A parameter tuning method for PQN model

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Abstract

The Piecewise Quadratic Neuron (PQN) model is a spiking neuron model that can be efficiently implemented on digital arithmetic circuits. In addition, this model can reproduce a variety if neuronal activities precisely with optimized parameter sets. In previous studies, we have optimized the parameters using meta-heuristic methods, which required a lot of computational time. In this paper, we proposed an parameter fitting method that takes into account the mathematical structure of the model and reproduces the electrophysiological activities of a target neuron with less computational time. We expect that this method can be used to construct silicon neuronal networks that faithfully replicate the nervous system. This method is expected to applicable to building silicon neuronal networks that faithfully replicate the nervous system.

Keywords: silicon neuronal network, spiking neuron model, PQN model, parameter fitting.

1. Introduction

The nervous system makes it possible for animals to process a variety of complex information. The silicon neuronal network (SNN) aims to achieve this information processing with low power consumption by mimicking the structure of the nervous system. The SNN is composed of the silicon neurons [1][2], which are digital or analog circuit units that simulates neuronal activities by solving the spiking neuron models. A variety of spiking neuron models have been used for the silicon neurons due to a trade-off between the reproducibility of neuronal activity and computational efficiency. For example, previous studies [3][4] used the ionic-conductance models. While these models can reproduce neuronal activities accurately, a large number of resources are required for the circuit implementation. On the other hand, integrate-and-fire (I&F)-based models are also widely used [5][6]. In I&F-based models, the variable corresponding to the membrane potential is reset to emulate the spike process when the neuron fires. This resetting has the advantage of low implementation cost, but makes the reproducibility of neuronal activities less than that of the ionic-conductance models.

We have proposed the Piecewise Quadratic Neuron (PQN) model, which is also known as the digital spiking silicon neuron models [7][8][9]. The feature of this

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model is that it uses piecewise quadratic functions. This model does not have a cubic term unlike other neuron models e.g. the FitzHugh-Nagumo [10] model and the Hindmarsh-Rose model [11], and can be implemented on the digital circuit with less circuit resources. In previous studies [12][13][14], the network containing the PQN model was constructed on a field-programmable gate array (FPGA) chip and the associative memory tasks were performed. In addition, the parameter fitting methods based on the meta-heuristic approach were proposed in previous studies [15][16], but they required a huge amount of computational time [17].

In this work, we propose the parameter fitting method focusing on the mathematical structure of the PQN model. And this method is applied to electrophysiological experimental data publicated in [18]. While the previous study [17] took several hours to determine the parameters, this method obtains the parameters within one minute.

The remainder of this paper is organized as follows. Section 2 explains about equations and the parameter fitting method for the PQN model. The results of parameter fitting are shown in Section 3 and the work is concluded in Section 4.

2. Methods

2.1. *PQN model*

Equations of the PQN model are given by

$$\begin{aligned}
\frac{dv}{dt} &= \frac{\phi}{\tau}(f(v) - n - q + I_0 + I_{\text{stim}}),\\
\frac{dn}{dt} &= \frac{1}{\tau}(g(v) - n),\\
f(v) &= \begin{cases} a_{fn}(v-b_{fn})^2 + c_{fn} & (v<0)\\ a_{fp}(v-b_{fp})^2 + c_{fp} & (v \geq 0)\end{cases},\\
g(v) &= \begin{cases} a_{gn}(v-b_{gn})^2 + c_{gn} & (v<r_g)\\ a_{gp}(v-b_{gp})^2 + c_{gp} & (v \geq r_g)\end{cases},\\
b_{fp} &= \frac{a_{fn}b_{fn}}{a_{fp}},\ c_{fp} = -\frac{a_{fn}^2 b_{fn}^2}{a_{fp}} + a_{fn}b_{fn}^2 + c_{fn},\\
b_{gp} &= -\frac{a_{gn}(-b_{gn}+r_g)}{a_{gp}} + r_g,\\
c_{gp} &= -\frac{a_{gn}^2(b_{gn}-r_g)^2}{a_{gp}} + a_{gn}(b_{fn}-r_g)^2 + c_{gn},
\end{aligned}$$

