

# Acoustic Impedance Measurement through the Modelling of Ultrasonic Wave Transmission\*

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## Abstract

In food industry, shortage of workers is a serious problem. Automation of food handling is a critical nowadays. To alleviate the damage during food picking by robotic hand, we propose non-contact acoustic impedance estimation with ultrasonic wave. We have the assumption of the correlation between hardness and acoustic impedance, and, built up ultrasonic transmission model considering attenuation by reflection and absorption, then, made an experiment to estimate the impedance. As the result, we succeeded in detecting acoustic impedance without contact.

*Keywords:* Acoustic Impedance, Ultrasonic Transmission, Hardness, Reflection, Absorption

## 1. Introduction

In Japan, the demand for prepared food has been increasing recently. The survey<sup>1</sup> says the expenditure for prepared food has been increasing since 2004 for all human ages. The rate of the increasing expenditure is 283.9[%] for the internet shopping, 28.0[%] for convenience stores. In domestic food industry, a few millions of lunch boxes are produced and consumed a day. Food packing stage requires human resources generally<sup>2</sup>. For alleviation of labor burden by human, automation of food packing is effective. however, picking food by solid hand causes of damages<sup>3</sup>. Hardness of food should be measured without contact, then, picking force should be optimized for the automation.

We assumed relation between acoustic impedance and hardness, then estimated the impedance through analysis of ultrasonic reflection. The impedance shows the

difficulty of acoustic transmission in a material, and can be estimated from reflection coefficient<sup>4</sup>. Confirming the relation of acoustic impedance and hardness realizes non-contact hardness measurement system.

## 2. Related Works

J. Machando et.al introduced ultrasonic wave into Non-Destructive Analysis for evaluation of wood strength<sup>5</sup>.

B. Cho and J. Irudayaraj measured cheese depth with ultrasonic wave and accuracy of the measurement was 99.98[%]. Moreover, they recorded more than 0.9 of correlation between cheese mechanic property and result of sonic velocity measurement<sup>6</sup>. S. Srivastava et.al introduced ultrasonic measurement for hardness estimation of tomatoes, and succeeded in detecting hardness changes during the growth<sup>7</sup>.

However, to our knowledge, no report has been found as measuring hardness unevenness. The unevenness should be considered because uniqueness of heating food is not guaranteed under cooking.

### 3. Proposed Method

#### 3.1. Outline

Fig.1 shows the process of our acoustic impedance estimation. In the system, receiver and transmitter are attached parallel, which enables observation of reflected wave. Here, the sample is assumed as a composition of multiple media with different acoustic impedances  $\zeta_k$  [Pa s/m] ( $k$  shows index of media), therefore, reflection occurs at  $z_k$  where  $\zeta_k$  changes. After the transmission, we calculate amplitudes of sound pressure  $P_{r,k}(z_0)$  in the reflected wave by maxima detection (Here,  $z_0$  denotes observation point.), to easily evaluate the attenuation between reflections. Then, reflection coefficient  $r_k$ , and attenuation coefficient  $\alpha_k$ , finally  $\zeta_k$  is estimated from the result of  $r_k$  estimation. Evaluation of the correlation between  $\zeta_k$  and hardness is our future work.

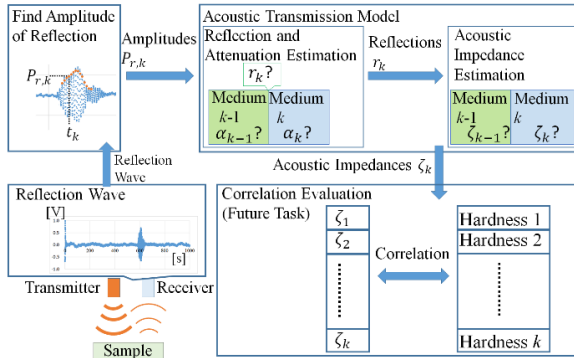


Fig. 1 Process of Acoustic Impedance Calculation

#### 3.2. Acoustic Transmission Model

##### 3.2.1. Condition at media boundary

In view of acoustic pressure sequence at media boundary  $z=z_k$ , the relation among incident wave (Amplitude:  $P_{i,k}(z_k)$  [Pa]), reflected wave (Amplitude:  $P_{r,k}(z_k)$  [Pa]), transmitted wave (Amplitude:  $P_{t,k}(z_k)$  [Pa]) is shown as Eq. (1). Here, wave frequency is  $\omega$  [rad/s], sonic velocity is  $c_{k-1}$  [m/s],  $c_k$  [m/s] each other in media  $k-1$  and  $k$ .

