Trajectory Tracking Control of Differential Wheeled Mobile Robots

Based on Rhombic Input Constraints

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Abstract

This paper focuses on the trajectory tracking control algorithm for differential wheeled mobile robots (DWMRs) based on rhombic input constraints. The kinematics and dynamics model of DWMRs are Established, and vector analysis method is used to design the controller when the linear velocity and angular velocity of DWMRs were not independent of each other. Through the trajectory tracking simulation of the 8-shaped curve, a good control performance is obtained.

Keywords: differential wheeled mobile robots, rhombic input constraints, trajectory tracking, vector analysis

1. Introduction

The tracking control of DWMRs has a very broad application background. There are many methods have been used in controller design for trajectory tracking. Sliding mode control,[1] backstepping control,[2] robust control,[3] fuzzy control,[4] active disturbance rejection control[5] etc. are used to solve this problem. From a practical perspective, the input constraints must be considered when designing controller, however most existing studies are assume that the input constraints of the robots' linear velocity v and angular velocity ω are independent of each other, that is, $|v| \le m_1$, $|\omega| \le m_2$, where m_1 and m_2 are positive constants. (Fig.1). A proof will be given later, the actual situation is that the input field of DWMRs is the rhombic area defined by $|v/m| + |\omega\, l/m| \le 1$, where m is the maximum velocity of the two driving wheels and l is half of the distance

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between them. If the controller in Ref.6 is applied to a differential drive robots, the rectangle estimation needs to be extracted in the rhombic input field. The largest rectangle estimate that can be calculated is determined by $|v| \le m/2$ and $|\omega| \le m/2l$, it is only half of the rhombic input field, which leads to the robots' mobility cannot be fully utilized. Rhombic input constraints are considered first time in Ref.7, it proposed a geometric analysis method to design time-varying feedback parameters.

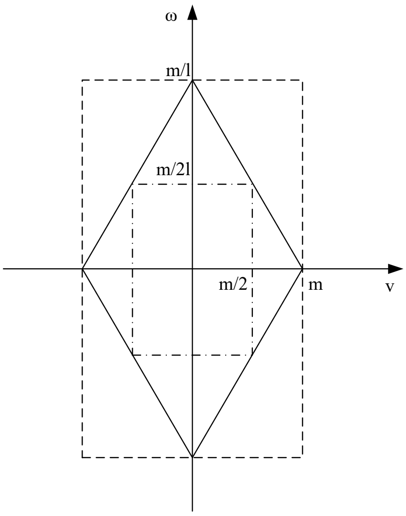


Fig. 1. Rectangular and diamond constraints

2. Problem Statement

2.1. *Rhombic Input Constraints*

Consider DWMRs shown in Fig.2, the driving wheels' velocities of the robots are v_l and v_r respectively. Assuming that the two driving wheels have the same performance, there maximum velocities are both m, that is $v_l \le m$ and $v_r \le m$. Usually v and ω of DWMRs are used as control inputs, and their relationship with the speed of the driving wheel is

$$v = (v_l + v_r)/2 \tag{1}$$

$$\omega = (v_r - v_l)/2l \tag{2}$$

Thus v and ω are constrained by

$$\begin{cases} -(m+v)/l \le \omega \le (m+v)/l, v \in [-m,0) \\ -(m-v)/l \le \omega \le (m-v)/l, v \in [0,m] \end{cases} \tag{3}$$

The above is collated into one expression:
Equation (3) can be sorted into one expression:

$$|v/m| + |\omega\, l/m| \le 1 \tag{4}$$

This constraints is shown in the Fig.1 as a rhombus with black solid line.

2.2. *Tracking Control Based on Rhombic Input Constraints*

The kinematics and dynamics equations of two-wheel differential mobile robots is

$$\dot{x} = v\cos\theta,\ \dot{y} = v\sin\theta,\ \dot{\theta} = \omega \tag{5}$$

(x, y) is the center point coordinates of DWMRs and θ is used to indicate its azimuth angle (see Fig.2).

