = PSPACE$. In our paper, since the size of

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matrix F in (12) is $n \times nsq$, the complexity of constructing the matrix-based detector (12) is $O(n^2sq)$, which is polynomial with respect to the size of system (7). On the other hand, in the worst case, the cardinality of the state set $\tilde{X}$ of detector G_{det} is 2^n. Therefore, the complexity of constructing matrix A is $O(2^n \times 2^n)$, which is exponential with respect to the size $\tilde{X}$. Overall, the total complexity of implementing Theorems 3.1-3.3 to verify aforementioned three types of detectability is $O(n^2sq + 2^{2n})$.

Remark 3.2 A Matlab toolbox on the numerical computation of the STP of matrices has been created at *http://lsc.amss.ac.cn/~dcheng/stp/STP .zip*. In this paper, the implementation of the following an example is based on this Matlab toolbox.

4. Illustrative example

4.1. Example 4.1

Consider a partially-observed DES shown in Fig.1, where $e_3 \in \Sigma_{uo}$.

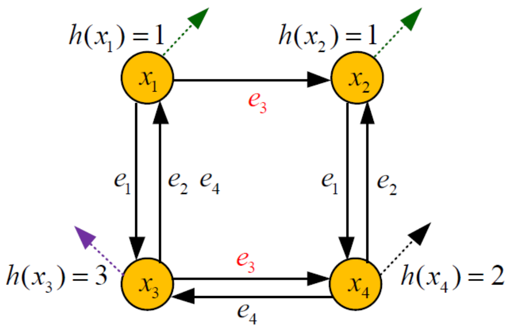

Fig.1 A partially-observed DES with output observations

Identifying $x_i \sim \delta_4^i$ $(1 \le i \le 4)$, $e_j \sim \delta_4^j$ $(1 \le j \le 4)$, $k \sim \delta_3^k$ $(1 \le k \le 3)$. Using Eq. (8), we obtain that $UR(x_i) = \{x_i\}$, $i = 1,2,3,4$. By Proposition 3.1, we obtain that the dynamics of detector G_{det}

$$\tilde{x}(t+1) = Fu(t)y(t)\,\tilde{x}(t). \quad (19)$$

Using Lemma 2.1, Eq.(19) becomes

$$\tilde{x}(t+1) = FW_{[4,12]}\tilde{x}(t)u(t)y(t), \quad (20)$$

where $F = [F_1, F_2, \cdots, F_{11}, F_{12}]$ with $F_i = 0_{4\times4}(i = 1,5,6,7,9,11)$, $F_2 = \delta_4[0,4,0,0]$, $F_3 = \delta_4[3,0,0,0]$, $F_8 = \delta_4[0,0,4,0]$, $F_{12} = \delta_4[0,0,0,3]$,

$$F_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad F_{10} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

We can obtain further that $\tilde{X} = \{z_1, z_2, z_3, z_4, z_5\}$, where $z_1 = \{x_1, x_2, x_3, x_4\}$, $z_2 = x_2$, $z_3 = x_3$, $z_4 = x_4$, $z_5 = \{x_1, x_2\}$, $X_{single} = \{z_2, z_3, z_4\}$, $\nu = \{2,3,4\}$,

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$\{\sigma | (\sum_{k=1}^{5} A^K)_{(\sigma,\sigma)} = 1\} = \{2,3,4,5\} \cap \nu \neq \emptyset$,

$\{\sigma | (\sum_{k=1}^{3} (P^T AP)^K)_{(\sigma,\sigma)} = 1\} = \{1,2,3\} \neq \emptyset, where\ P = \delta_5[2,3,4]$,

$\{\sigma | (\sum_{k=1}^{2} (Q^T AQ)^K)_{(\sigma,\sigma)} = 1\} = \emptyset, where\ Q = \delta_5[1,5]$.

By Theorems 3.1-3.3, the system shown in Fig.1 is strongly periodic detectable and weakly (periodic) detectable.

5. Conclusion

In this paper, we developed a matrix-based methodology to verify various types of detectability for partially-observed nondeterministic DESs. By resorting to our previous work [6], several matrix-based criterions for verifying three types of detectability were derived. These verification criterions all of are based on closed-forms

The proposed methodology in this paper is only viewed as a start of the related issues for partially-observed DESs, it may be provide a new theoretical framework for verifying and synthesizing of other types of system-theoretic property. For instance, extending the developed methodology to trajectory detectability[8] for partially-observed DESs modeled by finite automata (resp., Petri nets[15]) is an interesting direction.

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