A perfect play in 4×12 board of Othello

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Abstract

In 2015, Hiroki Takeshita started to analyze a reduced-form Othello board with a 4x6 board. Later, he analyzed the 4×8 and 4×10 boards and discovered perfect play. The analysis of 4×12 board Othello was sub-problemed by Takeshita and further sub-problemed by Toya Shotaro in 2019. In this paper, we present the solution to the remaining subproblem of 4 × 12 board Othello and the discovery of perfect play.

Keywords: combinatorial theory, combinatorial optimization, perfect play, miniature Othello.

1. Introduction

In 1993, the mathematician Joel Feinstein discovered that in a reduced form 6×6 Othello board, white wins when both players do their best, and announced this in the newsletter of the British Othello Association[1]. It was published in the newsletter of the Othello Society of Great Britain. The number of search phases reached about 40 billion, and the search period was described as two weeks. In 2015, in Hiroki Takeshita's senior thesis, he performed the same analysis of the 6×6 board Othello as Feinstein did and confirmed that Feinstein's result was correct. Since then, Takeshita has been working on the analysis of reduced board Othello and announced that he has succeeded in the complete analysis of 4×4, 4×6, 4×8, and 4×10 boards[2, 3].

The search space for the next 4×12 board was expected to be about 1000 times larger than the 4×10 board. Since it takes time to analyze such a huge number of phases, we tried to reduce the search space by referring to the perfect play of 4×10 board. The result of the search from the 7th moves onward, when the procedure up to the 6th move of the 4×12 board is the same as that of the 4×10 board, is that Black wins[4]. The result of the search after the 7th move was confirmed that Black won. In 2019, Shotaro Toya solved one of the subproblems.

In this study, we will analyze the subproblemized procedure and prove that Black wins the 4×12 board Othello. Since 13 out of 14 subproblems have already been analyzed by Takeshita, we will analyze the remaining subproblem. This subproblem has a larger search space than the other subproblems and takes more time to analyze, so it is further subdivided by Toya.

2. Othello*

The rules of Othello are as follows[5]. The two opponents are divided into two groups, one playing the white stones and the other the black stones, and place the stones as shown in Fig. 1. The player who chooses the black stone is the first to move. The opponent must place a stone in an empty square between the stones of his own color and the stones of his opponent's color on his turn. It can be placed vertically, horizontally, or diagonally. The stone of your opponent's color becomes a stone of your own color. After placing a stone, it is your opponent's turn to play. If there is no square that can hold a stone of the opponent's color, it is the opponent's turn without placing a stone (pass). The game ends when the board is completely filled with stones or when both opponents are unable to place any stones. At the end of the game, the side with more stones of its own color on the board is the winner.

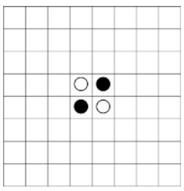

Fig. 1. Othello's setup.

Othello is classified as a two-player zero-sum finite definite perfect information game. As such, it can result in either a must-win first move, a must-win second move, or a draw[6].

3. Techniques

The program used for the analysis in this study is based on the negative-max-alpha-beta method with depth-first search. The negative-max method is an algorithm for finding the best order that exists in a game tree, with the same computational process for black and white. The best order in this research is the sequence of steps when you assume that you play the best move, and your opponent plays the worst move for you. The negative-max method requires that all nodes be explored. It can be used in conjunction with the alpha-beta method to reduce the computation time by not searching for moves that will never be selected.

4. Experiments

Hiroki Takeshita predicted that the perfect play of 4×12 board from the first move to the sixth move is identical to that of 4×8 and 4×10 boards. Fig. 2. shows the procedure of the perfect play with the 6th move fixed, and Fig. 3. shows the procedure from the first move to the 6th move.

The result of the perfect play with the 6th move fixed was "Black 42, White 0". If we can show the same best result before the sixth move, we can prove that the procedure including the fixed six moves is perfect play. There are 95 problems that are necessary to prove that the game is

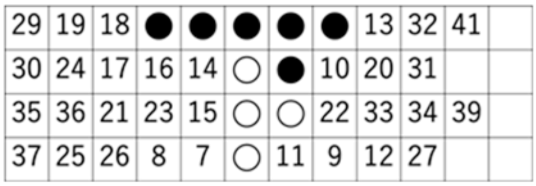

Fig. 2. A perfect play fixed from move 1 to move 6.

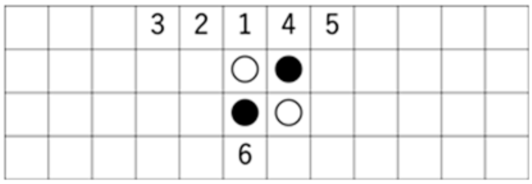

Fig. 3. A position fixed from move 1 to move 6.

played perfectly up to the 6th move as well. the 14 subproblems are shown in Table 1., and the board with the corresponding coordinates is shown in Fig. 4. Table 1. shows the 14 subproblems, and Fig. 4. shows the board with the corresponding coordinates. The blacked-out areas in Table 1. are problems that have already been analyzed.

*Othello is a registered trademark.

Table 1. Executing sub-problems to prove our perfect play.

Depth 6	Depth 4	Depth 2
f1e1d1g1h1e3	**f1e1d1e3f4g1**	**f1e3f4e1d1g1**
f1e1d1g1h1h2	**f1e1d1f4e2g1**	**f1e3f4e1d1h2**
	f1e1d1f4e2h1	**f1e3f4g1h1h2**
	f1e1d1h2h3f4	**f1g1h1e1d1e3**
	f1e1d1h2h3h4	**f1g1h1e1d1f4**
		f1g1h1e1d1h2
		f1g1h1e3g4h3

Of the 14 problems, those in Table 1. that are not blacked out are unsolved problems. Fig. 5. shows the board of an unsolved problem, where the X mark indicates a square where a black stone can be placed.

a1	b1	c1	d1	e1	f1	g1	h1	i1	j1	k1	l1
a2	b2	c2	d2	e2	○	●	h2	i2	j2	k2	l2
a3	b3	c3	d3	e3	●	○	h3	i3	j3	k3	l3
a4	b4	c4	d4	e4	f4	g4	h4	i4	j4	k4	l4

Fig. 4. Coordinates on 4×12 board.

5. Conclusion

As a result of analyzing the unsolved problem shown in Table 1., we found "Black 47 White 0". Since all problems in Table 1. were found to be perfect play, Takeshita's perfect play shown in Fig. 2. and Fig. 3. was proved. Fig. 7. shows the procedure of the perfect play discovered in this study.

					●	●	●				
				X	●	●		X			
				○	○	○	○				
			X	X	X	●	X	X			

Fig. 5. The board of the unsolved problem of 4×12 board Othello

In the 4×12 board Othello, it turned out that black wins as a result of both sides continuing to play their best moves. This is the same result as for the 4×6, 4×8, and 4×10 boards, so it is expected that black will also win in the 4×14 board.

●	●	●	●	●	●	●	●	●	●	●	●
●	●	●	●	●	●	●	●	●	●	●	●
●	●	●	●	●	●	●	●	●	●	●	●
●	●	●	●	●	●	●	●	●	●	●	

Fig. 6. Endgame board for perfect play of unsolved problems.

31	32	22	27	8	1	2	3	10	33	42	43
28	23	24	19	13	○	●	17	34	26	38	39
30	16	15	12	4	●	○	6	25	35	40	41
29	21	20	9	7	14	5	11	18	36	37	

Fig. 7. Procedure of perfect play for unsolved problems.

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