State-space modeling of motion of fingers measured by Leap Motion Controller

Ryuichi Usami
Graduate School of Humanities and Social Sciences, Hiroshima University,
1-1-1 Kagamiyama, Higashi-Hiroshima city, Hiroshima, 739-8524 Japan

Hideyuki Tanaka
Graduate School of Humanities and Social Sciences, Hiroshima University,
1-1-1 Kagamiyama, Higashi-Hiroshima city, Hiroshima, 739-8524 Japan
E-mail: tanakalpha@hiroshima-u.ac.jp

Abstract

This paper considers modeling of the motion of experts in sports to demonstrate their motion to beginners, studying the dynamics of a transient response from one position to another, such as a crouching start of a short-distance race. A modeling algorithm is developed to remove personal habits of experts from their motion and to present simple models to learners according to their learning stages. It is applied to fingers motion captured by the Leap Motion Controller.

Keywords: Leap Motion Controller, state-space representation, step response, modeling of motion of fingers,

1. Introduction

Motion of players in sports has been measured by sensors. For example, it has been reported that motion sensors were used for capturing the movements of the golf swing[1] and that pressure sensor systems were presented for monitoring the ankle supination torque during sport motions[2]. A wearable inertial sensor network and its associated activity recognition algorithm were presented for accurately recognizing human daily and sport activities[3]. Motion capture also has been used for analyzing sports performance; see a systematic review of the recent developments of motion capture systems for the analysis of sport performance[4]. Moreover, a system that is applicable to marker-less sports movement analysis has been presented[5].

Demonstrating models to learners has proven to be particularly effective in enhancing motor learning[6]. Presenting a model of an expert to beginners is therefor expected to be effective in learning the motion. However, in sports, advanced techniques and skills are too complex for beginners, as well as individual habits. We hence believe that a simple model that removes these factors would make it easier for beginners to learn the motion. We therefore introduce a step-by-step simplification of the behavior of an expert in this research.

In this study, we make mathematical models to give a simplified model for beginners and a complicated model for advanced learners. To this end, we introduce state-space representation driven by a step input, often used for dealing with dynamics in control engineering[7]. We use state-space representations, because it can deal with dynamics of multiple input and multiple output, and we can obtain a simplified model by reducing the order of the state-space representation. We can model a transient response from one position to another by means of a step input, e.g. a crouching start of a short-distance race. We will model a single grasping motion of a hand as a simple example of motion that moves from one position to another, because motions of finger joints are related to each other and are suitable for studying modeling a transient response of sports movements.

As a motion capture device, we use the Leap Motion Controller (LMC), an optical hand tracking module that captures the movements of hands[8]. Accuracy and robustness of the LMC has been analyzed[9], and the performance of the LMC was evaluated with the aid of a professional, high-precision, fast motion tracking system[10].

2. Measuring data

We describe how to measure the finger motion. Figs.1 and 2 show the start (the open hand) and the end (the grasped hand) of transient motion, respectively. The LMC can measure 21 finger joints in a three-dimensional cartesian coordinate system, meaning that the LMC obtains 63 elements at a sampling time. Figs. 3 and 4 show the measuring points and the coordinate system, respectively. We define the output $y(t)$ as a vector with 63 rows as

$$y(t) = \left[p_x(t)^\top \quad p_y(t)^\top \quad p_z(t)^\top\right]^\top,$$

where $p_x(t), p_y(t), p_z(t) \in R^{21}$ are positions in the X-Y-Z, coordinate of 21 joints of fingers. The positions of finger joints are measured at the equal sampling time between the time interval from the open hand to the grasped hand. The time interval is around 1 second, and 100 data is sampled in 1 second. We use Processing for measuring the grasping motion[11,12].

Fig.1. start of measurement

Fig.2. end of measurement

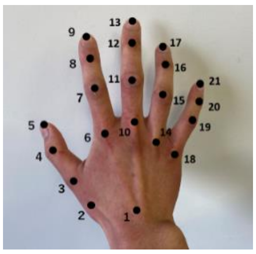

Fig.3. measuring points

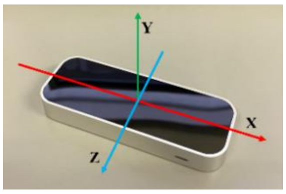

Fig.4. coordinate system

3. Problem setting

Suppose that the sampling time is h and let y_k be the coordinate $y(t)$ at $t = kh$:

$$y_k = y(kh). \tag{1}$$

We model y_k by means of the distance-time state-space representation:

$$x_{k+1} = Ax_k + Bu_k, \tag{2a}$$

$$y_k = Cx_k + \zeta, \tag{2b}$$

where x_k is a state of the system, $x_0 = 0$, ζ is a constant vector, and u_k is as follows:

$$u_k = \begin{cases} 0 & (k = -1, -2, \dots) \\ 1 & (k = 0, 1, 2, \dots) \end{cases} \tag{3}$$

We suppose that $k = 0$ when the transient motion starts and consider the following problems.

