# The Spherical Robot Transfer Problem with Minimal Total Kinetic Energy*

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**Abstract**

Previous spherical mobile robots were driven by two rollers with a fixed rotational axis, which restricts the angular velocity vector of the sphere to two dimensions. Three-dimensional freedom is expected to improve the rotational diversity of the sphere. This study proposes a spherical mobile robot with a variable roller-rotational axis that allows three-dimensional freedom of movement. Furthermore, the kinetic energy of transporting the sphere by the rollers is minimized by an optimization procedure.

*Keywords*: Angular velocity vector of the sphere, Angle of sphere rotational axis, Total kinetic energy

## 1. Introduction

Many types of robot, such as omnidirectional mobile robots and sphere-transported robots, are based on spherical motions. Therefore, various roller arrangements have been proposed for mobile robot applications.

**Table 1** shows the usage statuses of single spherical robots operated by different mechanisms, and the dimensions of the existence spaces of their angular velocity vectors. **Figure 1** shows the numbers of rollers $N_w$ arranged per wheel in each mechanism, and their contact types.

In the ACROBAT-S [1] mechanism with $N_w = 2$, the caster of each sphere is driven by two roller drives (see **Figure 1** (a)). The wheel chair mechanism [2] has three spheres (**Figure 1** (b)). The rollers are arranged on the equator, generate an angular velocity vector on the horizontal plane, and can move in all directions. The angular velocity vector of the sphere has two-dimensional freedom. This situation is theoretically considered in [3]. In the ball dribbling mechanism [4], the rollers are arranged in the upper hemisphere, where they hold the balls by friction (See **Figure 1** (c)).

Among the three-roller cases ($N_w = 3$), OWMPs [5] deployed in highway maintenance move the spheres within three constrained rollers (See **Figure 1** (d)).The rollers are arranged on the equator parallel to the horizontal plane, and the sphere can be rotated in all directions by generating an angular velocity vector on the

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plane (note that the existence space of the angular velocity vector is two-dimensional).

A ball balanced robot [6] has three unconstrained rollers (omni-rollers) arranged in a regular triangular configuration (See **Figure 1** (e)). CPU-Ball Bot [7] has four unconstrained rollers fixed in a square configuration (See **Figure 1** (f)). Such three-dimensional freedom of

**Table 1** Existence space of angular velocity vector for sphere mobile robot

| Mechanism | statuses | Rotational Dimension |
|---|---|---|
| ACROBAT-S[1] | Caster | 2 |
| Wheel chair [2] | Wheel | 2 |
| Ball dribbling mechanism [4] | Sphere conveyance | 2 |
| OWMPs [5] | Wheel | 2 |
| Ball Balanced robot [6] | Wheel | 3 |
| CPU-Ball Bot [7] | Wheel | 3 |

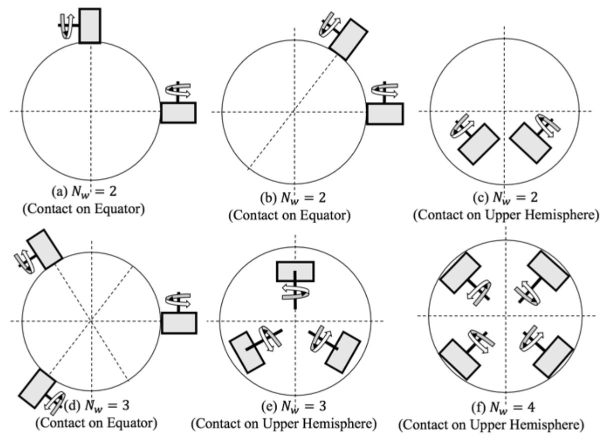



**Figure 1** Type of roller arrangement for sphere mobile robot

the angular velocity vector of a sphere will improve the motion diversity of the sphere. However, a mechanism adapting two constrained rollers is suitable in a spherical object conveyance. It is desired to transport with high kinetic energy efficiency.

