Robust Control of Nonholonomic Wheeled Mobile Robot with Hybrid Controller Approach

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Abstract

This study proposes a control strategy to solve the nonholonomic mobile robot trajectory tracking problem on the basis of Cerebellar Model Articulation Controller (CMAC). Mobile robot needs two controllers to provide the control demands. One controller is mathematically described in terms of robot's kinematics; while the other is given by dynamics equations. To implement the speed control to track the reference trajectory, we apply the Lyapunov theory to obtain the virtual speed control command. On the other hand, we use cerebellum controller to approach to the non-linearity and uncertainty of the dynamics model. Furthermore, we combine the speed error to construct a torque controller, which can online real-time compensate the influences made by uncertainties. The observer is used to estimate the external disturbance, so that the controller has more ability to reject external disturbance. The convergence and stability of the system is determined by the Lyapunov stability criterion after linearizing the system. Our simulation is performed in Matlab/Simulink environment, and the results verify the effectiveness of the controller algorithm.

Keywords: nonholonomic mobile robot; Cerebellar Model Articulation Controller; Lyapunov stability criterion; disturbance observer.

1. Introduction

Many literatures which study nonholonomic mobile robots are usually focus on wheeled mobile robots (WMR). In the theoretical research of WMR's motion control, in general, only pure rolling condition is considered. In other words, it is assumed that no slip condition (including lateral and longitudinal sliding) occurs. This ideal constraint is essentially a kind of nonholonomic constraint, therefore, WMR is a typical case of the nonholonomic system. In this paper, the control of WMR is studied. According to different control objectives, the control problems of nonholonomic systems can be divided into three categories[1] [2]: position stabilization, trajectory tracking and path following. However, in the content of research report[3], the emphasis the Brockett's theorem proves that there is no asymptotic stable fixed point where pure state feedback does not exist [1]. In the report, the nonholonomic mobile robot control is divided into two categories, one for fixed point asymptotic stabilization relying on highly nonlinear techniques, and the other based on more classical linear and nonlinear techniques for asymptotic stabilization of feasible and persistently exciting trajectories.

Position stabilization control refers to the design of a feedback controller, which can actuate and stabilize the system from a given initial state to an arbitrary target state. It is also called the posture stabilization, posture regulation, and set-point regulation in some references. The position, state, posture, set-point, and so on, describe position and attitude of a mobile robot by a set of generalized coordinates. In research, the origin point usually be set as the target state.

Trajectory tracking control refers to a controller design which can command the robot arriving and tracking a specific trajectory from a given initial state in an inertial coordinate system. On the other hand, path following control means to control the robot to arrive and keep following a specific geometric path from a given initial state in an inertial coordinate system. Frankly speaking, trajectory tracking control demands the robot to track a specific time-varying trajectory, however, path following control only follows a designed path regardless of the arriving time of the specific position. Hence, we can regard the path following problem as a special case of the trajectory tracking problem since the former is much easier to be deal with compared to the later.

The organization of this paper is in the following. Section 1 introduces the historical review of nonholonomic WMR motion control, including research significance and the main difficulties. The current studies about point stabilization, trajectory tracking, and path following control of nonholonomic WMR are summarized. The research motivation and main structure of this paper are illustrated in the final paragraph. These two algorithms are used to design the online compensation system for uncertainties and the controller. An innovative control concept is proposed in Section 2, which combines kinematics control and dynamics control. The cerebellar neural network weight updating algorithm and kernel space algorithm are introduced in Section 3. The effective control results given by the robust cerebellar neural network self-adjusting trajectory tracking controller are demonstrated in Section 4. Section 5 gives the conclusions and the future work.