Problem 1: Find (A, B, C, ζ) within the degrees of freedom of similarity transformations, given y_k in (2) and (3).

Problem 2: Suppose that the positions of joints fingers are measured as in (1). Find a mathematical model for y_k in (2) and (3).

The complexity of the model of the finger motion in (2) is determined by the size of the matrix A or the order of the system. The lower the order, the simpler the model is. The higher the order, the more accurately the finger motion can be modeled, and the closer motion is generated to the original one.

By choosing the order of the model (2), we can generate samples of the motion of the experts for different level of learners. It is expected that beginners will be able to learn simple motions generated by simpler dynamical models, whereas advanced learners will be able to learn more complex motions. It should be noted that the dynamical system (2) can model the transition from one point to another, though we model finger motion in this study.

4. Solution via deterministic realization

We solve Problem 1 by using the deterministic realization algorithm[13] and apply the algorithm to Problem 2. The deterministic realization algorithm is suitable for the purpose of this study, because the order of the model can be systematically determined by the singular value decomposition (SVD)[7]. Since the deterministic realization algorithm was developed for obtaining a state-space model of an impulse response, we modify the algorithm to obtain a state-space model of the finger motion.

We consider Problem 1. Let us describe y_k for (2) as

$$y_k = \sum_{j=1}^{k} CA^{j-1}B + \zeta, \tag{4}$$

and let v_k be as follows ($k \geq 1$):

$$v_k = y_k - y_{k-1}. \tag{5}$$

The signal v_k is the difference of the output y_k. From (4), v_k satisfies the following equation:

$$v_k = CA^{k-1}B. \tag{6}$$

For positive integers $\tau > n$ and $N > n$, define the Hankel matrix $H \in R^{\tau p \times N}$ from v_k as follows:

$$H = \begin{bmatrix} v_1 & v_2 & \cdots & v_{N+1-\tau} \\ v_2 & v_3 & \cdots & v_{N+2-\tau} \\ \vdots & \vdots & \vdots & \vdots \\ v_\tau & v_{\tau+1} & \cdots & v_N \end{bmatrix}. \tag{7}$$

We also define the extended observability and reachability matrices as follows:

$$\mathcal{O}_\tau = [C^\top \quad (CA)^\top \quad \cdots \quad (CA^{\tau-1})^\top]^\top, \tag{8a}$$

$$\mathcal{C}_\mathcal{N} = [B \quad AB \quad \cdots \quad A^{N-1}B]. \tag{8b}$$

From (6), H satisfies the following equation:

$$H = \mathcal{O}_\tau \mathcal{C}_\mathcal{N}. \tag{9}$$

Let us compute the SVD of H:

$$H = U\Sigma V^\top \approx [U_1 \quad U_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^\top \\ V_2^\top \end{bmatrix} = U_1 \Sigma_1 V_1^\top, \tag{10}$$

where Σ is a diagonal matrix, and U and V are orthogonal matrices satisfying $U^\top U = I$ and $V^\top V = I$. From (9), $\mathcal{O}_\tau$ and $\mathcal{C}_\mathcal{N}$ are expressed as

$$\mathcal{O}_\tau = U_1 \Sigma_1^{\frac{1}{2}}, \qquad \mathcal{C}_\mathcal{N} = \Sigma_1^{\frac{1}{2}} V_1^\top. \tag{11}$$