In this study, we optimize the total kinetic energy of two rollers. Section 2 calculates the angle of the rotational axis of the sphere, and theoretical formula of minimizes the sum of the kinetic energies of the two rollers. Section 3 presents a simulation of total kinetic energy, and Section 4 summarizes the results and suggests ideas for future work.

## 2. Total kinetic energy

### 2. 1 Angular velocity vector of the sphere

The center $\boldsymbol{O}$ of a sphere with radius $r$ is fixed as the origin of the coordinate system $\Sigma - xyz$ . The $i^{\text{th}}$ constraint roller ($i$ = 1 or 2) is in point contact with the sphere at a position vector $\boldsymbol{P_i}$ and is arranged such that the center of mass of the roller $\boldsymbol{P_i}$ and $\boldsymbol{O}$ are on the same line. $\boldsymbol{\omega}$ denotes the angular velocity vector of the sphere. $\boldsymbol{\eta_i}$ denotes the unit vector along the rotational axis of constraint roller. sphere direction $\varphi$ ($0° \le \varphi < 360°$) is the angle from $x$-axis.

Now, given the sphere mobile velocity $\boldsymbol{V}$ (the center velocity of sphere)

$$\boldsymbol{V} = \|\boldsymbol{V}\|[\cos\varphi \quad \sin\varphi \quad 0]^T \tag{1}$$

$\boldsymbol{\acute{\omega}}$ which is perpendicular to $\boldsymbol{V}$ is determined as follow.

$$\boldsymbol{\acute{\omega}} = \frac{\|\boldsymbol{V}\|}{r}[-\sin\varphi \quad \cos\varphi \quad 0]^T \tag{2}$$

And. $\boldsymbol{\acute{\omega}}$ is orthogonal projection of $\boldsymbol{\omega}$ with respect to $xy-$ plane.The angle of sphere rotational axis $\rho$ ($-90° \le \rho \le 90°$) is the angle between $\boldsymbol{\omega}$ and $\boldsymbol{\acute{\omega}}$ . Therefore, $\boldsymbol{\omega} = [\omega_x, \omega_y, \omega_z]^T$ has one-dimensional of freedom with parameter $\rho$ ($\omega_z = \|\boldsymbol{V}\| \tan\rho / r$).

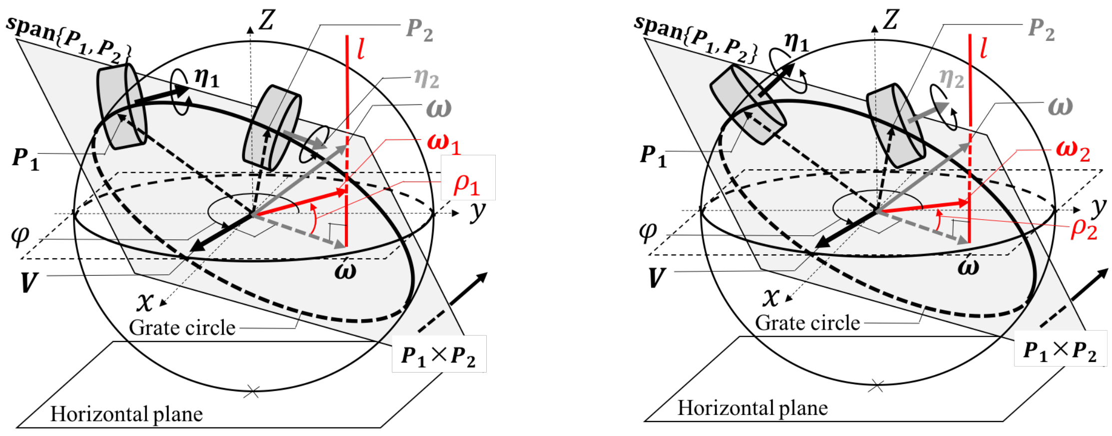
$$\boldsymbol{\omega} = \frac{\|\boldsymbol{V}\|}{r}[-\sin\varphi \quad \cos\varphi \quad \tan\rho]^T \tag{3}$$

### 2. 2 Kinetic energy of the two rollers

Consider two rollers with radius $R$, moment $I$, and roller's angular velocity $\omega_i$ at $\boldsymbol{P_i}$. The summed kinetic energy of the rollers is given by Eqs.(4).

