A Distributed Optimal Formation Control for Multi-Agent System of UAVs

Jichao Zhao[1], Fengzhi Dai[1, 3 *], Yunzhong Song[2]

[1] *College of Electronic Information and Automation, Tianjin University of Science and Technology, 300222, China;*

[2] *School of Electrical Engineering and Automation, Henan Polytechnic University, 454003, China;*

[3] *Tianjin Tianke Intelligent and Manufacture Technology CO., LTD, China*

*E-mail: * daifz@tust.edu.cn*

www.tust.edu.cn

Abstract

In this paper, the distributed optimization problem for multi-agent system (MAS) formation control of unmanned aerial vehicles (UAVs) is suggested here. The situation that the internal states of a single UAV can be made fully available was aimed at, the internal optimal control law of a single UAV is designed using the optimal control theory. To cope with the obstacle that each agent in the system can only communicate with its neighbor agents, the distributed formation control law of the system is introduced based on the communication topology of the system, and further the stability of the system is analyzed in helps of graph theory. The validity of the suggested scheme is verified by both the numerical simulation and UAV platform.

Keywords: multi-agent system, UAV, optimal control, distributed formation control

1. Introduction

As the capabilities of robots continue to be improved, the application fields of robots are also expanding at the same time. However, just like humans, a single robot will show lower abilities in many cases, and in this way, the coordination of multiple agents is required to play a more significant role. The distributed coordination and cooperation ability of the agents is the foundation of the multi-agent system (MAS), that is the key to exert excess advantages to the MAS, and it is also the embodiment of the intelligence of the entire MAS.

The problem of multi-agent coordinated control includes consensus control [1], rendezvous control [2], formation control [3], and such et al. Besides, there is optimization control [4] based on the above-mentioned coordinated control. However, the optimization of a MAS has a strong dependence on the system communication network. Because the optimal control law needs to be able to obtain all the states of the collective individuals in the system in real-time, otherwise the conditions of optimal control can not be met. To deal with this problem, we designed a distributed optimal formation control, which is invariant to the structure of the system network. And this result is applied to the formation control of multiple unmanned aerial vehicles (UAVs).

The main contribution of this paper mainly includes three aspects: 1) A formation control protocol based on static consensus protocol is designed. 2) The optimal control law when the performance function of a single agent is optimal is studied. 3) Combining the formation control protocol and the optimal control law, a distributed optimization formation control protocol is designed. This protocol cannot be affected by the internal communication topology of the MAS. Even if some agents cannot communicate with each other, the system can complete formation task.

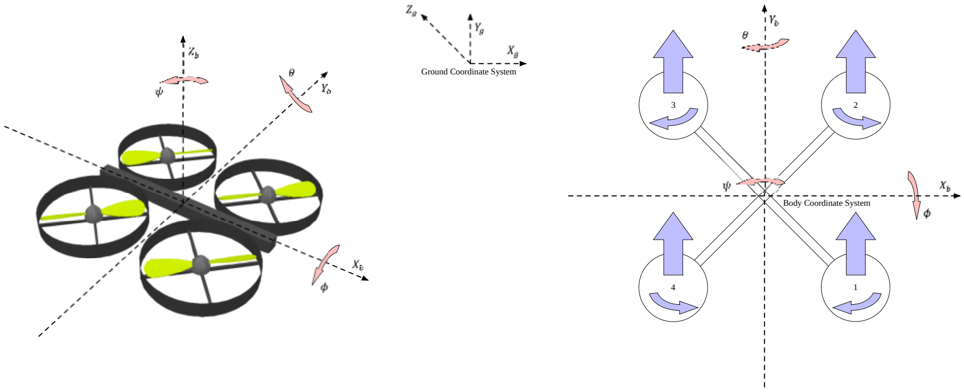


Fig.1. The illustration of UAV kinematics

2. Preliminaries

This section introduces the preliminary knowledge for studying UAV systems, including the use of graph theory to describe the communication relationships within the system, the dynamic model of a single UAV, and the state-space equation of the UAV system.