From (8), C and B can be obtained as follows:

$$C = \mathcal{O}_\tau(1{:}p, :), \qquad B = \mathcal{C}_\mathcal{N}(:, 1), \tag{12}$$

where we use the colon notation[14]. Let us define $\mathcal{O}_\tau^\downarrow$ and $\mathcal{O}_\tau^\uparrow$ as follows:

$$\mathcal{O}_\tau^\downarrow = \mathcal{O}_\tau(1{:}p(\tau-1), :), \tag{13a}$$

$$\mathcal{O}_\tau^\uparrow = \mathcal{O}_\tau(\tau+1{:}p\tau, :). \tag{13b}$$

From (8a), we have

$$\mathcal{O}_\tau^\downarrow = [C^\top \quad (CA^\top) \quad \cdots \quad (CA^{\tau-2})^\top]^\top, \tag{14a}$$

$$\mathcal{O}_\tau^\uparrow = [(CA)^\top \quad (CA^2)^\top \quad \cdots \quad (CA^{\tau-1})^\top]^\top, \tag{14b}$$

and hence

$$\mathcal{O}_\tau^\downarrow A = \mathcal{O}_\tau^\uparrow. \tag{15}$$

We can thus obtain A by solving the least-squares method. Let the estimates of A, B, C and ζ be denoted as $\hat{A}$, $\hat{B}$, $\hat{C}$, and $\hat{\zeta}$, respectively. By setting the initial state as $\hat{x}_0 = 0$, we compute η_k for $k = 0, \cdots, N$ as follows:

$$\hat{x}_{k+1} = \hat{A}\hat{x}_k + \hat{B}u_k, \tag{16a}$$

$$\eta_k = \hat{C}\hat{x}_k, \tag{16b}$$

where u_k is the step input in (3). We then have $\eta_k = \sum_{j=1}^{k-1} \hat{C}\hat{A}^{j-1}\hat{B}$ and obtain the following equations from (16):

$$\zeta = y_k - Cx_k = y_k - \textstyle\sum_{j=1}^{k} CA^{j-1}B.$$

We compute an estimate of ζ, by averaging $y_k - \eta_k$:

$$\hat{\zeta} = \frac{1}{N+1} \sum_{k=0}^{N} (y_k - \eta_k). \tag{17}$$

Thus, estimates of (A, B, C, ζ) for Problem 1 are obtained as $(\hat{A}, \hat{B}, \hat{C}, \hat{\zeta})$, and those of $\hat{y}_k$ in (2) are given by

$$\hat{x}_{k+1} = \hat{A}\hat{x}_k + \hat{B}u_k, \tag{18a}$$

$$\hat{y}_k = \hat{C}\hat{x}_k + \hat{\zeta}. \tag{18b}$$

We summarize the above procedure for Problem 1 as the following algorithm:

Step 1: Calculate v_k in (5).
Step 2: Construct the block Hankel matrix H in (7).
Step 3: Compute the SVD in (10).
Step 4: Determine $\mathcal{O}_\tau$ and $\mathcal{C}_\mathcal{N}$ as in (11).
Step 5: Compute the estimate $(\hat{A}, \hat{B}, \hat{C})$ of (A, B, C) from (12) and (15).
Step 6: Obtain an estimate $\hat{\zeta}$ of ζ as (17) and calculate $\hat{y}_k$ in (18).

By applying this algorithm to Problem 2, we have mathematical models for motions of fingers. The model is simplified by the SVD of the Hankel matrix in (10), and the order is determined by the number of the dominant singular values. We hence select the order by choosing the number of the non-zero diagonal elements of H. We thus simplify the state-space model (3) and obtain simplified transient motion from the open hand to the grasped hand. The lower the order of state-space model is, the simpler the transient motion becomes.

5. Experimental results

This section discusses obtained models. We use the first 0.7 seconds of the transition from the open hand to the grasped hand, focusing on the dynamics of the start of the motion and removing the data for the end of 0.3 seconds. The sampling time is $h = 0.01$(s). In Figs. 5-8, we show $p_y(t)$ at the measuring point 13, which is a tip of the middle finger and has one of the largest motions among the measuring points of fingers. The horizontal and vertical axes express the time (s) and the positions (mm) of $p_y(t)$, respectively. Red lines in Figs. 5-8 indicate the position of the original motion, whereas blue ones in Figs. 5-8 show models at the order of 4, 7, 12, and 18, respectively. Figs. 5-8 demonstrate that the output of models (blue lines) become closer to the original motion (red lines) as the order is higher; compare blue lines in Figs. 5 and 8 and notice that the motion drawn by blue line in Fig 5 is much simpler than that in Fig 8. We can thus demonstrate fingers motions with different simplifications.