2. Kinematic Model and Controller Design

2.1 Model of WMR

The dynamics equations of a nonholonomic mobile robot with n-dimensional state space and subject to m constraints read [4-7]

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}}(t)+C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}+G(\boldsymbol{q})+F(\dot{\boldsymbol{q}})+\boldsymbol{\tau}_d=B(\boldsymbol{q})\boldsymbol{\tau}-A^T(\boldsymbol{q})\lambda \quad (1)$$

$$\dot{\boldsymbol{q}}=S(\boldsymbol{q})\boldsymbol{u} \quad (2)$$

where $M(\boldsymbol{q})\in\mathbb{R}^{n\times n}$ is a symmetric, positive definite inertia matrix, $C(\boldsymbol{q},\dot{\boldsymbol{q}})\in\mathbb{R}^{n\times n}$ is the centripetal and Coriolis matrix, $G(\boldsymbol{q})\in\mathbb{R}^{n}$ is the gravitational vector, and $F(\dot{\boldsymbol{q}})\in\mathbb{R}^{n}$ is the surface friction. To a mobile robot moving on a smooth plane, vectors $G(\boldsymbol{q})$ and $F(\dot{\boldsymbol{q}})$ are equal to zero. $\boldsymbol{\tau}_d\in\mathbb{R}^{n}$ denotes bounded unknown disturbances, $\boldsymbol{\tau}\in\mathbb{R}^{p}$ denotes the control input, $B(\boldsymbol{q})\in\mathbb{R}^{n\times p}$ denotes the input transformation matrix, $\lambda\in\mathbb{R}^{m}$ is the constraint force vector, and $A(\boldsymbol{q})\in\mathbb{R}^{m\times n}$ is the constraint matrix.

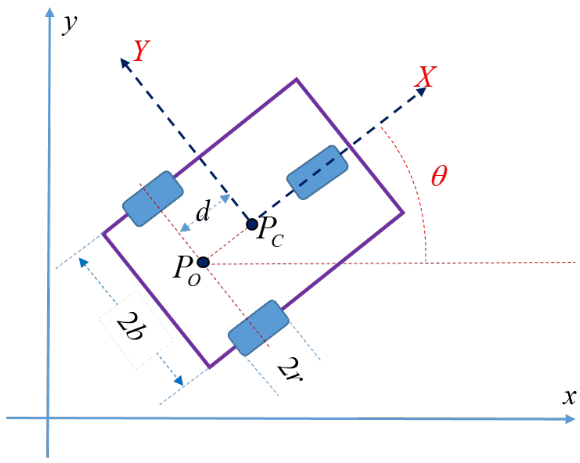

Figure 1 WMR nonholonomic mobile robot.

Figure 1 illustrates a three wheels mobile robot, in which d is the distance between robot's mass center P_C, P_O denotes the geometric center, $2b$ is the distance between two driving wheels, and r denotes the wheel radius. $\boldsymbol{q}=[x \quad y \quad \theta]^T$ represents robot's position and orientation, $\boldsymbol{u}=[v \quad \omega]^T$ represents velocity and angular velocity, $\boldsymbol{\tau}=[\tau_1 \quad \tau_2]^T$ is control torque, J denotes the inertia moment, and m_a denotes the mass of the WMR. The parameters in each matrix appeared in Eqs. (1) and (2) are

$$M(\boldsymbol{q})=\begin{bmatrix} m_a & 0 & m_a d\sin\theta \\ 0 & m_a & -m_a d\cos\theta \\ m_a d\sin\theta & -m_a d\cos\theta & J \end{bmatrix},$$

$$C(\boldsymbol{q},\dot{\boldsymbol{q}})=\begin{bmatrix} 0 & 0 & m_a d\dot{\theta}\cos\theta \\ 0 & 0 & m_a d\dot{\theta}\sin\theta \\ 0 & 0 & 0 \end{bmatrix}, \quad B(\boldsymbol{q})=\frac{1}{r}\begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ b & -b \end{bmatrix},$$

$$A^T(\boldsymbol{q})=\begin{bmatrix} -\sin\theta \\ \cos\theta \\ -d \end{bmatrix}, \quad S(\boldsymbol{q})=\begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix}$$

