Analysis and Circuit Design of a Novel 4D Chaotic System

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Abstract

In this paper, a novel 4D chaotic system is proposed. First, the basic dynamic characteristics of this system are analyzed theoretically. Second, dynamical properties of the system are investigated by phase portraits, Poincaré sections, Lyapunov exponent spectrum and bifurcation diagram. In addition, an analog circuit of the system is designed and simulated by Multisim, a circuit simulation software. The experimental results show that the circuit simulation results are consistent with the numerical simulation results. Therefore, the existence of chaotic attractor in the system is verified from the physical level.

Keywords: Novel 4D chaotic system; Dynamic characteristics; Simulation analysis; Analog circuit design.

1. Introduction

Since 1963, when Lorenz discovered the butterfly effect [1], more and more scholars have studied chaos. In recent decades, with the development of mathematical theory, chaos research has also made great progress with emerging a number of newly built systems. For example, in 1993, Chua et al. designed Chua circuits in which the mathematical model became a very classical chaotic system [2]. And Sprott J et al. proposed Sprott system in the following year [3]. In 1999, Chen system was successfully constructed by Chen et al. [4] In the early 21century, Lü et al. discovered the Lü system [5]. A three-dimensional continuous autonomous chaotic system was constructed by Liu et al. called Liu system [6]. Qi et al. proposed a four-dimensional chaos system, namely Qi system [7]. Except to studying how to construct a system with better dynamic performance, on the basis of existing systems, many scholars also investigate and realize chaotic system through numerical analysis, analog circuit implementation, digital circuit implementation and other methods [8,9,10,11]. In recent years, chaos theory has been applied in many fields, including information studying encryption and secure communication [12,13,14], optoelectronics [15], astrophysics [16], electromagnetic mechanics [17] and economics [18]. Through chaos theory, we can not only further investigate the nonlinear characteristics and their generation mechanism in these fields, but also provide a theoretical basis for the construction of new chaotic systems.

In this paper, a novel 4D chaotic system is proposed. Firstly, the dissipation and equilibrium of the system are theoretically analyzed. Then, simulation analysis is carried out through phase portraits, Poincaré section diagrams, Lyapunov exponent spectrum and bifurcation diagram to analyze the dynamic characteristics of the system in detail. Finally, in order to illustrate the existence of chaotic attractor in the system from the physical level, the circuit design and simulation are carried out with the circuit simulation software Multisim. Moreover, the existence of chaotic attractor in the system is further verified in the experiment.

2. 4D System Model

In this paper, a novel 4D chaotic system is proposed. The system equation of motion is as follows:

$$\begin{cases} \dot{x} = -cx + ay + xz + yw \\ \dot{y} = -ax + bw \\ \dot{z} = -x^2 + w^2 \\ \dot{w} = -xy - by - zw + u \end{cases} \tag{1}$$

In formula (1), x, y, z, w are the system state variables, a, b, c are the system adjustable parameters, u is the external input of the system, and all four numbers are positive.

2.1 Dissipation

The divergence of formula (1) is:

$$\nabla V=\frac{\partial \dot{x}}{\partial x}+\frac{\partial \dot{y}}{\partial y}+\frac{\partial \dot{z}}{\partial z}+\frac{\partial \dot{w}}{\partial w}=-c+z-z=-c \qquad (2)$$

The parameter c is positive, so the divergence of system (1) is negative. This indicates the system (1) is a dissipative system.

2.2 Equilibrium point

Let all the left sides of equation (1) are 0, then equation (1) becomes:

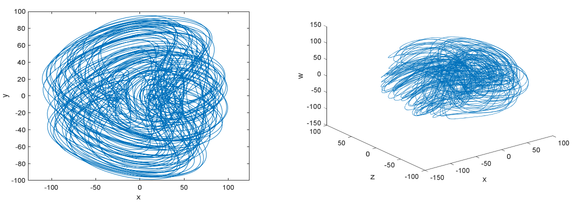
$$\begin{cases}0=-cx+ay+xz+yw\\0=-ax+bw\\0=-x^2+w^2\\0=-xy-by-zw+u\end{cases} \qquad (3)$$

Through formula (3), calculate the equilibrium point of system (1). It is found that there is no equilibrium point in the system. A chaotic attractor generated by a system without equilibrium point is a hidden chaotic attractor.