2.1. Graph theory

In order to describe the relationship of UAV systems, graph theory is used [1]. $A = [a_{ij} \in \{0,1\}]$ is the adjacency matrix of the graph G, indicating whether there is information exchange from node j to i or not. The matrix D is the out-degree matrix. The neighbors of node i are N_i. The Laplacian matrix is defined as

$$L = D - A \tag{1}$$

2.2. UAV model description

As shown in Fig.1, the UAV model can be simplified as equation (2) [5].

$$\begin{aligned} \ddot{x} &= g\theta \\ \ddot{y} &= -g\phi \\ \ddot{z} &= u_h/m - g \\ \ddot{\phi} &= u_\phi/I_x \\ \ddot{\theta} &= u_\theta/I_y \\ \ddot{\psi} &= u_\phi/I_z \end{aligned} \tag{2}$$

where x, y, z are positions in the ground coordinate system along X_g, Y_g, Z_g, respectively, ϕ, θ, ψ are roll angle, pitch angle, yaw angle of the UAV, respectively. I_x, I_y, I_z are inertia moments along X_b, Y_b, Z_b in the body coordinate system, respectively. And u_h is the lift from four propellers.

For the sake of easy computation, system (2) is transformed into the state-space format, listed in equation (3).

$$\dot{X} = AX + BU \tag{3}$$

where $X = [x \quad y \quad z \quad \dot{x} \quad \dot{y} \quad \dot{z} \quad g\theta \quad -g\phi \quad 0 \quad g\dot{\theta} \quad -g\dot{\phi} \quad 0]^{\mathrm{T}}$, $U = [u_\phi \quad u_\theta \quad 0]^{\mathrm{T}}$,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \otimes I_3, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \otimes I_3,$$

$\otimes$ is the Kronecker product.

When there have N UAVs, the state-space equation (3) can transform into the following form.

$$\dot{\tilde{X}} = I_N \otimes A\,\tilde{X} + I_N \otimes B\tilde{U} \tag{4}$$

where $\tilde{X} = [X_1^{\mathrm{T}}, X_2^{\mathrm{T}}, \dots, X_N^{\mathrm{T}}]^{\mathrm{T}}, \tilde{U} = [U_1^{\mathrm{T}}, U_2^{\mathrm{T}}, \dots, U_N^{\mathrm{T}}]^{\mathrm{T}}$, I_N represent an N-dimensional identity matrix.

The desired formation state h is defined as follows

$$h = \begin{bmatrix} h^x \\ h^y \\ h^z \end{bmatrix} \in \mathbb{R}^3 \tag{5}$$

Take the formation states (5) into the position states, three new error position states come naturally as (6).

$$\begin{bmatrix} \delta^x \\ \delta^y \\ \delta^z \end{bmatrix} = \begin{bmatrix} x - h^x \\ y - h^y \\ z - h^z \end{bmatrix} \tag{6}$$

Then the formation control problem is turned into finding a protocol $\tilde{U}$ to drive the error vector δ to zero, which indicates that

$$\lim_{t\to\infty}\|\delta_i - \delta_j\| = 0, \quad \lim_{t\to\infty}\|v_i\| = 0,$$
$$\lim_{t\to\infty}\|\Omega_i\| = 0, \quad \lim_{t\to\infty}\|\dot{\Omega}_i\| = 0, \quad i = 1,2,\cdots,N. \tag{7}$$

3. Main results

This section starts with the design of the formation control protocol. After that, the optimal control law of a single UAV is provided in detail. Finally, the optimal control law is extended to the formation control protocol in collection.

Lemma 1 [6]**.** For a $N \times N$ Laplacian matrix L, $Ne^{-Lt}, t > 0$ is a random matrix with positive diagonal elements. If L has a unique zero eigenvalue, $\mathrm{Rank}(N) = N-1$, then its left eigenvalue has $v = [v_1 \quad v_2 \quad \cdots \quad v_N]^{\mathrm{T}} \geq 0$ and $1_N^{\mathrm{T}} v = 1, L^{\mathrm{T}} v = 0$, where $t \to \infty, e^{-Lt} \to 1_N v^{\mathrm{T}}$.

