

Distributed Rotating Encirclement Control of Strict-Feedback Multi-Agent Systems Using Bearing Measurements

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Abstract

This paper investigates the distributed multi-target rotating encirclement formation problem of strict-feedback multi-agent systems using the targets' bearing angles and the agents' known positions, where all agents are forced to achieve even circular formation around the targets' geometric center. Firstly, an estimator is proposed for each agent to localize the neighbor targets. Secondly, based on the trajectory planning method, a reference trajectory is constructed by three estimators, which are used to obtain the targets' geometric center, the reference rotating radius and angular. Then, the proposed adaptive neural dynamic surface control law forces each agent to move along the reference trajectory, which satisfies the multi-target rotating encirclement formation conditions.

Keywords: Strict-feedback multi-agent systems, rotating encirclement control, target localization, trajectory planning, trajectory tracking

1. Introduction

Recent years the rotating encirclement formation problem of multi-agent systems have attracted considerable attention due to its significant potential applications in both military and civilian areas such as surveillance, search-and-rescue, reconnaissance, etc¹. Many interesting results have been achieved for the rotating formation or surrounding/encirclement control problem¹⁻⁷.

As one of the most important high-order systems, the strict-feedback system can be used to model a variety of physical systems including robotic manipulators, vessel, unmanned aerial vehicle and so on⁸. And recent years have witnessed the emergence of researches with respect to the strict-feedback single/multi-agent system⁸⁻¹¹. However, there is no research to date on the rotating encirclement control of high-order multi-agent system.

Motivated by above discussion, for the first time, we consider the multi-target rotating encirclement formation problem of strict-feedback multi-agent systems, and only bearing measurements of targets can be obtained. To this end, we divide the problem into three subproblems: target localization, trajectory planning and trajectory tracking. Four estimators are designed to construct a reference trajectory for each agent, and an adaptive neural dynamic surface control law is proposed to make the agent move along the desired trajectory.

2. Preliminaries and Problem Statement

2.1. Graph Theory

Let $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A}, \mathcal{B})$ be a weighted undirected graph corresponding to n agents and m targets, where $\mathcal{V} = \{v_1, v_2, \dots, v_n, s_1, \dots, s_m\}$ denotes the set of vertexes, $\mathcal{E} \subset$

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$\mathcal{V} \times \mathcal{V}$ denotes the set of edges, $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$ denotes the weighted adjacency matrix of targets, $\mathcal{B} = [b_{ik}] \in \mathbb{R}^{n \times m}$ denotes the weighted adjacency matrix from targets to agents. Let $d(v_i, v_j)$ denote the shortest distance from the vertex v_i to v_j , for instance, $d(v_i, v_j) = 1$ if $(v_i, v_j) \in \mathcal{E}$. The neighbor agents set of the agent v_i is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} | (v_i, v_j) \in \mathcal{E}\}$ and the neighbor targets set of the agent v_i is denoted by $\mathcal{N}_i^T = \{s_k \in \mathcal{V} | (v_i, s_k) \in \mathcal{E}\}$. The neighbor agents set of the target s_k is denoted by $\mathcal{N}_k^J = \{v_i \in \mathcal{V} | (v_i, s_k) \in \mathcal{E}\}$. And let $|\cdot|$ denote the number of elements in the set \cdot .

2.2. Problem Statement

Consider a multi-agent system consisting of n agents (Index set $\mathcal{I} = \{1, 2, \dots, n\}$) and m stationary targets (Index set $\mathcal{T} = \{1, 2, \dots, m\}$) with bearing-only measurements, where the dynamic of agent v_i is in the following q_i -order strict-feedback form.

$$\begin{cases} \dot{x}_{ij} = f_{ij}(\bar{x}_{ij}) + x_{ij+1} \\ \dot{x}_{iq_i} = f_{iq_i}(\bar{x}_{iq_i}) + u_i \\ y_i = x_{i1} \end{cases} \quad (1)$$

where $\bar{x}_{ij} = [x_{i1}^T, \dots, x_{ij}^T]^T$, and $\bar{x}_{iq_i}, y_i, u_i \in \mathbb{R}^2$ represent the states, output and control input of agent v_i , respectively. $f_{ij}(\bar{x}_{ij})$ is an unknown continuous nonlinear function.

The *objective* of this note is to design the distributed control scheme using bearing-only measurements of targets and the neighbor position information of agents, such that strict-feedback agents are capable of achieving the multi-target rotating encirclement formation, which is properly formulated by Definition 1 using the polar coordinate transformation using the polar coordinate transformation $y_i = \bar{r} + [l_i \cos(\theta_i), l_i \sin(\theta_i)]^T$.