$$\begin{aligned} E &= I(\omega_1^2 + \omega_2^2) \\ &= \frac{I}{R^2}(\|\boldsymbol{\omega} \times \boldsymbol{P}_1\|^2 + \|\boldsymbol{\omega} \times \boldsymbol{P}_2\|^2) \end{aligned} \tag{4}$$

Using the angular velocity vector of a sphere, we now derive the kinetic energy of **Type-I** (with both rotational axes fixed on the same plane) and **Type-II** (with variable rotational axes) configurations(indicated in **Figure 2** (a)(b)).




(a) (b)

**Figure. 2** Existence of the sphere angular velocity vector with respect to sphere movile velocity.(a) $\boldsymbol{\omega_1}$ is determined ($\boldsymbol{\omega_1}$ is lay on intersection $\mathbf{span}\{\boldsymbol{P_1},\boldsymbol{P_2}\}$ and line $\boldsymbol{l}$ . (b) $\boldsymbol{\omega_2}$ is determined as minimizes the summed kinetic energies of the two rollers.

***(A) Case of Type- I***

As shown in **Figure 2** (a), the end point of $\boldsymbol{\omega_1}$ can be determined as the intersection of $l$ (the line perpendicular to the horizontal plane passing through end point $\acute{\boldsymbol{\omega}}$) and $\mathbf{span}\{\boldsymbol{P_1},\boldsymbol{P_2}\}$. Therefore, The angle of sphere rotational axis is determined as $\rho = \rho_1$.

$$\rho_1 = \tan^{-1}\left[\frac{(\boldsymbol{P_1}\times\boldsymbol{P_2})_x \sin\varphi - (\boldsymbol{P_1}\times\boldsymbol{P_2})_y \cos\varphi}{(\boldsymbol{P_1}\times\boldsymbol{P_2})_z}\right] \tag{5}$$

Substituting Eqs.(5) into Eqs.(3), we obtain $\boldsymbol{\omega} = \boldsymbol{\omega_1}$ as Eqs.(6).

$$\boldsymbol{\omega_1} = \tag{6}$$

$$\frac{\|\boldsymbol{V}\|}{r}\left[-\sin\varphi\,,\cos\varphi\,,\frac{(\boldsymbol{P_1}\times\boldsymbol{P_2})_x \sin\varphi - (\boldsymbol{P_1}\times\boldsymbol{P_2})_y \cos\varphi}{(\boldsymbol{P_1}\times\boldsymbol{P_2})_z}\right]^T$$

Substituting Eqs.(6) into Eqs.(4), we obtain $E = E_1$ as Eqs.(7).

$$E_1 = \frac{I}{R^2}(\|\boldsymbol{\omega_1}\times\boldsymbol{P}_1\|^2 + \|\boldsymbol{\omega_1}\times\boldsymbol{P}_2\|^2) \tag{7}$$

***(B) Case of Type-II***

As shown in **Figure 2** (b), we determine $\rho = \rho_1$, $\boldsymbol{\omega} = \boldsymbol{\omega_2}$ that minimizes the summed kinetic energies of the two rollers, and calculate the minimum energy ($E = E_2$).

To this end, we first express $\boldsymbol{\omega}$ as the sum of $\acute{\boldsymbol{\omega}}$ and $\omega_z\boldsymbol{e_3}$.

$$\boldsymbol{\omega} = \acute{\boldsymbol{\omega}} + \omega_z\boldsymbol{e_3} \tag{8}$$

where

$$\boldsymbol{e_3} = [0, 0, 1]^T,\ \ \acute{\boldsymbol{\omega}} = [\omega_x, \omega_x\,, 0]^T \tag{9}$$

And.