where *v* is a state variable corresponding to the neuronal membrane potential. *n* is a recovery variable. Parameters φ and τ control time constants of variables. I_{stim} represents the input stimulus and parameter I_0 is a bias current. Parameters a_x, b_x, c_x and r_g, where *x* is *fp*, *fn*, *gp*, or *gn*, control the shapes of the *v*- and *n*-nullclines. Parameters b_{fp}, c_{fp}, b_{gp} and c_{gp} are determined so that the nullclines are continuous and smooth.

2.2. *Parameter fitting*

Figure 1 shows an example of neuronal activities simulated by the PQN model in response to a sustained current stimulus. In this study, for the efficient parameter fitting, we focus on these three features of the spike waveform: the distance from the minimum value to the threshold, the distance from the maximum value to the threshold, and the inter-spike interval. We defined the threshold as the value of the point at which the slope changes most rapidly before and after. The values of these three features are dependent on these three parameters, a_{fn}, ϕ, and I_0, respectively (Fig. 2). The a_{fn} controls a slope of the *v*-nullcline where *v* is less than 0. For example, when a_{fn} is increased, the slope becomes larger and the distance from the minimum value to the

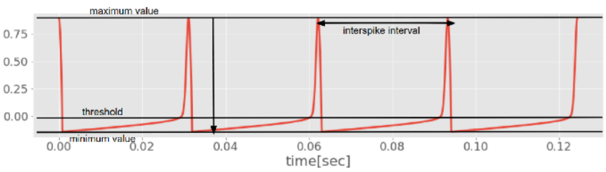

Figure 1: Typical spikes in the PQN model.

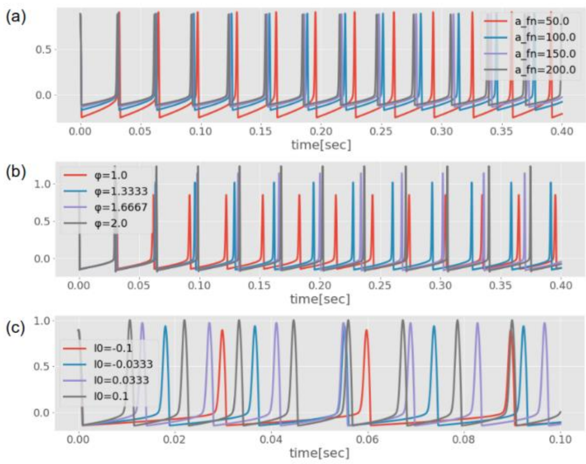

Figure 2: Effects of each parameter in the PQN model.

threshold becomes shorter (Fig. 2(a)). Note that when a_{fn} is changed, a_{gn}, b_{fn}, b_{gn}, c_{fn}, and c_{gn} are modified according to the following equation in order to minimize the effect on other features as much as possible.

$$a'_{fn} = ra_{fn},\ a'_{gn} = ra_{gn},\ b'_{fn} = b_{fn}/r,$$
$$b'_{gn} = r_g - (r_g - b_{gn})/r,\ c'_{fn} = a_{fn}b_{fn}^2 + c_{fn} - a'_{fn}b'^2_{fn},$$
$$c'_{gn} = a_{gn}(r_g - b_{gn})^2 + c_{gn} - a'_{gn}(r_g - b'_{gn})^2,$$

where *r* is the rate of change in a_{fn} and *x'*, where *x* is a_{fn}, a_{gn}, b_{fn}, b_{gn}, c_{fn}, or c_{gn}, means the value of *x* after this modification. The distance from the maximum value tothe threshold is dependent on ϕ. Figure 2(b) shows that as ϕ is increased the amplitude is also increased. In addition, by increasing I_0, the inter-spike interval becomes smaller (Fig. 2(c)).