Definition 1⁵. The multi-agent system is said to achieve the *multi-target rotating encirclement formation* if

$$\lim_{t \rightarrow \infty} \left[l_i - \lambda \max_{k \in \mathcal{T}} \|r_k - \bar{r}\| \right] = 0 \quad (2)$$

$$\lim_{t \rightarrow \infty} \left[\theta_i - \theta_j - \frac{2\pi(i-j)}{n} \right] = 0 \quad (3)$$

$$\lim_{t \rightarrow \infty} [\dot{\theta}_i - \omega] = 0 \quad (4)$$

Where $i, j \in \mathcal{I}$, r_k and $\bar{r}(t) = 1/m \sum_{k \in \mathcal{T}} r_k$ denote the position of the k -th target and the geometric center of all targets respectively. The design parameter $\lambda > 1$ determines the radius of the desired rotation formation. And ω is the desired angular velocity.

To facilitate the later control design and analysis, we make some reasonable assumptions.

Assumption 1. All agents are connected in some undirected communication topologies and each target connects to at least one agent via the directed edge.

Assumption 2. The radius of the desired rotation formation is bounded, i.e., there exists a positive constant d^* satisfying $\max_{k \in \mathcal{T}} \|r_k - \bar{r}\| \leq d^*$.

Assumption 3. The desired angular velocity ω and angular acceleration $\dot{\omega}$ are continuous and bounded, i.e., there exists positive constants ω^* and ω_d^* such that $\|\omega\| \leq \omega^*, \|\dot{\omega}\| \leq \omega_d^*$.

3. Control Design

In this section we present in detail the distributed multi-target rotating encirclement control scheme, which includes three parts: target localization, trajectory planning and trajectory tracking.

3.1. Target Localization

To estimate the neighbor target's position of agent v_i with bearing-only measurements, the following estimator is proposed according to Ref. 7.

$$\dot{\hat{r}}_{ik} = \alpha_{ik} (I - \varphi_{ik} \varphi_{ik}^T) (x_{i1} - \hat{r}_{ik}) \quad (5)$$

Where $k \in \mathcal{N}_i^T$, α_{ik} is a positive design parameter, and φ_{ik} is the unit vector from x_i to r_k , i.e., $\varphi_{ik} = (r_k - x_i) / \|r_k - x_i\|$.

3.2. Trajectory Planning

For each agent v_i , to plan a reference trajectory satisfying the Definition 1, we design the following distributed estimators to obtain the estimations p_i, \hat{l}_i and $\hat{\theta}_i$ of the desired geometric center \bar{r} , polar radius l_i and polar angle θ_i , respectively.

$$\begin{cases} \dot{p}_{ik} = \beta_i \sum_{j \in \mathcal{N}_i} a_{ij} [p_{jk} - p_{ik}] \\ \quad + \beta_i b_{ik} [\hat{r}_{ik} - p_{ik}] \\ p_i = \frac{1}{m} \sum_{k=1}^m p_{ik} \end{cases} \quad (6)$$

$$\begin{cases} \dot{\rho}_{i1} = \gamma_{i1} \max_{k \in \mathcal{N}_i^T} (\|\hat{r}_{ik} - p_i\|) - \rho_{i1} \\ \dot{\rho}_{i2} = \gamma_{i2} \max_{j \in \mathcal{N}_i \cup \{i\}} (\|\rho_{j1}\|) - \rho_{i2} \\ \vdots \\ \dot{\rho}_{iM} = \gamma_{iM} \max_{j \in \mathcal{N}_i \cup \{i\}} (\|\rho_{jM-1}\|) - \rho_{iM} \\ \hat{l}_i = \lambda \rho_{iM} \end{cases} \quad (7)$$

$$\begin{cases} \dot{\hat{\theta}}_i = \delta_i \sum_{j \in \mathcal{N}_i} a_{ik} \left[\hat{\theta}_j - \hat{\theta}_i - \frac{2\pi(j-i)}{n} \right] + \omega \end{cases} \quad (8)$$

Where $\beta_i, \gamma_{i1}, \dots, \gamma_{iM}, \delta_i$ are positive design parameters, and $M = \max_{i,j \in \mathcal{I}} \{d(i, j)\}$, which can be chosen as $M = n - 1$ if it is not prior information.