We can express nonholonomic constraints as

$$A(\boldsymbol{q})\dot{\boldsymbol{q}}=0 \quad . \tag{3}$$

Set a *n-m* dimensional full rank matrix $S(\boldsymbol{q})$ as a base set in null space $\boldsymbol{A}(\boldsymbol{q})$ such that

$$A(\boldsymbol{q})S^T(\boldsymbol{q})=0 . \tag{4}$$

We can obtain an auxiliary velocity control input $\boldsymbol{u}\in\mathbb{R}^{n-m}$ from Eqs. (2) and (4),

$$\dot{\boldsymbol{q}}=S(\boldsymbol{q})\boldsymbol{u} . \tag{5}$$

From Eq. (5) we can have

$$\ddot{\boldsymbol{q}}=S(\boldsymbol{q})\dot{\boldsymbol{u}}+S(\dot{\boldsymbol{q}})\boldsymbol{u} \quad . \tag{6}$$

By inserting Eq. (6) into Eq. (1) and multiplying by $S^T(\boldsymbol{q})$ to cancel the constraint matrix $A^T\lambda$, we obtain the dynamic equation of the nonholonomic mobile robot:

$$\boldsymbol{S}^T M\boldsymbol{S}\dot{\boldsymbol{u}}+\boldsymbol{S}^T\left(M\dot{\boldsymbol{S}}+C\boldsymbol{S}\right)\boldsymbol{u}+\boldsymbol{S}^T\boldsymbol{\tau}_d=\boldsymbol{S}^T B\boldsymbol{\tau} \tag{7}$$

After variable replacements, Eq. (7) becomes

$$\bar{M}(\boldsymbol{q})\dot{\boldsymbol{u}}+\bar{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\boldsymbol{u}+\bar{\boldsymbol{\tau}}_d=\bar{\boldsymbol{\tau}} \tag{8}$$

where $\bar{M}(\boldsymbol{q})\in\mathbb{R}^{p\times p}$ is the symmetric positive definite inertia matrix, $\bar{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\in\mathbb{R}^{p\times p}$ is matrix for centripetal and Coriolis forces, $\boldsymbol{S}^T\boldsymbol{\tau}_d=\bar{\boldsymbol{\tau}}_d\in\mathbb{R}^p$ contains unstructured unmodeled dynamic bounded unknown perturbations, and $\boldsymbol{S}^T\boldsymbol{B}\boldsymbol{\tau}=\bar{\boldsymbol{\tau}}\in\mathbb{R}^p$ is control input matrix ; in which $\bar{M}=\begin{bmatrix} m_a & 0 \\ 0 & J-m_a d^2\end{bmatrix}$, $\bar{C}=0_{2\times 2}$, $\bar{\boldsymbol{\tau}}_d=S^T(\boldsymbol{q})\boldsymbol{\tau}_d$, and $\bar{\boldsymbol{\tau}}=S^T(\boldsymbol{q})B(\boldsymbol{q})$.

2.2 Kinematic Controller Design

$\boldsymbol{q}_r=\begin{bmatrix} x_r & y_r & \theta_r\end{bmatrix}^T$ is the reference orientation of the WMR

$$\begin{bmatrix}\dot{x}_r\\ \dot{y}_r\\ \dot{\theta}_r\end{bmatrix}=\begin{bmatrix}\cos\theta_r & -d\sin\theta_r\\ \sin\theta_r & d\cos\theta_r\\ 0 & 1\end{bmatrix}\begin{bmatrix} v_r\\ \omega_r\end{bmatrix} \tag{9}$$

where x_r , y_r , θ_r , v_r and ω_r are the expectations of x, y ,θ, v and ω.

