# Weighted Multiple Model ADRC for Uncertain Linear Systems

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### Abstract

For uncertain linear systems such as parameter jumping, this paper presents a weighted multiple model adaptive control frame work, which uses fixed model set to cover the uncertainties of the real plant to be controlled, and for each local model with minor uncertainties, the corresponding local controller is designed based on active disturbance rejection controller (ADRC) approach. Some simulations have been conducted based on MATLAB to verify the effectiveness of the proposed weighted multiple model ADRC adaptive control algorithm.

Keywords: Multiple model adaptive control, ADRC, weighting algorithm.

## 1. Introduction

For systems with large uncertainties, such as, model structure change or parameter jumping, conventional adaptive control can no longer meet the performance needs of industrial control systems. Therefore, the control research of uncertain systems has important theoretical significance and practical significance. Weighted multiple model adaptive control (WMMAC) [1-12] can effectively control complex and uncertain systems with prescribed performance indexes. Aiming at the uncertain system, this paper proposes a weighted multiple model adaptive control approach with active disturbance rejection control (ADRC) as local control strategy, which was originally proposed by Professor Jingqing Han [13-20].

Aiming at a class of uncertain linear controlled plants with nonlinear disturbances, this paper combines the active disturbance rejection controller and the weighted multiple model adaptive control system to design a weighted multiple model ADRC control system for the second-order time-delay system and a weighted multiple model ADRC control system for the third-order uncertain system, so that to completed the control of these two uncertain linear systems with nonlinear disturbances. The simulation results show that compared with a single ADRC control system, the weighted multiple model ADRC control system can better meet the control

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performance requirements and has stronger tracking ability and robustness when the parameters are uncertain in a large range or with multiple jumps.

The block diagram of the proposed system is shown in Fig. 1.



Figure 1 The block diagram of the proposed system

## 2. Local controller design based on ADRC

Consider a plant with small uncertainties

$$\begin{cases} x_1 = x_2 \\ \dot{x}_2 = x_3 + b0u(t) \\ \dot{x}_3 = \dot{f}(x_1, x_2, \omega(t), t) = h(t) \\ y = x_1 \end{cases}$$
(1)

where f(.) represents unknown unmodeled dynamics or 'total disturbance', u(t) control input,  $\omega(t)$  external disturbance, b0 control gain, y(t) system output.

First, we need to design the extended state observer (ESO)

$$\begin{cases} \varepsilon_{1} = z_{1} - y \\ \dot{z}_{1} = z_{2} - \beta_{01}\varepsilon_{1} \\ \dot{z}_{2} = z_{3} - \beta_{02}\varepsilon_{1} + b_{0}u \\ \dot{z}_{3} = -\beta_{03}\varepsilon_{1} \end{cases}$$
(2)

Its discrete-time version has the following form

$$\begin{cases} \varepsilon_{1} = z_{1}(k) - y(k) \\ z_{1}(k+1) = z_{1}(k) + h(z_{2}(k) - \beta_{01}\varepsilon_{1}) \\ z_{2}(k+1) = z_{2}(k) + h(z_{3}(k) - \beta_{02}\varepsilon_{1} + b_{0}u) \\ z_{3}(k+1) = z_{3}(k) - h\beta_{03}\varepsilon_{1} \end{cases}$$
(3)

Where  $\mathcal{E}_1$  is observation error,  $\beta_{01}$ ,  $\beta_{02}$ , and  $\beta_{03}$  are the gains of the observer,  $z_1$ ,  $z_2$ , and  $z_3$  are the estimations of y,  $\dot{y}$ , and f(.), respectively. Second, we need to design the control law based on the ESO

$$u_0 = k_p e_1 - k_d z_2 \tag{4}$$

where  $e_1 = v_1 - z_1$ , v is the reference input of the system,  $k_p$  and  $k_d$  are adjustable parameters. Based on (4), the local control law is given by

$$u = \frac{u_0 - z_3}{b_0}$$
(5)

## 3. Multiple model ADRC

Consider time-varying uncertain plant

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 + b(t)u(t) \\ \dot{x}_3 = \dot{f}(x_1, x_2, d(t), \omega(t), t) = h(t) \\ y = x_1 \end{cases}$$
(6)

Then we need to design multiple model ADRC, which uses multiple models (model set) to cover the uncertainties of b(t), for each (local) model, a local controller (ADRC controller) is designed off-line, and the global control law is weighted sum (on-line) of all the local controllers. That is

$$u(t) = \sum p_i(t)u_i(t)$$

where  $u_i(t)$  is calculated according to (5).

The weighting algorithm is described as follows (Suppose the number of local controllers is N)



where  $e_i$  is the output error of each local model, i.e.,  $e_i(k) = y(k) - y_i(k)$ .

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The calculation of weights values is carried out at each time instant k, and through a ZOH (Zero Order Hold),  $p_i(t)$  can be obtained from  $p_i(k)$ .

## 4. Simulation results

To verify the effectiveness of the proposed multiple model ADRC framework, many simulations have been conducted with  $\omega(t) = 0.1 \sin t$  in equation (6), based on MATLAB/ Simulink.

Suppose the plant will change from Model 1 (corresponding to b(t) = 0.6 in equation (6)) to Model 2 (corresponding to b(t) = 1 in equation (6)) at time 30s, and finally change to Model 3 (corresponding to b(t) = 1.5 in equation (6)) at time 60s. The model set include 4 models, i.e., Model1, Model 2, Model 3, and Model 4 (corresponding to b(t) = 2 in equation (6)). The step responses of multiple model ADRC system and single ADRC system are shown in Fig. 2.



Figure 2. Step response comparison between multiple model ADRC and single ADRC

Weight values of multiple model ADRC are shown in Figure 3.



### Figure 3. Weight values of multiple model ADRC

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#### 5. Conclusion and future work

This paper presented a multiple model ADRC framework for uncertain dynamic time-varying system. Simulation results verified the effectiveness of the proposed methods including weighting algorithm and local controller scheme. In the future research work, we will focus on model set selection for time-varying system with more than one variable.

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