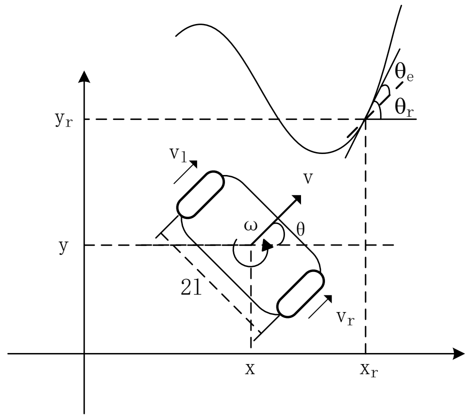


Fig. 2. Trajectory tracking of DWMRs

Assumption 1. The input constraint of DWMRs is (4), and its reference trajectory satisfies:

$$\dot{x}_r = v_r\cos\theta_r,\ \dot{y}_r = v_r\sin\theta_r,\ \dot{\theta}_r = \omega_r \tag{6}$$

and

$$|v_r/m| + |\omega_r l/m| \le 1 - l\varepsilon/m \tag{7}$$

Among them, $(x_r, y_r, \theta_r, v_r, \omega_r)$ is the target values of $(x, y, \theta, v, \omega)$, where ε is a constant satisfies $0 < \varepsilon < m/l$.

Remark 1. We introduce a constant ε in equation (7) to ensure that the reference trajectory can be tracked by DWMRs under the input constraints (4)

Fig.2 shows that system errors of DWMRs are defined as:

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \tag{8}$$

The tracking errors system can be obtained by deriving the two sides of the above formula (8)

$$\begin{aligned} \dot{x}_e &= v_r\cos\theta_e - v + \omega y_e \\ \dot{y}_e &= v_r\sin\theta_e - \omega x_e \\ \dot{\theta}_e &= v_r\omega_r - \omega \end{aligned} \tag{9}$$

Now our goal is to design the controller to make the system errors x_e, y_e and θ_e tend to zero, and the control variables v and ω must meet the constraints (4).

3. Controller Design Based on Rhombic Input Constraints

Before our controller design work begins, we first understand two important lemmas

Lemma 1.[8] $f:[0,\infty)\to R$ is first-order continuous differentiable and $\lim_{t\to\infty} f(t)$ is a finite value, if $\dot{f}(t), t\in[0,\infty)$ is uniformly continuous, then $\lim_{t\to\infty}\dot{f}(t)=0$.

Lemma 2.[7] There is a scalar function $\rho(x), x\in[0,\infty)$, which satisfies the following properties:

1. $\rho(x)$ is a continuous and non-decreasing function;
2. $\rho(0)=0$, and $0<\rho(x)\le 1$ for $x\in(0,\infty)$;
3. $\lim_{x\to 0^+}\rho'(x)=\rho_0$, which ρ_0 is a positive constant.

Define $\psi(x)$ as

$$\psi(x)=\begin{cases}\rho(x)/x & x\in(0,\infty)\\ \rho_0 & x=0\end{cases} \tag{10}$$

Then, for $\forall\sigma\in(0,\infty)$, there always exist α and β, such that $\alpha\le\varphi(x)\le\beta$ holds for $x\in[0,\sigma]$, where both α and β are positive constants.

$\rho(x)=\tanh(x)$ is a function that satisfies the above conditions.

There are many results about the design of the tracking controller of the differential drive robots in the existing literature. In this paper we choose the controller in Ref.9.

$$\begin{aligned} v &= v_r\cos\theta_e + k_x x_e \\ \omega &= \omega_r + k_y v_r y_e\frac{\sin\theta_e}{\theta_e} + k_\theta\theta_e \end{aligned} \tag{11}$$

where k_x, k_y and k_θ are positive constants. Through formula (11), we can easily see that too large errors will cause the control variables v and ω to be too large, and then the control variables will exceed the constraints. In this way, the control commands cannot be executed well.

Lemma 3.[7] For controller (11), if following conditions are met:

1) $\underline{k}_x\le k_x\le\overline{k}_x, \underline{k}_y\le k_y\le\overline{k}_y, \underline{k}_\theta\le k_\theta\le\overline{k}_\theta$

2) k_y is differentiable and $\dot{k}_y\ge 0$.

where $\underline{k}_x,\overline{k}_x,\underline{k}_y,\overline{k}_y,\underline{k}_\theta,\overline{k}_\theta$ are positive constant values, Then, trajectory tracking errors of DWMRs will converge to zero, that is x_e, y_e, θ_e will converge to zero.