In the process of the parameter fitting, we firstly tuned a_{fn} by comparing the difference from the minimum value to the threshold. Then, ϕ is tuned in order to adjust the distance from the maximum value to the threshold. Finally, the inter-spike interval is fit by changing I_0. We repeated this procedure three times, and totally required 55.4 seconds in average. In this experiment, we used the computer with Intel Core i7-8700 CPU. And cython library (version 0.29.24) was used in the source code.

3. Results

Figure 3 compares the initial waveform and the waveform simulated by obtained parameters. Table 1 shows the mean squared error (MSE) between electrophysiological data and simulated data for three neurons in [20]. MSE is given by: where *T* is the number of time

$$\mathrm{MSE} = \sum_{t=1}^{T} (V_{\mathrm{measured}}(t) - V_{\mathrm{PQN}}(t))^2,$$

steps. $V_{\mathrm{measured}}(t)$ and $V_{\mathrm{PQN}}(t)$ are values of measurement and simulated data at time *t*, respectively. In all three data, the value of MSE became smaller than before the parameter fitting.

4. Conclusion

In this work, we proposed the parameter fitting method focusing on the mathematical structure of the PQN model. And we used this method to reproduce the electrophysiological data of three different neurons. The results showed that the MSE were reduced through the fitting. In our future work, we will study how the parameters effect on the time from the start of the spike until the spike reaches the highest point and the time from the spike reaching its highest point until it reaches the minimum value. By fitting these times, MSE is expected to be more reduced.

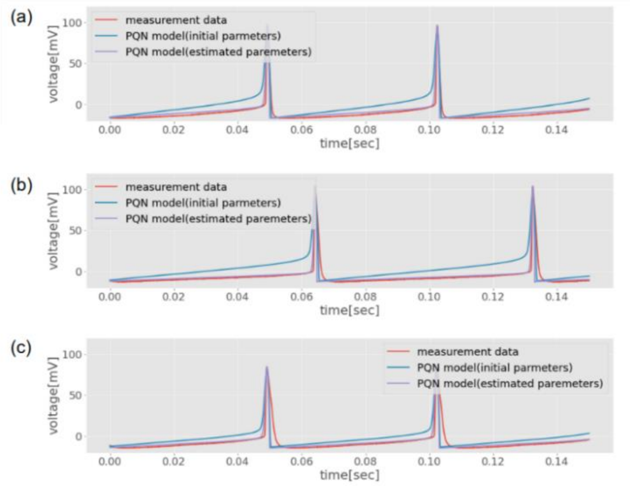

Figure 3: Results of parameter fitting. The specimen ids of the data (a), (b) and (c) are 613438337, 614635228, 614726150, respectively [20].

Table 1: Mean squared error.

specimen id	initial parameters	estimated parameters
613438337	0.006368	0.003469
614635228	0.009431	0.005571
614726150	0.011699	0.009618

5. Appendix

Table 2: initial parameters

a_{fp}	-2	b_{fp}	1	c_{fp}	1.48	r_g	0
a_{fn}	50	b_{fn}	-0.04	c_{fn}	-0.6	ϕ	0.8
a_{gp}	2	b_{gp}	-0.98	c_{gp}	-2.44	τ	0.001
a_{gn}	49	b_{gn}	-0.04	c_{gn}	-0.6	I_0	-

Table 3: estimated parameters

id	613438337	614635228	614726150
a_{fp}	-2	-2	-2
a_{fn}	121.13	238.99	140.87
a_{gp}	2	2	2
a_{gn}	118.71	234.21	138.05
b_{fn}	-0.016511	-0.0083686	-0.014198
b_{gn}	-0.016511	-0.0083686	-0.014198
c_{fn}	-0.55302	-0.53674	-0.54840
c_{gn}	-0.55396	-0.53800	-0.54943
ϕ	1.2637	1.4695	1.0491
I_0	-0.059050	-0.10996	-0.14165

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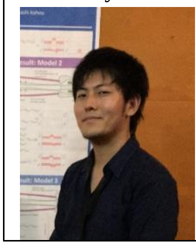

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