2.3 Basic dynamic simulation analysis

2.3.1Phase portraits and Poincaré mapping diagram

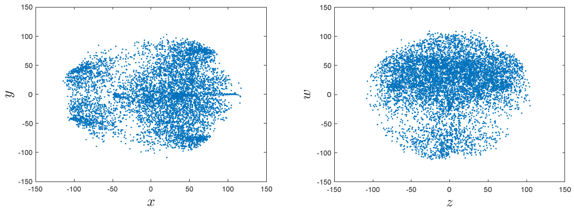
The parameters and external input are a = 50, b = 25, c = 0.47, u = 70, the initial values are (1,1,1,1), the phase portraits of system (1) are shown in Fig.1.



(a) (b)

Fig.1 Phase portraits of system (1) with a = 50, b = 25, c = 0.47, u = 70: (a) $x-y$; (b) $x-z-w$.

As can be seen from Fig.1, chaos is generated in the system with the selected parameters and initial values. In order to further illustrate that the state is chaotic indeed, the Poincaré mapping diagrams under the same parameters and initial values are given. w = 0 and y = 0 are the selected sections, and the results are shown in Fig.2(a) and Fig.2(b).



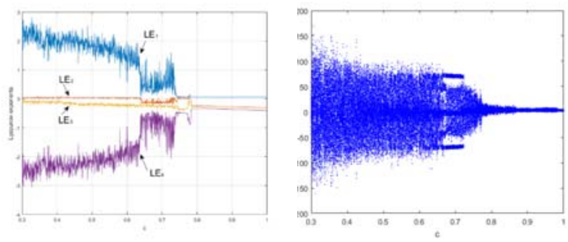
(a) (b)

Fig.2 Poincaré sections of system (1): (a) w = 0; (b) y = 0.

As can be seen from the Poincaré sections, the map of system (1) under these two sections are many points with dense distribution and fractal structure. This indicates the chaotic behavior occurs in this system.

2.3.2 Lyapunov exponent spectrum and bifurcation diagram

The most common way to determine whether a system is chaotic is to determine the number of positive Lyapunov exponent of a system. For a 4D system, if there are two positive Lyapunov exponents, the system is hyperchaotic. If there is one positive Lyapunov exponent, the system is chaotic. As analyzing this chaotic system, the fixed parameter is a = 50, b = 25, u = 70, and the initial values are (1,1,1,1). The Lyapunov exponent spectrum and bifurcation diagram of system (1) changing with the parameter c in the range (0.3,1) are made. The results are as shown in Fig.3(a) and Fig.3(b) respectively.



(a) (b)

Fig.3 (a): Lyapunov exponent spectrum and (b): bifurcation diagram of system (1).

By combining the Lyapunov exponent spectrum and bifurcation diagram, it can be analyzed that when $c\in(0.3,0.74)$, system (1) is chaotic; When $c\in(0.74, 0.783)$, system (1) is quasi-periodic. When $c\in(0.783, 1)$, system (1) is periodic. Select c = 0.47, c = 0.7224, c = 0.76 and c = 0.9 of system (1) in these three ranges respectively. Make the corresponding phase portraits, the results are as shown in Fig.4. When c = 0.47, the four Lyapunov exponents are LE_1 = 2.5001, LE_2 = 0, LE_3

= –0.2192, LE_4 = –2.8070, which accord with the feature of chaotic state. When c = 0.7224, the four Lyapunov exponents are LE_1 = 1.2961, LE_2 = 0, LE_3 = –0.2706, LE_4 = –1.7442, which accord with the feature of chaotic state. When c = 0.76, the four Lyapunov exponents are LE_1 = 0, LE_2 = 0, LE_3 = –0.3503, LE_4 = –0.4908, which accord with the feature of quasi-periodic state. When c = 0.9, the four Lyapunov exponents are LE_1 = 0, LE_2 = –0.2664, LE_3 = –0.3201, LE_4 = –0.3752, which accord with the feature of periodic state. These features indicate that the dynamic behavior of the system is very divers.