$$P_{i,k}(z_k)e^{j\omega(t - \frac{z-z_k}{c_{k-1}})} + P_{r,k}(z_k)e^{j\omega(t + \frac{z-z_k}{c_{k-1}})} = P_{t,k}(z_k)e^{j\omega(t - \frac{z-z_k}{c_k})}. \quad (1)$$

##### 3.2.2. Attenuation by Reflection

As Fig. 2 shows, samples with uneven hardness is assumed to have multiple media of different  $\zeta_k$ , thus, reflection occurs at a boundary where difference of  $\zeta_k$  is observed. As Eq. (2) shows<sup>4</sup>, small difference of  $\zeta_k$  causes of the attenuation by reflection. The acoustic pressure is attenuated to  $r_k$  times of  $P_{i,k}(z_k)$  after the reflection.

$$r_k = \frac{\zeta_{k-1} - \zeta_k}{\zeta_{k-1} + \zeta_k} = \frac{P_{r,k}(z_k)}{P_{i,k}(z_k)}. \quad (2)$$

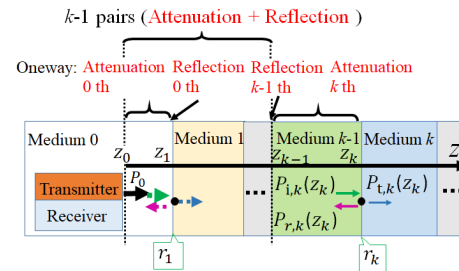


Fig. 2 Acoustic Transmission and Reflection in Media

The reflection coefficient of  $k-1$  from  $k$  should be considered by changing the numerator of Eq. (2) into  $\zeta_k - \zeta_{k-1}$ . As the result, the coefficient is expressed in  $-r_k$ .

##### 3.2.3. Attenuation by Absorption

The attenuation is caused by viscosity of media and heat transmission in the event of longitudinal wave penetration<sup>8</sup>, whose magnitude is equal to attenuation coefficient  $\alpha$  [Np/m]<sup>8-9</sup>. Supposing  $P_0$  [Pa] is original acoustic pressure, and, the wave moves  $z$  [m], the pressure  $P$  diminishes as Eq. (3) shows.

$$P = P_0 e^{-\alpha z}. \quad (3)$$

We disregard wave diffusion<sup>10</sup> because the angle of wave expansion is enough small as our instrument.

##### 3.2.4. Integration of Reflection and Absorption

As Fig. 2 shows, when  $P_0$  [Pa] is emitted at  $z=z_0$  [m] of media 0, penetration and reflection occurs at each boundary  $z_k$ . During the transmission in media  $k$ , absorption attenuation occurs in  $\alpha_k$  [Np/m]. By the arrival at media  $k$  ( $z=z_k$ ), totally,  $k$  absorptions and  $k-1$  reflections occur. The state of incident wave (Amplitude:  $P_{i,k}(z_k)$ ) into media  $k$  is expressed by Eq. (4).

$$P_{r,k}(z_k)e^{j\omega t} = \{P_0 e^{-\alpha_{k-1}(z_k - z_{k-1})} \prod_{k'=1}^{k-1} e^{-\alpha_{k'-1}(z_{k'} - z_{k'-1})} (r_{k'} + 1)\} e^{j\omega t} \quad (4)$$

Immediately after the reflection, the acoustic pressure of reflected wave is  $P_{r,k}(z_k)$  [Pa]. By the arrival at the receiver ( $z=z_0$ ), totally,  $k$  absorptions and  $k-1$  reflections occur again.  $P_{r,k}(z_k)$  is diminished to  $P_{r,k}(z_0)$ . This state is expressed in Eq. (5). All signs of all  $r_k$  s are opposite to Eq. (4). Eq. (4)(5) is summarized to Eq. (6)(7).

$$P_{r,k}(z_0)e^{j\omega t} = \{P_{r,k}(z_k)e^{-\alpha_{k-1}(z_k - z_{k-1})} \prod_{k'=1}^{k-1} e^{-\alpha_{k'-1}(z_{k'} - z_{k'-1})} (-r_{k'} + 1)\} e^{j\omega t}. \quad (5)$$

$$P_{r,k}(z_0)e^{j\omega t} = \{r_k P_0 e^{-2\alpha_{k-1}(z_k - z_{k-1})} \prod_{k'=1}^{k-1} (1 - r_{k'}^2) e^{-2\alpha_{k'-1}(z_{k'} - z_{k'-1})}\} e^{j\omega t}. \quad (6)$$

$$\frac{P_{r,k}(z_0)}{P_{r,k-1}(z_0)} = (1 - r_{k-1}^2) \frac{r_k}{r_{k-1}} e^{-2\alpha_{k-1}(z_k - z_{k-1})}. \quad (7)$$

