# No Free Lunch Principle in Agent Swarm Systems: One Case Study

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### Abstract

This note comes with information flooding of the control action of the leader in leader follower framework. In this situation, even observer for the leader velocity is built in a distributed style, the agent swarm system in a whole could not be classified into distributed one. One case study example will be borrowed to demonstrate the truth of the no free lunch principle in agent swarm systems.

Keywords: free lunch principle, agent swarm systems, distributed observer, leader-follower framework

## 1. Introduction

The agent swarm systems are recognized widely and this trend is still flooding<sup>1</sup>. Networks of networks for leader-follower system was explored by Song<sup>2</sup>, which emphasizes interdependence among the agents. After that, the root node agents and the leaf node agents, could flock into each other, were touched upon further<sup>3</sup>.

For agent swarm systems, on account of the interwoven between information and control, complexities are enlarged unprecedented<sup>4-5</sup>. Leader-follower agent swarms tracking control, was focused upon by several scientists<sup>6-9</sup>.

To be set apart from the main results, this note will contribute to the neglected part of the dual roles of information and control. It is well known that the

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distributed control is overly dependent on the neighbor based information exchange among agents, local control based regulation is always the main theme of the agent swarm systems. While besides that, the moving agents, especially the active leader one, must have its own velocity and added control action. For second order leader-follower agent swarms, the velocity of the leader agent was paid much more attention and its observers design is necessarily executed for its unavailable peculiarity for not all the follower agents. To be distinct, the added control action of the leader agent is assumed possible for all the follower agents, and this assumption is overly strong to be possible, for the availability of the added control action of the leader node is always impossible unless there is some broadcasting mechanism about that signal. This note will concentrate on the complexity balance between information and control of the agent swarm systems, the simplicity of one side will deteriorate on the other side. That is no free lunch principle in agent swarm systems.

The note will be arranged as follows. At start, problem formulation will be proposed, then after the main results will be presented in the second part. Followed will be some conclusions.

## 2. Distributed Observers Design for Leader-Follower Agent Swarm systems

This part will begin with distributed observers design for leader-follower agent swam systems.

### 2.1. Problem formulation

Assume the agent swarm system is composed with one leader agent and n follower agents. Here, the graph G is used to describe the graph of the follower agents, which is undirected graph. We use G denotes the graph with the leader agent. Some agents in the follower ones will connect the leader one with directed arcs. Obviously, the G can be named as connected if at least one of the follower agents is connected with the leader one. To highlight the main theme of this note, the topology of the agent system is assumed to be static. One form of the leader-follower agent topology of this note can be listed as Figure 1. In Fig.1, we have two different connections from the leader agent to the follower agents, one is characterized as solid line with arrow, the other are characterized as discontinuous lines with arrows. Here, the solid line is employed to represent the

physical proximity of the interacting agents, while the discontinuous lines are for information flooding from the leader to the followers, broadcasting can be one kind of mechanisms.



Fig.1 Network consists of 4 nodes

### 2.2. The Main Results

Leader agent is assumed to be active. The dynamic of the leader agent is expressed as:

$$\begin{cases} \dot{x}_0 = v_0, x_0 \in R^m, \\ \dot{v}_0 = u_0, u_0 \in R^m, \\ y = x_0, \end{cases}$$
(1)

Where  $x_0$ ,  $v_0$  and y are position velocity and the only measurable variable, respectively. The dynamic of the follower agents is described as:

$$\begin{cases} \dot{x}_{i} = v_{i}, x_{i} \in R^{m}, \\ \dot{v}_{i} = u_{i}, u_{i} \in R^{m}, i = 1, ..., n, \end{cases}$$
(2)

The distributed observer demonstrated was in the following form:

$$\begin{cases} x_{ij} = x_i - x_j, x_{i0} = x_i - x_0, \\ u_i = u_0 - k(v_i - \hat{v}_i) \\ -l(\sum_{j \in N_i(t)} a_{ij} x_{ij} + \sum_{i \in N_0(t)} b_i x_{i0}), \\ \dot{\hat{v}}_i = u_0 - (l / k^2) (\sum_{j \in N_i(t)} a_{ij} x_{ij} + \sum_{i \in N_0(t)} b_i x_{i0}), \end{cases}$$
(3)

Where  $N_i(t), N_0(t)$  denotes the neighbour agents of the follower agent *i* and the leader agent.  $a_{ij}, b_i$  is related with the topology of <u>G</u>. If agent *j* is in the neighbourhood of agent *i*, then  $a_{ij} = 1$ . Similarly, if the follower agent *i* is connected with the leader agent directly, then  $b_i > 0$ .

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**Comment 1**: The protocol of (3) for (1) and (2) was put forward by Hong et al in [7], concentrated by Sarras in [8], and replied by Hong et al again in [9].

**Comment 2**: The seminal work of Hong et al in [7] declare that their protocol to be superior to the other ones in its simplicity in observer structure, while the dynamic of the system is in second order, the observer in distributed form can be in first order. That avoids the intricacy of the full order observer.

**Comment 3**: The communication between Hong et al and Sarras et al was only constrained in the coefficient of the protocol, without doubt, the selection of the coefficient is vitally important, and to do that, the research was promoted a lot, that can be seen clearly in [8] and [9].

**Comment 4**: To be set apart from the already existed results, no free lunch principle in leader follower agent swarms would be introduced, and without losing the simplicity of (3).