Controller (11) can be designed by using the vector analysis method, define the controller v,ω as a vector

$$\overrightarrow{OD}=\begin{bmatrix} v\\ \omega\end{bmatrix}$$

then by defining other vectors as:

$$\overrightarrow{OA}=\begin{bmatrix} v_r\cos\theta_e\\ \omega_r\end{bmatrix}, \overrightarrow{AB}=\begin{bmatrix} 0\\ k_y v_r y_e\frac{\sin\theta_e}{\theta_e}\end{bmatrix}$$
$$\overrightarrow{BC}=\begin{bmatrix} k_x x_e\\ 0\end{bmatrix}, \overrightarrow{CD}=\begin{bmatrix} 0\\ k_\theta\theta_e\end{bmatrix} \tag{12}$$

Then we can get the vector representation of the controller

$$\overrightarrow{OD}=\overrightarrow{OA}+\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD} \tag{13}$$

In order to make the controller meet the rhombic input constraints, we need to analyze each vector in turn.

$$\left|\frac{v_r\cos\theta_e}{m}\right|+\left|\frac{\omega_r l}{m}\right|\le\left|\frac{v_r}{m}\right|+\left|\frac{\omega_r l}{m}\right|\le 1-\frac{l\varepsilon}{m} \tag{14}$$

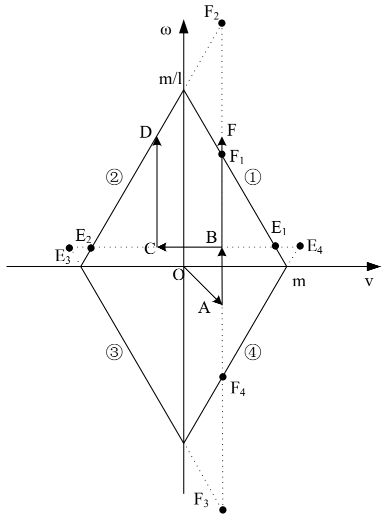


Fig. 3. Vector method design controller

Equation (14) shows that $\overrightarrow{OA}$ satisfy the rhombic input constraints, without loss of generality, we represent $\overrightarrow{OA}$ as shown in Fig.3, and because the length of $\overrightarrow{AB}$ is proportional to k_y, we can definitely find a k_y to make $\overrightarrow{AB}$ within the rhombic input constraints. Similarly, we can also find suitable k_x and k_θ, so $\overrightarrow{BC}$ and $\overrightarrow{CD}$ can meet the rhombic constraints respectively. Obviously, the controller $\overrightarrow{OD}$ will definitely meet the rhombic input constraints.

Since the requirement for k_y is $\dot{k}_y\ge 0$, we intuitively thought of designing if from Lyapunov function $V(t)$

$$V(t)=\frac{1}{2}(x_e^2+y_e^2+\frac{\theta_e^2}{k_y}) \tag{15}$$

Let k_y be

$$k_y = \frac{\lambda\varepsilon}{m\sqrt{2V(t)+\mu^2}} \tag{16}$$

Where λ and μ are constants, $0<\lambda<1$ and $\mu>0$. According to (15) (16), we can get

$$k_y = \frac{-m\theta_e^2+\sqrt{m^2\theta_e^4+4\lambda^2\varepsilon^2(x_e^2+y_e^2+\mu^2)}}{2m(x_e^2+y_e^2+\mu^2)}>0 \tag{17}$$

$$\dot{k}_y = \frac{2k_x k_y^2 x_e^2+2k_\theta k_y\theta_e^2}{2k_y(x_e^2+y_e^2+\mu^2)+\theta_e^2} \tag{18}$$