$$\begin{aligned}\boldsymbol{\omega}\times\boldsymbol{P}_i &= (\acute{\boldsymbol{\omega}} + \omega_z\boldsymbol{e_3})\times\boldsymbol{P}_i \\ &= \acute{\boldsymbol{\omega}}\times\boldsymbol{P}_i + \omega_z(\boldsymbol{e_3}\times\boldsymbol{P}_i)\end{aligned} \tag{10}$$

$$\|\boldsymbol{\omega}\times\boldsymbol{P}_i\|^2 \tag{11}$$

$$= \langle\acute{\boldsymbol{\omega}}\times\boldsymbol{P}_i + \omega_z(\boldsymbol{e_3}\times\boldsymbol{P}_i)\,,\acute{\boldsymbol{\omega}}\times\boldsymbol{P}_i + \omega_z(\boldsymbol{e_3}\times\boldsymbol{P}_i)\rangle$$

$$= \omega_z^2\|\boldsymbol{e_3}\times\boldsymbol{P}_i\|^2 + 2\omega_z\langle\acute{\boldsymbol{\omega}}\times\boldsymbol{P}_i, \boldsymbol{e_3}\times\boldsymbol{P}_i\rangle + \|\acute{\boldsymbol{\omega}}\times\boldsymbol{P}_i\|^2$$

Using, Eqs.(11), it is represented as quadratic function with respect to $\omega_z$ as Eqs.(12)

$$\|\boldsymbol{\omega}\times\boldsymbol{P}_i\|^2 + \|\boldsymbol{\omega}\times\boldsymbol{P}_i\|^2 = \tag{12}$$

$$\begin{aligned}&(\|\boldsymbol{e_3}\times\boldsymbol{P}_1\|^2 + \|\boldsymbol{e_3}\times\boldsymbol{P}_2\|^2)\omega_z^2 \\ &+ 2(\langle\acute{\boldsymbol{\omega}}\times\boldsymbol{P}_1, \boldsymbol{e_3}\times\boldsymbol{P}_1\rangle + \langle\acute{\boldsymbol{\omega}}\times\boldsymbol{P}_2, \boldsymbol{e_3}\times\boldsymbol{P}_2\rangle)\omega_z \\ &+ \|\acute{\boldsymbol{\omega}}\times\boldsymbol{P}_1\|^2 + \|\acute{\boldsymbol{\omega}}\times\boldsymbol{P}_2\|^2\end{aligned}$$

Hence, focusing the coefficients of Eqs.(12), $E$ takes minimal value $E_2$ as Eqs.(13).

$$E_2 = (\|\acute{\boldsymbol{\omega}} \times \boldsymbol{P}_1\|^2 + \|\acute{\boldsymbol{\omega}} \times \boldsymbol{P}_2\|^2)\,\frac{I}{R^2} \qquad (13)$$

$$-\frac{(\langle \acute{\boldsymbol{\omega}} \times \boldsymbol{P}_1, \boldsymbol{e}_3 \times \boldsymbol{P}_1\rangle + \langle \acute{\boldsymbol{\omega}} \times \boldsymbol{P}_2, \boldsymbol{e}_3 \times \boldsymbol{P}_2\rangle)^2}{\|\boldsymbol{e}_3 \times \boldsymbol{P}_1\|^2 + \|\boldsymbol{e}_3 \times \boldsymbol{P}_2\|^2}\frac{I}{R^2}$$

where

$$\omega_z = -\frac{\langle \acute{\boldsymbol{\omega}} \times \boldsymbol{P}_1, \boldsymbol{e}_3 \times \boldsymbol{P}_1\rangle + \langle \acute{\boldsymbol{\omega}} \times \boldsymbol{P}_2, \boldsymbol{e}_3 \times \boldsymbol{P}_2\rangle}{\|\boldsymbol{e}_3 \times \boldsymbol{P}_1\|^2 + \|\boldsymbol{e}_3 \times \boldsymbol{P}_2\|^2} \qquad (14)$$

$$\rho_2 = \tan^{-1}\left[-\frac{\langle \acute{\boldsymbol{\omega}} \times \boldsymbol{P}_1, \boldsymbol{e}_3 \times \boldsymbol{P}_1\rangle + \langle \acute{\boldsymbol{\omega}} \times \boldsymbol{P}_2, \boldsymbol{e}_3 \times \boldsymbol{P}_2\rangle}{\|\boldsymbol{e}_3 \times \boldsymbol{P}_1\|^2 + \|\boldsymbol{e}_3 \times \boldsymbol{P}_2\|^2}\right] \qquad (15)$$