### 3.3. Solution

#### 3.3.1. Amplitude of Reflected Wave $P_{r,k}(z_0)$

To obtain the Amplitudes, we introduce Gradient method. From intersections with  $z$ -axis, the reflection climbs up then descends. In our condition, Large gradient (more than  $T_\varepsilon$ ) should be observed at the intersections where maxima searching starts. the derivative of reflection should be  $T_\varepsilon$  or less, and the reflection should be  $T_p$  or more where candidates of  $P_{r,k}(z_0)$  locate.

#### 3.3.2. Coefficient $\alpha_k$ and $r_k$

Though all  $P_{r,k}(z_0)$  are solved, unknown  $\alpha_{k-1}$ ,  $r_{k-1}$ ,  $r_k$  are included in one equation (Eq. (7)). We introduce Monte Carlo Method to find the solutions. The relationship between  $\alpha_{k-1}$ ,  $r_{k-1}$ ,  $r_k$  is expressed in Eq. (8)

From the Natural Logarithm of Eq. (7).

$$\alpha_{k-1} = \frac{1}{2(z_k - z_{k-1})} \ln \left| (1 - r_{k-1}^2) \frac{r_k}{r_{k-1}} \frac{P_{r,k-1}(z_0)}{P_{r,k}(z_0)} \right|. \quad (8)$$

Here,  $\alpha_{k-1} \geq 0$  should be satisfied.  $\alpha_{k-1} < 0$  means amplification, which leads a contradiction against attenuation. We treat only positive for  $r_k$ , then, necessary requirement of  $r_k$  is stated in Eq. (9) for  $\alpha_{k-1} \geq 0$ .

$$\frac{P_{r,k}(z_0)}{P_{r,k-1}(z_0)} \frac{r_{k-1}}{(1 - r_{k-1}^2)} \leq r_k < 1. \quad (9)$$

According to Eq. (9), left side of the inequality exceeds one depending on the combination of  $P_{r,k}(z_0)$  and  $r_k$ . To solve the contradiction, Eq. (10) should be satisfied.

$$\frac{P_{r,k+1}(z_0)}{P_{r,k}(z_0)} \frac{r_k}{(1 - r_k^2)} < 1. \quad (10)$$

As the result of Eq. (10), (11) is obtained as a solution.

$$0 < r_k < \frac{-P_{r,k+1}(z_0) + \sqrt{P_{r,k+1}(z_0)^2 + 4P_{r,k}(z_0)^2}}{2P_{r,k}(z_0)}. \quad (11)$$

$r_k=0, 1$  should be excluded so that antilogarithm of Eq. (8) is one or more. The upper limit in Eq. (11) is considered so as not to make contradiction for  $r_{k+1}$ . Finally, necessary requirement of  $r_k$  is defined as the overlapped range of Eq. (9) and (11).

## 4. Experiment

### 4.1. Method

In this experiment, we confirmed possibility of finding  $\zeta_k$  through reflected wave analysis. In the environment (Fig. 3), transmitter and receiver were attached parallel. Sponges were prepared to remove echoes. The distance between transmitter and sample surface is  $L_{ss}$ [mm]. As the setting,  $L_{ss}=100$ [mm], frequency  $\omega=800\pi$ [krad], sample width  $W_s = 100$ [mm], depth  $H_s=10$ [mm] (We selected ABS). We emitted ultrasonic wave and analyzed reflected wave by Monte Carlo Method generating random values of  $r_k$  within the range Eq. (9), (11), and obtained  $\alpha_{k-1}$  following Eq. (8). Then we averaged  $r_k$  and  $\alpha_{k-1}$  for the same  $k$ . Acoustic impedance  $\zeta_0$  is known to  $4.1 \times 10^2$ [Pa s/m]<sup>11</sup>, and, true  $\zeta_k$  of ABS is  $2.4 \times 10^6$ [Pa s/m]<sup>12</sup>.  $N_k$ ,  $N_{rd}$  (=500) is the number of amplitudes,  $r_k$  subsets (Each subset includes  $r_1, r_2, \dots, r_{N_k-1}$ ) each other. Because  $H_s$  is sufficiently thin, we ignored the difference of sonic velocity  $c$  in each layer. We calculated distance from  $z_0$  to  $z_k$  by Eq. (12).  $t_k$  is the time to receive  $P_{r,k}(z_0)$ .