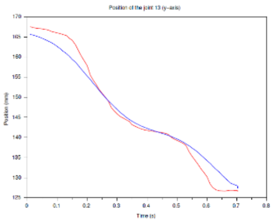

Fig.5. model ($n = 4$)

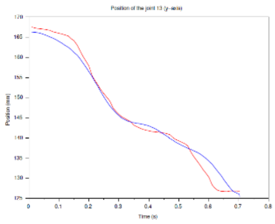

Fig.6. model ($n = 7$)

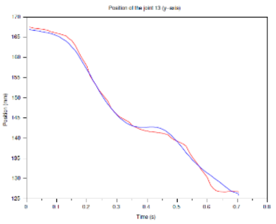

Fig.7. model ($n = 12$)

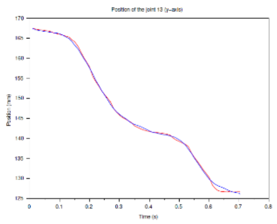

Fig.8. model ($n = 18$)

6. Conclusions

In this study, we have made mathematical models of fingers motions with the aim of simplifying the motion of experts in sports. We solved the problem of modeling fingers motions based on a step response of a state-space representation, by modifying the deterministic realization algorithm. The experimental results showed that the modeling method satisfied the purpose of simplifying complex fingers motions reducing the order of the state-space representation. By choosing the order, we could select the model of finger motion from simple one to more accurate one that is close to the original motion.

References

1. H. Ghasemzadeh, V. Loseu, E. Guenterberg, and R. Jafari, “Sport training using body sensor networks: A statistical approach to measure wrist rotation for golf swing”, Proceedings of the Fourth International Conference on Body Area Networks, 2009.
2. D. T.-P. Fong, Y.-Y. Chan, Y. Hong, P. S.-H. Yung, K.-Y. Fung, and K.-M. Chan, “A three-pressure-sensor (3PS) system for monitoring ankle supination torque during sport motions”, Journal of Biomechanics, Volume 41, Issue 11, pp. 2562-2566, 2008.
3. Y.-L. Hsu, S.-C. Yang, H.-C. Chang, and H.-C. Lai, “Human daily and sport activity recognition using a wearable inertial sensor network”, IEEE Access, vol. 6, pp. 31715-31728, 2018.
4. B. Pueo and J. M. J.-Olmedo, “Application of motion capture technology for sport performance analysis”, editor: Federación Española de Asociaciones de Docentes de Educación Física (FEADEF), Retos, 32, pp. 241-247, 2017.
5. B. Rosenhahn, T. Brox, U. G. Kersting, A. W. Smith, J. K. Gurney, and R. Klette, “A system for marker-less motion capture”, Künstliche Intelligenz, vol. 1, pp 46-52, 2006.
6. G. H. Sage, “Motor learning and control -A neuro-psychological approach-“, Wm. C. Brown Publishers, 1984.
7. T. Katayama, “Subspace methods for system identification”, Springer-Verlag London Limited, 2005.
8. ultraleap, “Leap Motion Controller”, https://www.ultraleap.com/product/leap-motion-controller/.
9. F. Weichert, D. Bachmann, B. Rudak, and D. Fisseler, “Analysis of the accuracy and robustness of the Leap Motion Controller”, Sensors, 13, pp. 6380-6393, 2013.
10. J. Guna, G. Jakus, M. Pogačnik, S. Tomažič, and J. Sodnik, “An analysis of the precision and reliability of the Leap Motion sensor and its suitability for static and dynamic tracking”, Sensors, 14, pp. 3702-3720, 2014.
11. Processing, https://processing.org.
12. ultraleap, “API Overview”, https://developer-archive.leapmotion.com/documentation/java/devguide/Leap_Overview.html.
13. B. L. Ho and R. E. Kalman, “Effective construction of linear state-variable models from input/output functions”, Regelungstechnik, vol. 14, no.12, pp. 545-548, 1966.
14. G. H. Golub and C. F. Van Loan, “Matrix computations, 4th Edition”, The Johns Hopkins University Press, 2013.

Authors Introduction

Mr. Ryuichi Usami

He received his bachelor's degree from the School of Education, Hiroshima University, Japan in 2021. He is currently a Master’s Program student of Graduate School of Humanities and Social Sciences in Hiroshima University, Japan.

Dr. Hideyuki Tanaka

He graduated master course at graduate school of engineering in Kyoto University and received Dr. (Eng.) from Kyoto University. He is a member of Division of Educational Sciences, Graduate School of Humanities and Social Sciences in Hiroshima University. He is a member of IEEE, SICE, and ISCIE.