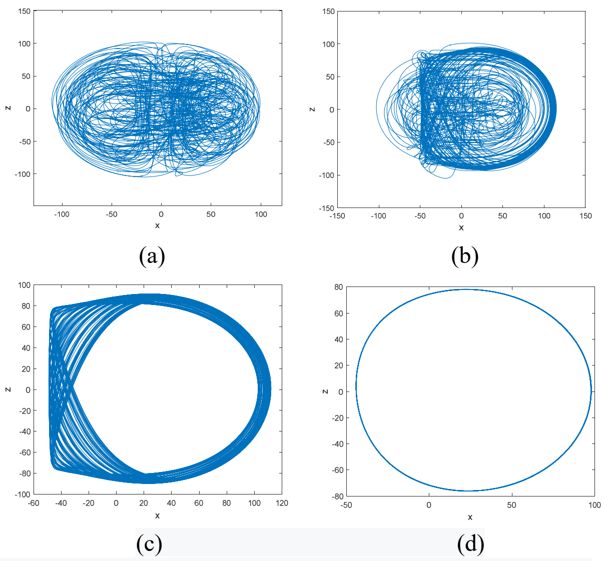


Fig.4 The phase portraits with different values of parameter c: (a) c = 0.47; (b) c = 0.7224; (c) c = 0.76; (d) c = 0.9.

3. Circuit design and implementation

Multisim software is used to design and realize an analog circuit of a system. The selected parameters are a = 50, b = 25, c = 0.47, u = 70, and the initial values are (1,1,1,1). The rated voltage of the power supply in the circuit is $\pm 15V$, so the linear dynamic range of the operational amplifier is $\pm 13.5V$. But the chaotic attractor range of the system is much larger than $\pm 13.5V$, therefore, the spatial proportional compression transformation should be carried out for the system (1). Let:

$$x = 10X, y = 10Y, z = 10Z, w = 10W \quad (4)$$

Put equation (4) into equation (1) and replace X, Y, Z and W with x, y, z and w again, then:

OS15_Xiaoyan Chen (5)

Now the circuit is designed with the improved method. The circuit is standardized in advance, and the parameters are applied into equation (5). Then, equation (5) becomes:

$$\begin{cases} \dot{x} = -0.47x - 50(-y) - 10(-x)z - 10(-y)w \\ \dot{y} = -50x - 25(-w) \\ \dot{z} = -10x^2 - 10(-w)w \\ \dot{w} = -10xy - 25y - 10zw - (-70) \end{cases} \quad (6)$$

In order to observe the phase portraits of system (1) in the oscilloscope, time scale transformation is required. Let $t = \tau_0 T, \tau_0 = 100$ and use t to replace T, so:

$$\begin{cases} \dot{x} = -47x - 5000(-y) - 1000(-x)z - 1000(-y)w \\ \dot{y} = -5000x - 2500(-w) \\ \dot{z} = -1000x^2 - 1000(-w)w \\ \dot{w} = -1000xy - 2500y - 1000zw - (-7000) \end{cases} \quad (7)$$

The circuit model built according to equation (7) is shown in Fig.5. The four parts of the circuit correspond to four first nonlinear differential equations respectively, and the circuit design is realized by using operational amplifiers—LF347N chips, multipliers—AD633 chips, resistors and capacitors [19,20].

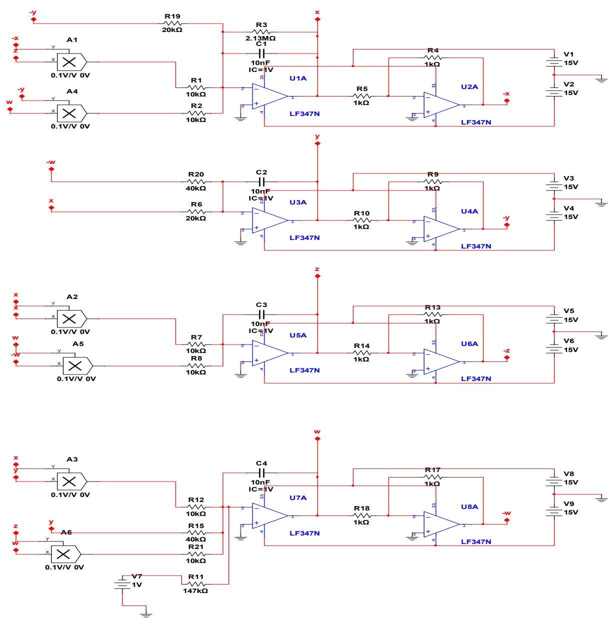


Fig. 5 Circuit diagram of system (1).