Then, with the polar coordinate transformation, the reference trajectory of agent v_i is provided as follows.

$$\hat{y}_i = p_i + [\hat{l}_i \cos(\hat{\theta}_i), \hat{l}_i \sin(\hat{\theta}_i)]^T \quad (9)$$

3.3. Trajectory Tracking

Similar to the backstepping-based DSC design procedure, we define dynamic surface errors as follows.

$$\begin{cases} z_{i1} = x_{i1} - \hat{y}_i \\ z_{ij} = x_{ij} - \hat{\eta}_{ij} \end{cases} \quad (10)$$

Where $\hat{\eta}_{ij}(t)$ is the first-order filter estimation of the virtual controller $\eta_{ij}(t)$ with the time constant $\tau_{ij} > 0$ and the filter error is denoted by $\tilde{\eta}_{ij} = \hat{\eta}_{ij} - \eta_{ij}$.

$$\tau_{ij} \dot{\hat{\eta}}_{ij} + \hat{\eta}_{ij} = \eta_{ij}, \hat{\eta}_{ij}(0) = \eta_{ij}(0) \quad (11)$$

Then, we will present the following virtual and actual controllers and adaptive law such that each agent moves along its desired reference trajectory.

$$\begin{cases} \eta_{ij} = -\kappa_{ij} z_{ij} - \hat{W}_{ij}^T S_{ij}(\zeta_{ij}) \\ u_i = -\kappa_{iq_i} z_{iq_i} - \hat{W}_{iq_i}^T S_{iq_i}(\zeta_{iq_i}) \end{cases} \quad (12)$$

$$\dot{\hat{W}}_{ij} = -\Gamma_{ij}^{-1} [\sigma_{ij} \hat{W}_{ij} - S_{ij}(\zeta_{ij}) z_{ij}^T] \quad (13)$$

Where $\Gamma_{ij} = \Gamma_{ij}^T > 0$ is an adaptive gain matrix, \hat{W}_{ij} and $S_{ij}(\zeta_{ij})$ represent the estimation of the optimal weight matrix W_{ij} and the basis function vector respectively. And κ_{ij}, σ_{ij} are positive design parameters.

4. Main Results

With the proposed control scheme in Section 3, we can easily obtain the following reasonable results.

Lemma 1. Consider the estimator (5) under Assumptions 1-2. Then for any $i \in \mathcal{I}, k \in \mathcal{T}$, the estimation position \hat{r}_{ik} will asymptotically converge to the actual position r_k of the k -th target.

Proof. The proof is similar to Theorem 3.1 in Ref. 7. \square

Then, we define the estimation of the targets' geometric center as follows.

$$\hat{\bar{r}} = \frac{1}{m} \sum_{i=1}^n \sum_{k \in \mathcal{N}_i^T} \frac{1}{|\mathcal{N}_i^T|} \hat{r}_{ik} \quad (14)$$

Apparently, $\hat{\bar{r}}$ will asymptotically converge to the actual geometric center \bar{r} .

Lemma 2. Consider the estimator (6) under Assumptions 1-2. For any $i \in \mathcal{I}$, the estimation position p_i will asymptotically converge to \bar{r} .

Proof. The proof is similar to Lemma 4 in Ref. 5. \square

Then, combining lemma 1 with lemma 2, we can conclude that the estimation position p_i of the i -th agent will asymptotically converge to the actual geometric center \bar{r} .

Lemma 3. Consider the estimator (7) under Assumptions 1-2. For any $i \in \mathcal{I}$, the following equation holds.

$$\lim_{t \rightarrow \infty} [\hat{l}_i - \lambda \max_{i \in \mathcal{I}, k \in \mathcal{N}_i^T} (\|\hat{r}_{ik} - p_i\|)] = 0 \quad (15)$$

In other words, the estimation \hat{l}_i of the polar radius will asymptotically converge to the above value.

Proof. The proof is similar to Lemma 5 in Ref. 5. \square

Furthermore, with Lemma 1 and Lemma 2, it is easy to see that $\lim_{t \rightarrow \infty} [\hat{l}_i - \lambda \max_{k \in \mathcal{T}} (\|r_k - \bar{r}\|)] = 0$, which implies that \hat{l}_i satisfies the condition (2).

Lemma 4. Consider the estimator (8) under Assumptions 1-3. For any $i, j \in \mathcal{I}$, the following equations hold.

$$\lim_{t \rightarrow \infty} [\hat{\theta}_i - \hat{\theta}_j - \frac{2\pi(i-j)}{n}] = 0 \quad (16)$$

$$\lim_{t \rightarrow \infty} [\hat{\theta}_i - \omega] = 0 \quad (17)$$

In other words, the estimation $\hat{\theta}_i$ of the polar angle satisfies conditions (3) and (4).

