Analysis of 5x5 board Quoridor

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Abstract

Quoridor is a perfect information game played by two or four players. The purpose of this study is to classify this game as a first move win, a second move win, or a draw. However, the analysis of the standard version of this game, 9x9 board, requires a huge amount of calculation, so in this paper, we show one of a perfect plays of a reduced version with a 5x5 board and one fence each.

Keywords: Perfect play, Retrograde Analysis, Quoridor, combinatorial theory

1. Introduction

Quoridor is a strategy game designed by Mirko Marchesi and released by Gigamic Games* in 1997. In the year it was released, Quoridor was selected for the Mensa Mind Game award†.

Quoridor is played on a 9x9 board with two or four players (sometimes unofficially three). Quoridor for two-player starts from the initial position in Fig.1 with 20 fences per player. Do either "move one of own pawn" or "install one fence" on one's turn, advance the turn in order, the player who first reaches any space on the opposite side of the game board wins.

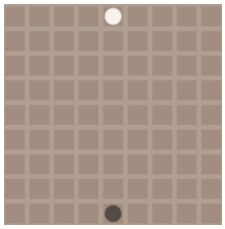

Fig 1. Initial position of Quoridor for two-player.

Quoridor for two-player is categorized into two-player zero-sum finite deterministic games of perfect information[1,2]. Games in this class are possible to look ahead in theory, thus if both players choose constantly the best move, these are classified into win, loss or draw game for the first player (the sequence obtained in this way is called perfect play). This game is still young among board games and has not been studied much, so it is not clear whether it is a win, a loss or a draw game for the first move player.

Therefore, this paper uses retrograde analysis to show one of a perfect plays of a reduced version with a 5x5 board and one fence each.

2. Quoridor

In this paper, we deal with a miniature board Quoridor. This section describes the rules of a 5x5 board for two-player with one fence each.

2.1. *Object of the Game*

Object of the game is the same as the standard version[3], to be the first to reach the line opposite to one's base line.

2.2. *Game Play (2 players)*

Each player in turn, chooses to move his pawn or to put up one of his fence. When he has run out of fences, the player must move his pawn.

At the beginning the board is empty. Choose and place your pawn in the center of the first line of your side of the board, your opponent takes another pawn and places it in the center of the first line of his side of the board (the one facing yours). Then take one fence each.

2.3. *Pawn moves*

The pawns are moved one square at a time, horizontally or vertically, forwards or backwards, never diagonally. The pawns must bypass the fences. If, while you move, you face your opponent's pawn you can jump over.

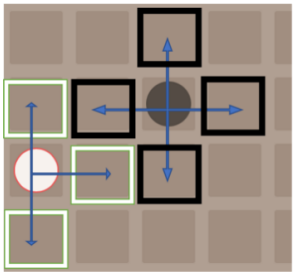

Fig 2. How to move pawn.

The white square is where the white pawn can move and the black square is where the black pawn can move.

2.4. *Positioning of the fences*

The fences must be placed between two sets of two squares. By placing fences, you force your opponent to move around it and increase the number of moves they need to make. However, you are not allowed to lock up to lock up your opponents pawn, it must always be able to reach it's goal by at least one square.

2.5. *Face to face*

When two pawns face each other on neighboring squares which are not separated by a fence, the player whose turn it is can jump the opponent's pawn (and place himself behind him), thus advancing an extra square.

If there is a fence behind the said pawn, the player can place his pawn to the left or the right of the other pawn.

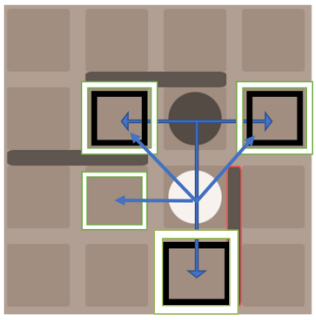

Fig 3. When two pawns are next to each other or when the path is blocked by a wall.

2.6. *End of the game*

The first player who reaches one of the five squares opposite his base line is the winner.

3. Retrograde Analysis

In this study, we conducted an experiment using receding analysis[4,5]. This method goes back one step at a time from the final stage where the victory or defeat is decided toward the initial board. In the process, if the previous move is connected to the victory phase, the victory information is received, and if all are connected to the defeat phase, the defeat information is received and the flow is repeated, so that the victory or defeat of the first phase can be known. The advantage of this method is that you can also consider the case of a tie, which involves

repeating the same move with each other, which is called "Sennichite".