3.1. Formation control

Referring to the previous work, the consensus protocols can be divided into two types: One is static and the other one is dynamic. Based on the static consensus protocol, the following formation control protocol is employed here as (8)

$$u_i = \alpha \sum_{j \in N_i} (\delta_j - \delta_i) - \beta v_i - \gamma_1 \Omega_i - \gamma_2 \dot{\Omega}_i \quad (8)$$

where α, β, γ_1 and γ_2 all of them are positive gains. $\delta_i = \left[\delta_i^x, \delta_i^y, \delta_i^z\right]^{\mathrm{T}}, v_i = [\dot{x}_i, \dot{y}_i, \dot{z}_i]^{\mathrm{T}}$,
$\Omega_i = [g\theta_i, -g\phi_i, 0]^{\mathrm{T}}, \dot{\Omega}_i = \left[g\dot{\theta}_i, -g\dot{\phi}_i, 0\right]^{\mathrm{T}}$.

Theorem 1. Assume G is the connected undirected graph. The system (4) can realize the formation as defined in (7) if protocol (8) satisfies the following conditions:

$$\alpha > 0, \beta > 0, \gamma_1 > 0, \gamma_2 > 0, \beta \gg \alpha,$$
$$\gamma_1, \gamma_2 > \beta \gg \alpha, \beta\gamma_1\gamma_2 > \beta^2 + \gamma_2^2 \alpha$$

Proof. Please referring to Reference item [5]. □

3.2. Optimal control

The solution of optimal control needs to be able to obtain all the state information in the system, which is often not satisfied in the multi-agent system. However, the optimal control law in a single agent can be studied to determine the optimal control law of the system.

Before giving the performance function, let $\bar{X} = [\delta^x \quad \delta^y \quad \delta^z \quad \dot{x} \quad \dot{y} \quad \dot{z} \quad g\theta \quad -g\phi \quad 0 \quad g\dot{\theta} \quad -g\dot{\phi} \quad 0]^{\mathrm{T}}$, then the performance function is defined as

$$J_i = \int_0^\infty \left[\bar{X}_i^{\mathrm{T}}(t) Q \bar{X}_i(t) + u_i^{\mathrm{T}}(t) R u_i(t)\right] dt \qquad (9)$$

Since the states of UAV on different coordinate axes are independent, we need to set weight matrices $Q = q * I_{12}, R = r * I_3$, where $q > 0, r > 0$.

Through the optimal control theory, the optimal control law of single agent is

$$u_i^* = -R^{-1} B^T P \bar{X} \qquad (10)$$

where P is the solution of the Riccati equation (11).

$$A^{\mathrm{T}} P + PA - PBR^{-1} B^{\mathrm{T}} P + Q = 0 \qquad (11)$$

3.3. Distributed optimization formation control

Through the above calculation, the optimal control law (10) can be obtained. Let $K = R^{-1} B^T P$, the dimension of K must be 3×12, and it also has the following form $K = [k_1 \quad k_2 \quad k_3 \quad k_4] \otimes I_3 =$

$$\begin{bmatrix} k_1 & 0 & 0 & k_2 & 0 & 0 & k_3 & 0 & 0 & k_4 & 0 & 0 \\ 0 & k_1 & 0 & 0 & k_2 & 0 & 0 & k_3 & 0 & 0 & k_4 & 0 \\ 0 & 0 & k_1 & 0 & 0 & k_2 & 0 & 0 & k_3 & 0 & 0 & k_4 \end{bmatrix}^{\mathrm{T}}$$

The multi-agent system of UAV is isomorphic, that is, the dynamic performance of all agents is the same, so the optimal control law can be directly applied to the UAV system. Therefore, the optimal formation control law is obtained as follows

$$u_i = k_1 \sum_{j \in N_i} (\delta_j - \delta_i) - k_2 v_i - k_3 \Omega_i - k_4 \dot{\Omega}_i \quad (12)$$

where k_1, k_2, k_3, k_4 come from matrix K.

Theorem 2. Assume G is connected undirected graph. The UAV system (4) can complete the formation while making the performance function (9) optimal if it uses the protocol (12).