If $k_x>0$ and $k_\theta>0$, then according to equations (17) and (18), $\dot{k}_y<0$ can be derived, and further from (15) we can get

$$\underline{k}_y = \frac{\lambda\varepsilon}{m\sqrt{2V(0)+\mu^2}} \le k_y \le \frac{\lambda\varepsilon}{m\mu} = \overline{k}_y \tag{19}$$

In this way, the vector $\overrightarrow{OB}$ can be expressed as:

$$\overrightarrow{OB} = \overrightarrow{OA}+\overrightarrow{AB} = (v_r\cos\theta_e, \omega_r + k_y v_r y_e\frac{\sin\theta_e}{\theta_e})^{\mathrm{T}} \tag{20}$$

Similar to $\overrightarrow{OA}$ satisfy the rhombic input constraints, we can easily verify that $\overrightarrow{OB}$ satisfies the rhombic input constraints through formulas (14), (15) and (16). Since $k_x, k_\theta>0$, so the directions of the vectors $\overrightarrow{BC}$ and $\overrightarrow{CD}$ are determined by the signs of x_e and θ_e, In order to expand the input field as much as possible while meeting the rhombic input constraints. First, we need to determine the triangle area $\triangle \mathrm{BEF}$ where the points of C and D are located, as shown in Fig.3, when $x_e<0$ and $\theta_e>0$, we take the constraint segment ② to determine the reference triangle $\triangle \mathrm{BE_2F_2}$, similarly, when $x_e>0$ and $\theta_e>0$, we take the constraint segment ① to determine the reference triangle $\triangle \mathrm{BE_1F_1}$, when $x_e<0$ and $\theta_e<0$, we get the reference triangle $\triangle \mathrm{BE_3F_3}$, and when $x_e<0$ and $\theta_e<0$, we get the reference triangle $\triangle \mathrm{BE_4F_4}$. Through the equations of the four constraint lines and the coordinates of point B, we can easily obtain the coordinates of point E as:

$$E:(\mathrm{sgn}(x_e)(m-\mathrm{sgn}(\theta_e)(\omega_r+k_y v_r y_e\frac{\sin\theta_e}{\theta_e})l), \quad w_r + k_y v_r y_e\frac{\sin\theta_e}{\theta_e}) \tag{21}$$

Similarly, we can get the coordinates of F as:

$$F:(v_r\cos\theta_e, \frac{\mathrm{sgn}(\theta_e)(m-\mathrm{sgn}(x_e)v_r\cos\theta_e)}{l}) \tag{22}$$

Where $\mathrm{sgn}(\cdot)$ is sign function

$$\mathrm{sgn}(x)\begin{cases}|x|/x & x\ne 0\\ 0 & x=0\end{cases} \tag{23}$$

Further we can get the expressions of $\overrightarrow{BE}$ and $\overrightarrow{BF}$ as

$$\begin{aligned}\overrightarrow{BE} &= \overrightarrow{OE}-\overrightarrow{OB}\\ &= (\mathrm{sgn}(x_e)(m-\mathrm{sgn}(\theta_e)(\omega_r+k_y v_r y_e\frac{\sin\theta_e}{\theta_e})l)\\ &\quad -v_r\cos\theta_e, 0)^{\mathrm{T}}\end{aligned} \tag{24}$$

$$\begin{aligned}\overrightarrow{BF} &= \overrightarrow{OF}-\overrightarrow{OB}\\ &= (0, \frac{\mathrm{sgn}(\theta_e)(m-\mathrm{sgn}(x_e)v_r\cos\theta_e)}{l}-\omega_r\\ &\quad -k_y v_r y_e\frac{\sin\theta_e}{\theta_e})^{\mathrm{T}}\end{aligned} \tag{25}$$

To design k_x and k_θ, let

$$\begin{aligned}\overrightarrow{BC} &= \frac{\rho(|x_e|)}{2}\overrightarrow{BE}\\ \overrightarrow{CD} &= \frac{\rho(|\theta_e|)}{2}\overrightarrow{BF}\end{aligned} \tag{26}$$