The equation of the four states realized by the circuit can be obtained from Fig.5. The result is such as equation (8):

$$\begin{cases} \dot{x} = -\dfrac{1}{R_3C_1}x - \dfrac{1}{R_{13}C_1}(-y) - \dfrac{1}{10R_1C_1}(-x)z \\ \quad -\dfrac{1}{10R_2C_1}(-y)w \\ \dot{y} = -\dfrac{1}{R_6C_2}x - \dfrac{1}{R_{20}C_2}(-w) \\ \dot{z} = -\dfrac{1}{10R_7C_3}x^2 - \dfrac{1}{10R_8C_3}w^2 \\ \dot{w} = -\dfrac{1}{10R_{12}C_4}xy - \dfrac{1}{R_{15}C_4}y - \dfrac{1}{10R_{21}C_4}zw \\ \quad -\dfrac{1}{R_{11}C_4} \end{cases} \tag{8}$$

By equating equation (7) and equation (8), it can be obtained that:

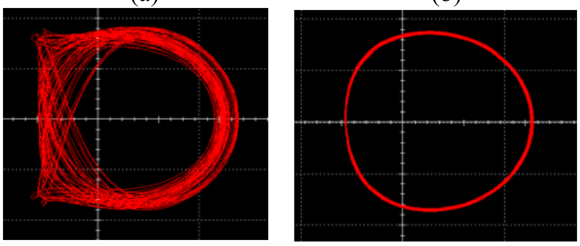
$$\begin{cases} \dfrac{1}{R_3C_1} = 47 \\ \dfrac{1}{R_{13}C_1} = \dfrac{1}{R_6C_2} = 5000 \\ \dfrac{1}{R_{20}C_1} = \dfrac{1}{R_{15}C_2} = 2500 \\ \dfrac{1}{10R_1C_1} = \dfrac{1}{10R_2C_1} = \dfrac{1}{10R_7C_3} = 1000 \\ \dfrac{1}{10R_8C_3} = \dfrac{1}{10R_{12}C_4} = \dfrac{1}{10R_{21}C_4} = 1000 \end{cases} \tag{9}$$

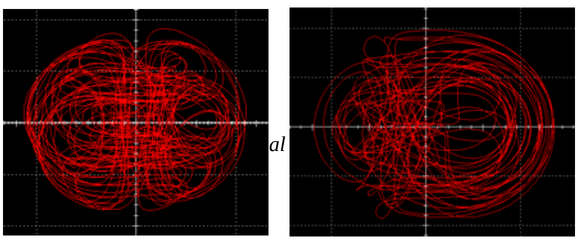
Let $C_1 = C_2 = C_3 = C_4 = 10nF$, then resistance values can be calculated:

$$R_3 = 2.13M\Omega,\ R_{13} = R_6 = 20K\Omega,\ R_{20} = R_{15} = 40K\Omega,$$
$$R_1 = R_2 = R_7 = R_8 = R_{12} = R_{21} = 10K\Omega$$

By the above circuit model design and built, perform circuit simulation experiment with Multisim. When c = 0.47, the phase portrait of system (1) observed on the virtual oscilloscope is shown in Fig.6(a). By the same method, when c = 0.7224, c = 0.76, c = 0.9, the phase portraits of system (1) observed on the virtual oscilloscope are shown in Fig.6(b), Fig.6(c), Fig.6(d) respectively. Compared with Fig.4, it can be seen that the circuit simulation results can consistent with the numerical simulation results.

(a) (b)





(c) (d)

Fig.6 Results of circuit simulation: (a) c = 0.47; (b) c = 0.7224; (c) c = 0.76; (d) c = 0.9.

4. Conclusion

In this paper, an innovative four-dimensional chaotic system is proposed. Theoretical analysis shows that this system is a dissipative system without any equilibrium point. The basic dynamic characteristics of the system are studied by using Lyapunov exponent spectrum and bifurcation diagram and so on. In addition, in order to illustrate the existence of chaotic attractor in the system from the physical level, Multisim software is used to design and implement the analog circuit of the chaotic system. The circuit simulation results are consistent with the numerical simulation results, so the physical realizability of the system is verified. There is no equilibrium point in this system, therefore the chaotic attractor generated by the system is hidden chaotic attractor, which is worthy of further study.

5. References

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