A. Kinematics Controller Design

Let us define tracking error as

$$\begin{bmatrix} x_e\\ y_e\\ \theta_e\end{bmatrix}=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix} x_r-x\\ y_r-y\\ \theta_r-\theta\end{bmatrix}=T_e\begin{bmatrix} x_r-x\\ y_r-y\\ \theta_r-\theta\end{bmatrix} \tag{10}$$

where $T_e=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$ (11)

With the differentiation of Eq. (10), we can have the attitude error with respect to time expressed as:

$$\begin{bmatrix}\dot{x}_e\\ \dot{y}_e\\ \dot{\theta}_e\end{bmatrix}=\begin{bmatrix} v_r\cos\theta_e\\ v_r\sin\theta_e\\ \omega_r\end{bmatrix}+\begin{bmatrix} -1 & y_e\\ 0 & -x_e\\ 0 & -1\end{bmatrix}\begin{bmatrix} v\\ \omega\end{bmatrix} \tag{12}$$

From the above analysis, we can define the trajectory tracking of the mobile robot kinematics model as finding the bounded input v and ω such that for any initial error, system (12) is bounded by the control input $(x_e, y_e, \theta_e)^T$ and satisfies the condition $\lim_{t\to\infty}\left\|(x_e, y_e, \theta_e)^T\right\|=0$ ·

Lemma 1: To any $\varphi\in\mathbb{R}$ and $\|\varphi\|\in\mathbb{R}$, the condition $f(\varphi)=\varphi\sin(arc\tan(\varphi))\geq 0$ is true, if and only if $\varphi=0$.

B. $\bar{\tau}_d$ Disturbance Observer Design

There are two main factors affecting the robustness of the WMR system. One is the interference from the outside world; while the other is the uncertainty of the internal parameters of the whole system. In fact, the WMR will be affected by friction and various noise such that there are some differences compared to the ideal system. The disturbance observer is designed to estimate the external disturbance, $\bar{\tau}_d$. The external disturbance estimator designed for the WMR system is usually in the form of [3,6,7]:

$$\begin{cases}\hat{\bar{\boldsymbol{\tau}}}_d=z+\boldsymbol{L}\boldsymbol{u}\\ \dot{z}=\boldsymbol{L}\bar{M}^{-1}z-\boldsymbol{L}\left(-\bar{M}^{-1}\boldsymbol{L}\boldsymbol{u}-\bar{M}^{-1}\bar{C}\boldsymbol{u}+\bar{M}^{-1}\bar{\boldsymbol{\tau}}\right)\end{cases} \tag{13}$$

where $\hat{\bar{\boldsymbol{\tau}}}_d$ denotes the estimation of the unknown external disturbance $\bar{\tau}_d$, z is the internal state of the nonlinear estimator, $\boldsymbol{L}$ is the parameter of the nonlinear estimator needed to be solved, which is usually expressed in terms of the constant matrix.

Some assumptions are needed to control the WMR to track the target trajectory. Let us state these assumptions in the following.

Assumption 1: The disturbances $\|\bar{\boldsymbol{\tau}}_d\|$ and $\|\dot{\bar{\boldsymbol{\tau}}}_d\|$ are bounded, and $\bar{\boldsymbol{\tau}}_d$ is constant in the steady state, i.e., $\lim_{t\to\infty}\|\dot{\bar{\boldsymbol{\tau}}}_d\| = 0$.

Assumption 2: The first derivative of the reference linear velocity v_r and angular velocity ω_r are bounded and $v_r > 0$.

Lemma 2: $L\bar{M}^{-1}$ is the Hurwitz matrix, and $\tilde{\bar{\tau}}_d$ is asymptote convergence, then we express $\bar{\boldsymbol{\tau}}_d = \hat{\bar{\boldsymbol{\tau}}}_d$.

C. The Output Layer Setting of the Cerebellar Model Controller

Consider Eqs. (12) and (14), and introduce a new control variable, $\bar{\boldsymbol{u}} = \boldsymbol{u}_c - \boldsymbol{u}$

$$\begin{aligned}\bar{M}\dot{\bar{\boldsymbol{u}}} &= -\bar{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\bar{\boldsymbol{u}} + \left(\bar{M}\dot{\boldsymbol{u}}_c + \bar{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\boldsymbol{u}_c\right) + \bar{\boldsymbol{\tau}}_d - \bar{\boldsymbol{\tau}} \\ &= -\bar{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\bar{\boldsymbol{u}} + \Gamma(\boldsymbol{u}_c,\dot{\boldsymbol{u}}_c) + \bar{\boldsymbol{\tau}}_d - \bar{\boldsymbol{\tau}} \end{aligned} \quad (14)$$