**Comment 5**: That availability  $u_0$  for all the follower agents was too strong to be true. In fact, some extra mechanism must be taken to guarantee its availability, broadcasting can be one of them, but broadcasting easily suffer from information leakage and adversarial attacks.

**Comment 6**: Control simplicity and information complexity are always stand by each other. The gain of the control simplicity comes along with information intricacy. That is the no free lunch principle in leader-follower agent swarm systems. Our improved protocol is like:

$$\begin{cases} x_{ij} = x_i - x_j, x_{i0} = x_i - x_0, \\ u_i = u_0 - k(v_i - \hat{v}_i) \\ -l(\sum_{j \in N_i(t)} a_{ij} x_{ij} + \sum_{i \in N_0(t)} b_i x_{i0}) + \overline{u}, \\ \dot{\hat{v}}_i = u_0 - (l / k^2) (\sum_{j \in N_i(t)} a_{ij} x_{ij} + \sum_{i \in N_0(t)} b_i x_{i0}) + \overline{u}, \end{cases}$$
(4)

Where we assume that  $u = u_0 + \overline{u}$ , and  $u_0$  is virtually possible to drive the controller and the observer of the follower agents,  $\overline{u}$  is added in to compensate this virtually dealt scheme,  $\overline{u}$  satisfies  $\||\overline{u}|| \le M \ge 0$ .

**Comment 7**: The suggested protocol for the follower agents about the controller function of the leader agent, which is in the closed control loop, is feasible. This can be said in the following reasons: At first the on board sensor, actuator for the second integrator in the same series can be guessed on some sense, though with some errors, and second, the error induced disturbance can again be estimated by customized observer.

**Lemma 1**: For leader (1) and followers (2), with  $\overline{u} = 0$ , the controller-observer pair (3) can produce

$$\lim_{t \to \infty} |x_i(t) - x_0(t)| = 0, \lim_{t \to \infty} |v_i(t) - v_0(t)| = 0;$$
(5)

**Theorem 1**: There exists a constant  $c_M > 0$  with  $\lim_{M \to 0} c_M = 0$ , such that

$$\lim_{t \to \infty} |x_i(t) - x_0(t)| \le c_M, \lim_{t \to \infty} |v_i(t) - v_0(t)| \le c_M;$$
(6)

**Proof:** In case of convenience, set  $\xi = (x_1,...,x_n)^T - x_0 \mathbf{1}, \eta = (v_1,...,v_n)^T - v_0 \mathbf{1},$  and  $\zeta = k(v_1,...,v_n)^T - kv_0 \mathbf{1},$  where  $\mathbf{1} = (1,...,1)^T \in \mathbb{R}^m$ . Then the closed system can be turned into:

$$\begin{cases} \dot{\xi} = \eta, \\ \dot{\eta} = -l(L+B)\xi - k\eta + \zeta + \overline{u}, \\ \dot{\zeta} = \frac{l}{k}(L+B)\xi + \overline{u}, \end{cases}$$
(7)

Assume  $z = \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix}$ , then in a compact form, (7) can be

casted into

$$\dot{z} = Fz = \begin{pmatrix} 0 & I & 0 \\ -lH & -kI & I \\ -\frac{l}{k}H & 0 & 0 \end{pmatrix} + \delta,$$

ſ

Where H = L + B,  $\delta = (0, \overline{u}, \overline{u}, 0, ..., 0)^T$ . For system (7), a control Lypunov function (CLF) can be constructed as  $V(z) = z^T(t)Pz(t)$ ,

where 
$$P = \begin{pmatrix} kI & I & -\frac{k}{2}I \\ I & I & -\frac{1}{2}I \\ -\frac{k}{2}I & -\frac{1}{2}I & \frac{k}{2}I \end{pmatrix}$$
, which can be

guaranteed positive if parameters l, k are properly chosen. To be detail, they satisfy the following inequalities  $l \ge 2/\lambda_{\min}, k \ge 4 + \lambda_{\max}$ , where  $\lambda_{\min}, \lambda_{\max}$  are the minimum and maximum eigenvalues of positive matrix H.

Consequently,

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$$V(z(t)) \le e^{-\beta t} V(z(0)) + \beta_0 M^2.$$
(8)

This implies (6).



Here, we use Fig.2 as the case study example, parameters are selected as:

 $u = u_0 + \overline{u}, u_0 = \cos t, \overline{u} = 0.02 \cos t, l = 40, b = 0.2, k = 200.$ 



The corresponding results are demonstrated as Fig.3. From simulation results, we can say the suggested scheme is effective, which is with the virtual control function estimation and perturbation term of amendment.

## 2.3. Further Comments

Inspirations can be always possible if we pay much more attention to the interplay of information and control in agent swarm systems. No free lunch principle can be one example of them, though sometimes it is subtle.

## 3. Conclusion

No free lunch principle can be a big help for us to promote the research about leader-follower agent swarm systems.

System intricacy, could not be easily mastered if we are kept away from the concrete systems. Case study like high performance vehicle driving<sup>10-11</sup>, can be the milestone systems. Time varying delays and other negative points, can also be the adversary factors. Adaptive scheme in Jia<sup>12</sup> provided a good example. Refined control of leader follower system according their roles of the leader agent, was recommended also<sup>7</sup>. So, balance between information and control can be an eternal topic in agent swarm systems.

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