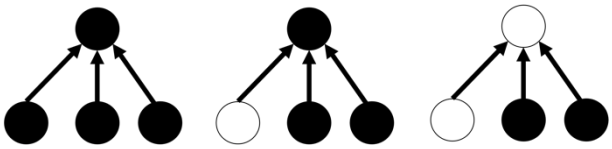

Fig 4. Retrograde Analysis

In Fig. 4, the circle represents a position of the game. The black circle and the white circle mean the black victory phase and the white victory phase, respectively. The lower circles represent the condition that can be transitioned from the upper circle.

In retrograde analysis of game, the result of the position is determined in the reverse order from the final position where the victory or defeat is decided to the initial position. For example, if the next move is only a black victory, that position is a black victory, see Fig.4.

4. Results

Some of the results of the regression analysis are shown in Fig. 5 which shows a game tree at a depth of 2 from the initial condition. "W" indicates that the victory of the first move is confirmed, and "L" indicates that the victory of the second move. The "P" part represents the movement of the pawn. The "F" part represents the placement of the fence. Pawn movements are divided into vertical movements and horizontal movements. Since there are many Fs, some are omitted.

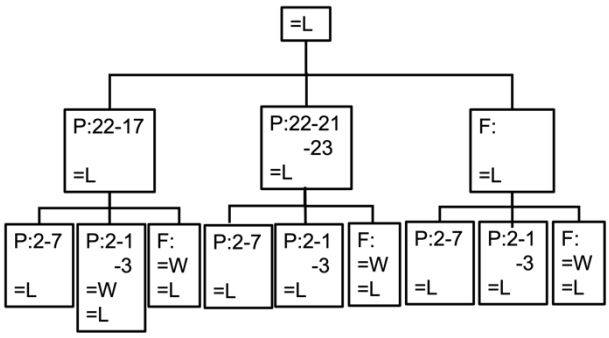

Fig 5. Some of the results of the regression analysis

For the position of the pawn in Fig. 5, the board number is specified as shown on the left in Fig. 6. Similarly, regarding the position of the fence, the position number is given to the fence as shown on the right in Fig. 6.

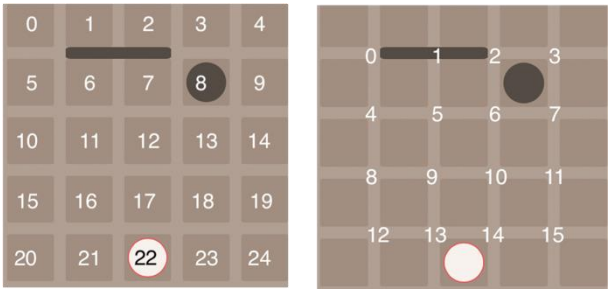

Fig. 6. Pawn and fence position number

Fig. 7 shows A perfect plays of Quoridor with a 5x5 board and one fence each. If the first move and the second move make the best move to each other, the first move must jump over or make a detour, so the first move loses by one move.

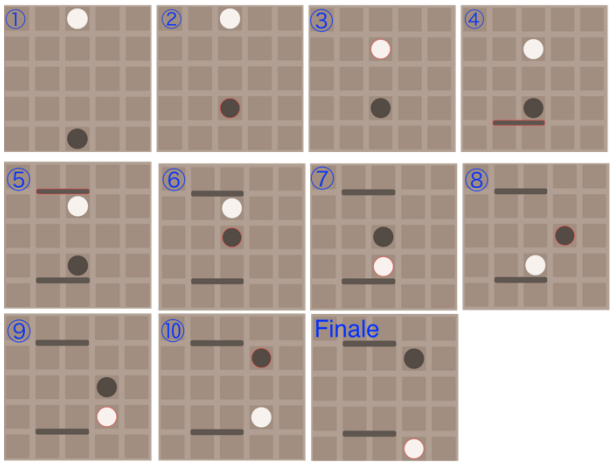

Fig. 7 A perfect plays of Quoridor with a 5x5 board and one fence each.

The number of positions obtained in this experiment was 298807 (There are $3.9905*10^{42}$ in the regular version[6].), excluding 768 positions not allowed by the rules. Furthermore, considering the turn of each position, the number is doubled to 597614. Only 20 of them were undecided.

5. Conclusion

After the experiment, I tried to play the 3x3 and 5x5 board with one fence each, and found that the game was won by the rear player in both cases. Since the only perfect play in this case is when the fence is placed horizontally in the center of the board where it can block the opponent's path, we can expect the same kind of

moves in the 7x7 and 9x9 board as in the 5x5 board, resulting in a backward win. On the other hand, in 6x5 and 8x5, where the length of the rows is even, the game is expected to be won by the first move. I would like to confirm these two points.

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