where $\boldsymbol{\Gamma}(\boldsymbol{u}_c,\dot{\boldsymbol{u}}_c) = \bar{M}\dot{\boldsymbol{u}}_c + \bar{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\boldsymbol{u}_c$ is the nonlinear function of the mobile robot, function $\boldsymbol{\Gamma}(\boldsymbol{u}_c,\dot{\boldsymbol{u}}_c)$ includes many parameters of the mobile robot such as mass, rotation inertia and so on. It is very difficult to determine these parameters, hence we use cerebellar model controller to approach, which can be expressed as

$$\boldsymbol{\Gamma}(\boldsymbol{u}_c,\dot{\boldsymbol{u}}_c) = Y(\boldsymbol{u}_c,\dot{\boldsymbol{u}}_c)\boldsymbol{W} \quad (15)$$

where $Y(\boldsymbol{u}_c,\dot{\boldsymbol{u}}_c)$ denotes the cerebellar network output, and $\boldsymbol{W}$ is the connection weighting of the cerebellar network output. Then we can rewrite Eq. (14) as

$$\bar{M}\dot{\bar{\boldsymbol{u}}} = -\bar{C}\bar{\boldsymbol{u}} + Y\boldsymbol{W} + \bar{\boldsymbol{\tau}}_d - \bar{\boldsymbol{\tau}} \quad (16)$$

3. Simulation Results and Analysis

In order to verify the actual control performance provided by the proposed algorithm, we use MATLAB / SIMULINK to execute the simulation. Parameters of the WMR are list as follows: the wheel radius, $r = 0.12\ m$, the distance between two driving wheels, $2b = 0.6\ m$, the mass of the WMR, $m_a = 4\ kg$, the distance between robot's mass center and geometric center, $d = 0.25\ m$ and the inertial moment, $J = 0.25\ kgm^2$. In the simulation, the parameters of the observer and controller are designed as $\boldsymbol{L} = \begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix}$, where $L_1 = L_2 = 12m$, and $k_x = k_y = k_\theta = 1$, $k_u = 0.3$ and $k_s = 0.8$. The initial position of the reference input is $[x_r(0) \quad y_r(0) \quad \theta_r(0)] = [0 \quad 0 \quad 0]$. The reference velocities are set as $v_r = 0.1\ m/s$, $\omega_r = 0.01\ rad/s$. The initial state of the WMR is given as $[x(0) \quad y(0) \quad \theta(0)] = [39.5m \quad 0.1m \quad 89.427\deg.]$ .The external disturbance $\bar{\boldsymbol{\tau}} = \left[1.2\sin(0.3t - \frac{\pi}{2}) \quad 0.51\sin(0.3t + \frac{\pi}{2}) \quad -1.31\sin(0.5t)\right]^T$ occurs after the WMR is moving. Learning rate of CMAC controller is $\boldsymbol{\eta} = 0.5$, and the inertia coefficient is $\boldsymbol{\alpha} = 0.5$.

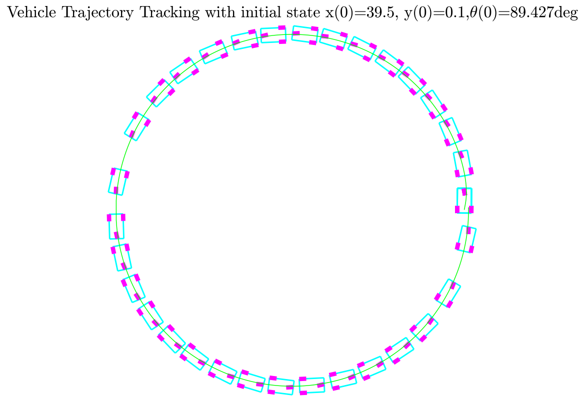

Figure 1 WRM trajectory tracking

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