Then, we get from (12)(24)(26), that

$$\begin{aligned}k_x &= \frac{\psi(|x_e|)}{2}(m-\mathrm{sgn}(\theta_e)(\omega_r+k_y v_r y_e\frac{\sin\theta_e}{\theta_e})l\\ &\quad -\mathrm{sgn}(x_e)v_r\cos\theta_e)\\ k_\theta &= \frac{\psi(|\theta_e|)}{2}(\frac{m-\mathrm{sgn}(x_e)v_r\cos\theta_e}{l}-\mathrm{sgn}(\theta_e)\\ &\quad (\omega_r+k_y v_r y_e\frac{\sin\theta_e}{\theta_e}))\end{aligned} \tag{27}$$

By formula (15)(16)(20)and *Lemma2* we can easily get

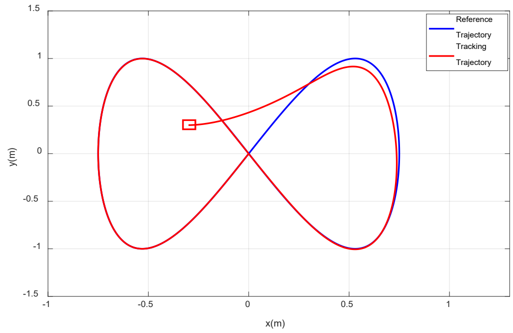
$$\begin{aligned}\underline{k}_x &\le \frac{\alpha(1-\lambda)\varepsilon l}{2} \le k_x \le \beta m \le \overline{k}_x\\ \underline{k}_\theta &\le \frac{\alpha(1-\lambda)\varepsilon}{2} \le k_\theta \le \frac{\beta m}{2} \le \overline{k}_\theta\end{aligned} \tag{28}$$

At this point, the k_x, k_y *and* k_θ meet the two conditions in *Lemma 3*, so the system error will converge to zero. And because of our vector method design the parameters ensure that the parameter control variables v and ω meet the rhombic input constraints too.

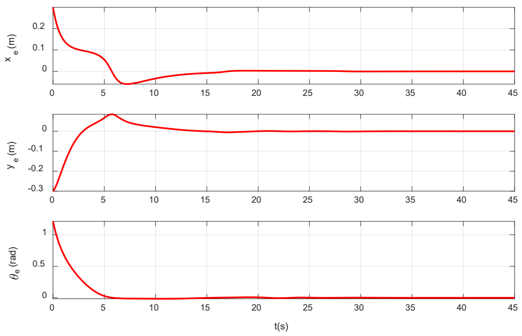
4. Simulation Results

In this section, we simulate and verify the effect of the controller. The maximum speed of the drive wheels is set to $m=0.4m/s$, the wheel spacing is set to $l=0.16m$, For setting some parameters of the controller, we choose $\rho(x) = \tanh(x),\ \varepsilon = 0.1,\ \lambda = 0.99,\ \mu = 0.01$.

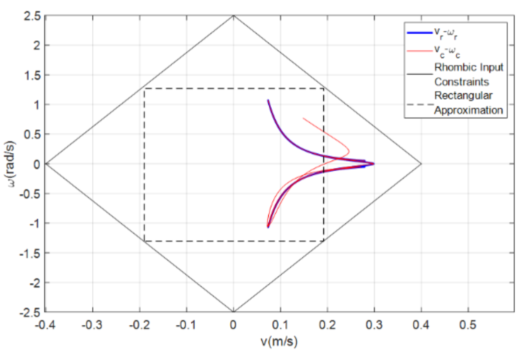
Fig.4(a) shows that DWMRs gradually tracks on the reference trajectory. The tracking errors x_e, y_e, θ_e are gradually converge to zero as shown in Fig.4(b), also we can guarantee the control variables v, ω satisfy the rhombic input constraints through Fig.4(c), and sometimes v can basically reach the bounds of rhombic input constraints.



(a) Tracking reference trajectory



(b) Tracking errors



(c) Input and constraints

Fig.4. Simulation results.

5. Conclusion

The trajectory tracking problem of DWMRs with rhombic input constraints is solved in this paper, compared with the controller based on rectangular input constraints. It can better play the robots' mobility and the controller not only solves the tracking problem, but also solves the stability problem, the simulation shows the controller is effective. Future work will focus on the controller design with uncertainty based on a more complex application environment.

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