Substituting Eqs.(15) into Eqs.(3), $\boldsymbol{\omega}_2$ as Eqs.(16).

$$\boldsymbol{\omega}_2 = \qquad (16)$$

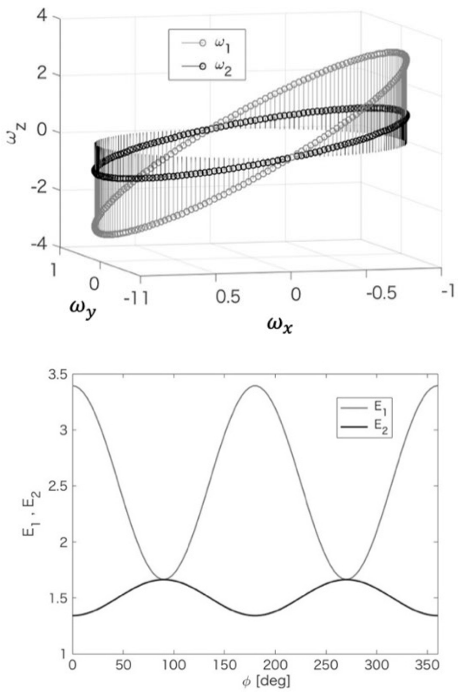
$$\frac{\|\boldsymbol{V}\|}{r}\left[-\sin\varphi, \cos\varphi, -\frac{\langle \acute{\boldsymbol{\omega}} \times \boldsymbol{P}_1, \boldsymbol{e}_3 \times \boldsymbol{P}_1\rangle + \langle \acute{\boldsymbol{\omega}} \times \boldsymbol{P}_2, \boldsymbol{e}_3 \times \boldsymbol{P}_2\rangle}{\|\boldsymbol{e}_3 \times \boldsymbol{P}_1\|^2 + \|\boldsymbol{e}_3 \times \boldsymbol{P}_2\|^2}\right]^T$$

## 3. Simulation

This section presents the simulation results $\boldsymbol{\omega}_1$ (See Eqs.(6)), $\boldsymbol{\omega}_2$ (See Eqs.(16)), $E_1$ (See Eqs.(7)) and $E_2$(See Eqs.(13)) with parameter $\varphi$ ($0° \le \varphi < 360°$) in the given sphere mobile speed: $\|\boldsymbol{V}\| = 1$ [m/s]. The conditions are as follows: $I = 1$, $R = 00.1$[m], $\|\boldsymbol{V}\| = 1$ [m/s], $r = 0.1$[m], $\theta_{1,1} = 215°$, $\theta_{1,2} = 60°$, $\theta_{2,1} = 325°$, $\theta_{2,2} = 60$ °.

### 3. 1 Trajectory of the end point of the angler velocity vector and Totally kinetic energy

As shown in **Figure 3** (a), the ellipsoid trajectory of $\boldsymbol{\omega}_2$ lies nearer to the $xy$-plane than that of $\boldsymbol{\omega}_1$, and both trajectories cross a common line parallel to the $y$-axis.




(a)

(b)

**Figure 3** Simulation result. (a) Trajectory of angular velocity vector of sphere. (b) Totally kinetic energy

As shown in **Figure 3** (b), $E_1$ is minimized at sphere direction angles $\varphi =$ 90° and 270°, and maximized at $\varphi = 0°$ and 180°. Meanwhile, $E_2$ is minimized at $\varphi = 0°$ and 180°, and maximized at $\varphi =$ 90° and 270°. And. $E_1$ and $E_2$ are same value at $\varphi =$ 90° and 270°.

## 4. Conclusion

We optimized the total kinetic energy of the two rollers' movement and calculated the angle of the rotational axis of the sphere, and theoretical formula and we present a simulation of total kinetic energy.

In future work, we plan to evaluate the kinetic energy of the roller arrangement at an arbitrary contact point on the upper hemisphere as an evaluation function.

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