Proof. The proof is similar to Lemma 6 in Ref. 5. \square

Thus, from the polar coordinate transformation of (9), we can easily conclude that the reference trajectory \hat{y}_i satisfies conditions of the multi-target rotating encirclement formation in Definition 1.

Then, we will carry on the stability analysis of the proposed control scheme (12) and (13), which drives the output y_i of the agent v_i to the reference trajectory \hat{y}_i . Since the neural network is capable of approaching any continuous nonlinear function with free precision (See Lemma 2 in Ref. 8 for details), we make the following reasonable approximation.

$$\begin{aligned} \phi_{i1} &= W_{i1}^T S_{i1}(\zeta_{i1}) + \varepsilon_{i1}(\zeta_{i1}) \\ &= f_{i1}(\bar{x}_{i1}) - \hat{y}_i \\ \phi_{ij} &= W_{ij}^T S_{ij}(\zeta_{ij}) + \varepsilon_{ij}(\zeta_{ij}) \\ &= f_{ij}(\bar{x}_{ij}) + \tilde{\eta}_{ij}/\tau_{ij} + z_{ij-1} \end{aligned} \quad (18)$$

Where $\|\varepsilon_{ij}(\zeta_{ij})\| < \varepsilon_{ij}^*$, ε_{ij}^* is an arbitrarily small constant and denote $\tilde{W}_{ij} = W_{ij} - \hat{W}_{ij}$.

Choose a common Lyapunov function candidate as $V_i = \frac{1}{2} \sum_{j=1}^{q_i} [z_{ij}^T z_{ij} + \text{tr}(\tilde{W}_{ij}^T \Gamma_{ij} \tilde{W}_{ij})] + \frac{1}{2} \sum_{j=2}^{q_i} \tilde{\eta}_{ij}^T \tilde{\eta}_{ij}$. Thus, by calculating the time derivative of V_i , we have

$$\begin{aligned} \dot{V}_i &= \sum_{j=1}^{q_i} z_{ij}^T [-\kappa_{ij} z_{ij} + \tilde{W}_{ij}^T S_{ij}(\zeta_{ij}) + \varepsilon_{ij}(\zeta_{ij})] \\ &\quad + \sum_{j=2}^{q_i} z_{ij-1}^T \tilde{\eta}_{ij} + \sum_{j=1}^{q_i} \text{tr}(\tilde{W}_{ij}^T \Gamma_{ij} \dot{\tilde{W}}_{ij}) \\ &\quad + \sum_{j=2}^{q_i} \tilde{\eta}_{ij}^T \dot{\tilde{\eta}}_{ij} \end{aligned} \quad (19)$$

The dynamic of the filter error $\tilde{\eta}_{ij}$ can be written as

$$\dot{\tilde{\eta}}_{ij} = -\frac{\tilde{\eta}_{ij}}{\tau_{ij}} + \pi_{ij}(\bar{z}_{ij}, \tilde{\eta}_{ij}, \tilde{W}_{ij-1}, Y_i) \quad (20)$$

Where we denote $\bar{z}_{ij} = [z_{i1}^T, \dots, z_{ij}^T]^T$, $\tilde{\eta}_{ij} = [\tilde{\eta}_{i2}^T, \dots, \tilde{\eta}_{ij}^T]^T$, $\tilde{W}_{ij} = [\tilde{W}_{i1}^T, \dots, \tilde{W}_{ij}^T]^T$ and $Y_i = [\hat{y}_i^T, \hat{y}_i^T, \hat{y}_i^T]^T$.

Therefore, substituting the adaptive law (13) and (20) into (19), we obtain

$$\begin{aligned}\dot{V}_i = & -\sum_{j=1}^{q_i} \kappa_{ij} z_{ij}^T z_{ij} - \sum_{j=1}^{q_i} z_{ij}^T \varepsilon_{ij}(\zeta_{ij}) \\ & + \sum_{j=1}^{q_i} \sigma_{ij} \text{tr}(\tilde{W}_{ij}^T \tilde{W}_{ij}) - \sum_{j=2}^{q_i} \frac{\tilde{\eta}_{ij}^T \tilde{\eta}_{ij}}{\tau_{ij}} \\ & + \sum_{j=2}^{q_i} z_{ij-1}^T \tilde{\eta}_{ij} + \sum_{j=2}^{q_i} \tilde{\eta}_{ij}^T \pi_{ij}\end{aligned}\quad (21)$$