$$z_k - z_0 = \frac{1}{2} c (t_k - t_0). \quad (12)$$

### 4.2. Result and Discussion

Fig. 4 shows the results of measurement. (a) shows  $v_r(z_0)$ , that is correspondent of reflected wave in voltage, where amplitude is shown in orange. (b)(c)(d) shows average of  $r_k$ ,  $\alpha_{k-1}$ ,  $\zeta_k$ , each other. In Fig. 5, the same averages ((a) $r_k$ , (b) $\alpha_{k-1}$ , (c) $\zeta_k$ ) are expressed in blue bar and the standard deviations are red.  $\zeta_k = 2.5 \times 10^6$ [Pa s/m] (at  $z_k=107.5$ [mm]) was the closest from the truth.

According to Fig. 5, 6,  $v_r(z)$ ,  $r_k$  and  $\zeta_k$  rose, while  $\alpha_{k-1}$  decreased in deeper layer of the sample. From the point,  $r_k$  also naturally increases as  $v_r(z)$  rises. However,  $\alpha_{k-1}$  is particularly large near the surface. The deviation of  $\alpha_{k-1}$  is outstanding in Fig. 5(b). We estimated  $r_1$  and  $\alpha_1$  without  $P_0$ , then, the accuracy of  $\zeta_1$  cannot be guaranteed.  $r_k$  and  $\zeta_k$  may change if truly observed value of  $P_0$  is used for the solution. Eq. (2) can be transformed into Eq. (13)

$$\zeta_k = \frac{1+r_k}{1-r_k} \zeta_{k-1} . \quad (13)$$

Thus,  $\zeta_k$  dramatically changes if  $r_k$  is near 1.0. Enhancing the accuracy of  $r_k$  is essential for higher accuracy of  $\zeta_k$ .

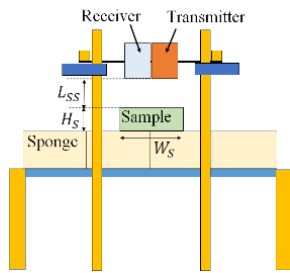


Fig. 3 Environment for the Experiment

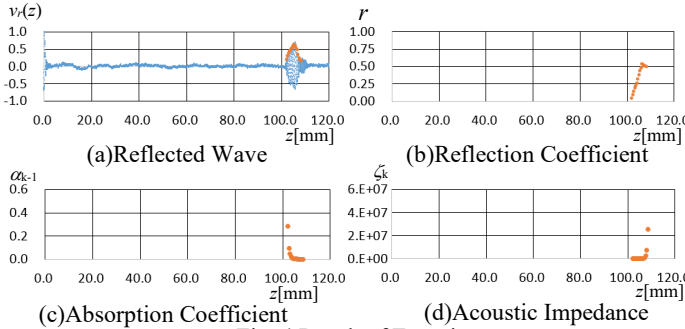


Fig. 4 Result of Experiment

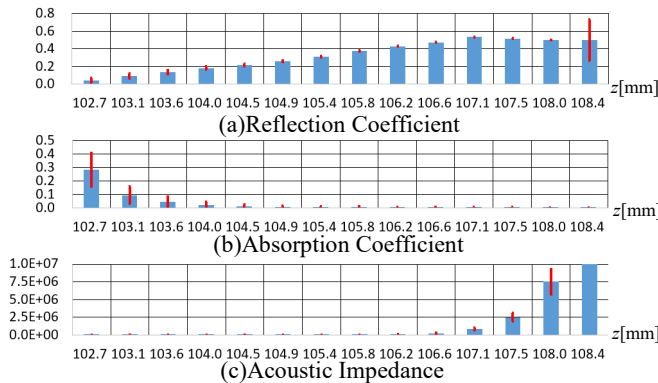


Fig. 5 Averages(Blue) and Standard Deviation (Red)

## 5. Conclusion

In this research, we proposed acoustic impedance measurement method with ultrasonic wave. As the result, reasonable solutions of reflection coefficients were obtained in deeper layer. then accuracy of acoustic impedance estimation was 0.1[Pa s/m] error for  $2.4 \times 10^6$ [Pa s/m] of true ABS acoustic impedance. However, as near the surface, the solution may change according to setting of emitted acoustic pressure. Enhancing reliability in reflection coefficient estimation is necessary for better accuracy of acoustic impedance.

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