Proof. Substitute (1) to (2), we can get

$$U = \Gamma \otimes I_3 \bar{X}$$
$$\Gamma = L \otimes [k_1 \quad 0 \quad 0 \quad 0] + I_3 \otimes [0 \quad k_2 \quad k_3 \quad k_4]$$

Then

$$\dot{\bar{X}} = A \bar{X} + B\, \Gamma \otimes I_3 \bar{X} = \bar{\Gamma} \bar{X}$$

Combining the proof of Theorem 1, $\bar{\Gamma}$ can be turned into a Jordan standard:

$$\bar{\Gamma} = P J P^{-1}$$

Let v_1^T is the first row of P^{-1}, the left eigenvalue of the zero eigenvalue, and ω_1 is the right eigenvector of the zero eigenvalue of the first column of P. So $v_1^{\mathrm{T}} \omega_1$ is equal to 1. By taking time to infinity, the state of the system becomes:

$$\lim_{t \to \infty} \bar{X} = \lim_{t \to \infty} e^{\bar{\Gamma} t} \bar{X}(0)$$
$$e^{\bar{\Gamma} t} \bar{X}(0) \to (\omega_1 v_1^{\mathrm{T}}) \bar{X}(0), \qquad t \to \infty$$

According to Lemma 1, as time tends to infinity, the system (4) is asymptotically consensus and the error vector is 0, that is, the system completes the formation control task. □

4. Simulations

In this section, numerical simulation and virtual platform simulation were carried out to verify the effectiveness of the designed formation control protocol. In this section, numerical simulation and virtual platform simulation were carried out to verify the effectiveness of the designed formation control protocol.

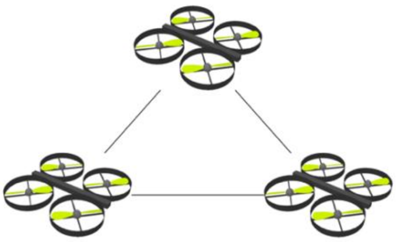


Fig.2. System communication topology diagram

4.1. Experiment 1

The numerical experiments have verified the formation control and distributed optimization formation control respectively, as shown in Fig.3 and Fig.4.

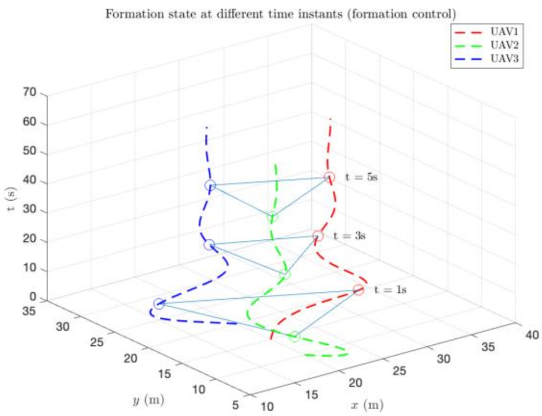


Fig.3. System states formation based on formation control

In the formation control, it can be observed that the system can complete the triangular formation, but there is a large error from the set position during the completion of the formation, and there is still an error at the 5th second. When using distributed optimal formation control, it can be observed that the system can quickly complete the triangular formation, and the error between the actual position and the set value is small.

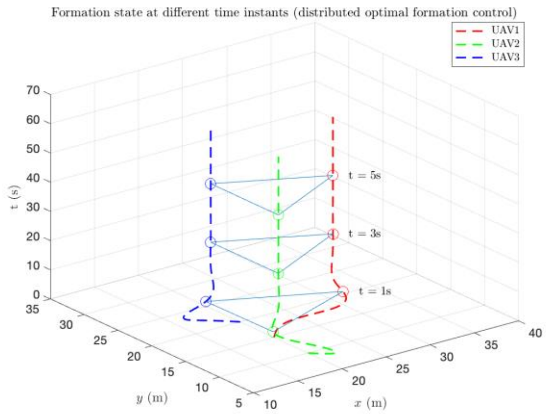


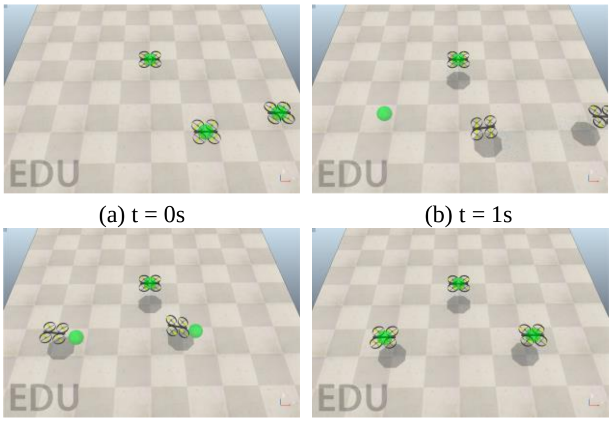
Fig.4. System states based on distributed optimization formation control

In addition, it is surprising that the optimal control not only reduces the loss of the system but also accelerates the convergence speed of the system, which is of great help in reducing the time it takes for the system to form a formation.