According to Assumption 2 and 3, we know that the desired rotating radius, angular velocity and angular acceleration are bounded. Then \hat{y}_i , $\dot{\hat{y}}_i$ and $\ddot{\hat{y}}_i$ are bounded, i.e., there exists a positive constant Y_i^* such that $\Xi_i = \{Y_i \| \hat{y}_i \| + \| \dot{\hat{y}}_i \| + \| \ddot{\hat{y}}_i \| \leq Y_i^*\}$. In addition, we denote that $\Pi_i = \{(\bar{z}_{iq_i}, \tilde{\eta}_{iq_i}, \tilde{W}_{iq_i}) | V_i \leq 2\mu_i\}$, where μ_i is a positive constant. Then, it is not hard to see that Ξ_i and Π_i are compact sets. Thus, there exists a positive constant π_{ij}^* satisfying $\|\pi_{ij}(\cdot)\| \leq \pi_{ij}^*$.

Moreover, with Young's inequality⁸, we have

$$\begin{cases} z_{ij}^T \varepsilon_{ij}(\zeta_{ij}) \leq (\varepsilon_{ij}^{*2} / 2\varrho_{ij}) \|z_{ij}\|^2 + \varrho_{ij} / 2 \\ \text{tr}(\tilde{W}_{ij}^T \tilde{W}_{ij}) \leq -(1/2) \|\tilde{W}_{ij}\|_F^2 + (1/2) \|W_{ij}\|_F^2 \\ z_{ij-1}^T \tilde{\eta}_{ij} \leq (1/2) \|z_{ij-1}\|^2 + (1/2) \|\tilde{\eta}_{ij}\|^2 \\ \tilde{\eta}_{ij}^T \pi_{ij} \leq (\pi_{ij}^{*2} / 2\Delta_{ij}) \|\tilde{\eta}_{ij}\|^2 + \Delta_{ij} / 2 \end{cases}\quad (22)$$

Make the following names.

$$\begin{cases} \varepsilon_{ij}^1 = \kappa_{ij} - \frac{1}{2} - \frac{\varepsilon_{ij}^{*2}}{2\varrho_{ij}}, \varepsilon_{iq_i}^1 = \kappa_{iq_i} - \frac{\varepsilon_{iq_i}^{*2}}{2\varrho_{iq_i}} \\ \varepsilon_{ij}^2 = \frac{\sigma_{ij}}{2}, \varepsilon_{ij}^3 = \frac{1}{\tau_{ij}} - \frac{1}{2} - \frac{\pi_{ij}^{*2}}{2\Delta_{ij}} \end{cases}\quad (23)$$

Then, \dot{V}_i can be rewritten as

$$\dot{V}_i \leq -c_{i0} V_i + c_{i1}\quad (24)$$

Where

$$\begin{cases} c_{i0} = \min\{2\varepsilon_{ij}^1, 2\varepsilon_{ij}^2, 2\varepsilon_{ij}^3\} \\ c_{i1} = \sum_{j=1}^{q_i} \frac{\varrho_{ij}}{2} + \sum_{j=1}^{q_i} \frac{\sigma_{ij} \|W_{ij}\|_F^2}{2} + \sum_{j=2}^{q_i} \frac{\Delta_{ij}}{2} \end{cases}\quad (25)$$

Theorem 1. Consider the multi-agent system (1) in the strict-feedback form with stationary multi-targets. Suppose that Assumptions 1-3 hold. For any bounded initial condition $V_i(0) \leq \mu_i$, if we choose design parameters satisfy $c_{i0} > 0$, then all agents will achieve the multi-target rotating encirclement formation with the proposed control scheme in Control Design.

Proof. By integrating both ends, it is obvious that the solution of (24) satisfies the following inequality.

$$V_i \leq \left[V_i(0) - \frac{c_{i1}}{c_{i0}} \right] e^{-c_{i0}t} + \frac{c_{i1}}{c_{i0}}\quad (26)$$

Then, with Lemma 1 in Ref. 8, we know that the tracking error z_{i1} is bounded, and the upper bound is associated with c_{i1}/c_{i0} . By reasonably selecting design parameters, z_{i1} can be sufficiently reduced to 0.

Combining with Lemma 1,2,3 and 4, we can easily conclude that our proposed control scheme will drive all agents achieve the multi-target rotating encirclement formation. \square

5. Conclusion

The collective multi-target rotating encirclement formation problem of strict-feedback multi-agent systems is investigated by dividing into three subproblems. Our proposed control scheme can solve this problem well.

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