4.2. Experiment 2

The simulation software CoppeliaSim/Vrep was also used to verify the flight of the UAV formation. To keep the system in view during the operation, we assume that a UAV remains stationary.

Observing the position of the system at different time instants, as shown in Fig.5, it can be seen that the system can finally complete the triangular formation. The full version of the experiment video can be seen at https://www.bilibili.com/video/BV1Br4y1D7VJ/.



(a) t = 0s (b) t = 1s

(c) t = 3s (d) t = 5s

Fig.5. The position of three UAVs at different time instants

5. Conclusions

In this paper, a multi-UAV system is established by analyzing the dynamic model of UAV. Based on the static consensus protocol, the formation control protocol is designed. Aiming at the problem that some UAVs in the system cannot obtain the status information of other UAVs, an optimal control law within a single UAV is designed. Finally, combining the formation control protocol and the optimal control law, a distributed optimization formation control protocol for the multi-UAV system is designed. This protocol is not interfered with by the communication topology of the multi-agent system and can optimize the performance function while the system completes the formation task.

In the next step, we plan to combine engineering applications and apply theoretical research results to practice.

Acknowledgements

This work was supported in part by the National Natural Science Foundation of China (No. 61340041, 61374079 and 61903126) and the Natural Science Foundation of Henan Province (182300410112).

References

1. J. Zhao, F. Dai, Y. Song, Analysis of the consensus protocol of heterogeneous agents with time-delays, in: *2021 Proceedings of International Conference on Artificial Life and Robotics (ICAROB)*, 2021, pp. 592-595.
2. Song Y, Zhao W. Multi-agent system rendezvous via refined social system and individual roles. *Wseas transactions on systems and control*, 2014, 9(1): 526-532.
3. Wang J, Han L, Dong X, et al. Distributed sliding mode control for time-varying formation tracking of multi-UAV system with a dynamic leader. *Aerospace Science and Technology*, 2021, 111: 106549.
4. Ren Y, Wang Q, Duan Z. Optimal Distributed Leader-following Consensus of Linear Multi-agent Systems: A Dynamic Average Consensus-based Approach. *IEEE Transactions on Circuits and Systems II: Express Briefs*, 2021.
5. J. Zhao, F. Dai, Y. Song, Consensus of Heterogeneous Mixed-Order Multi-agent Systems Including UGV and UAV, *Proceedings of 2021 Chinese Intelligent Systems Conference* 805 (2022) 202–210.
6. Zhu J, Tian Y P, Kuang J. On the general consensus protocol of multi-agent systems with double-integrator dynamics. *Linear Algebra and its Applications*, 2009, 431(5-7): 701-715.

Authors Introduction

Mr. Jichao Zhao

He is a third-year master candidate in Tianjin University of Science and Technology. His research area is about multi-agent system, control engineering.

Dr. Fengzhi Dai

He received an M.E. and Doctor of Engineering (PhD) from the Beijing Institute of Technology, China in 1998 and Oita University, Japan in 2004 respectively. His main research interests are artificial intelligence, pattern recognition and robotics. He worked in National Institute of Technology, Matsue College, Japan from 2003 to 2009. Since October 2009, he has been the staff in College of Electronic Information and Automation, Tianjin University of Science and Technology, China.

Dr. Yunzhong Song

He is affiliated with the School of Electrical Engineering and Automation, Henan Polytechnic University as a full professor. He received a PhD degree from the Department of Systems Science, Zhejiang University, P. R. China in 2006. His research interest covers multi-agent systems analysis and control, complex networks as well as nonlinear systems analysis and control. Up to now, he has published about 100 papers in peer referred journals and conference proceedings. To be specific, the work initiated by Professor SONG such as consensus of multi-agent systems with hybrid order dynamics, and social role based multi-agent systems flocking as well as hybrid-mode multi-agent systems regulation have been recognized.