

# Effects of Variable Arm Length on UAV Control Systems

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## Abstract

Quadrotor is a type of unmanned aerial vehicle that has been widely used in many applications, such as, policing, surveillance, aerial photography and agriculture. Conventionally, the control of quadrotor flight direction is accomplished by varying speeds of motors or manipulating torques. In this paper, a novel mechanism is proposed. The mechanism uses stepper motors to control the arm length for changing flight directions, while maintaining motors' speed at constant. A mathematical model has been created. The analysis results have shown that varying arm length can effectively control the moment of bending of quadrotors. Increasing the length of arms can result in the increase of the moment of bending without changing speed of motors, thus saving energies. Experimental results have shown that the new mechanism is able to carry more payloads which the motor speed can be utilized fully at 100% while the flight direction is been controlled by changing of the arm length compared to conventional flight control mechanisms.

*Keywords:* UAV, Quadrotor, Moment of Bending, Arm length

## 1. Introduction

UAV is a short name for "Unmanned Aerial Vehicle" defined as aircrafts without the onboard presence of pilots [1]. UAV is commonly used in military and police forces in situations where the risk in sending a human piloted air craft is unacceptable. In many developed countries, UAV is also used to perform tasks, for example intelligence, surveillance, and reconnaissance missions. Small UAVs can also be used for entertainment industry, such as, aerial filming, aerial photography and others. Most of the UAVs are made in quadrotor due to its easy to design, small rotors, and excellent manoeuvrability [2]. The study requires analyzing the relationship between UAV's motor support bar length and the UAV control systems. Quadrotor is a type of UAV that is lifted and propel by four rotors. The propellers are connected with two pairs of support bars. The flight direction of quadrotor can be controlled by increasing or decreasing the speed of motors [3].

Larger motor required quite high current, which are difficult to control using current or voltage regulating circuitry. Also, power usage of the UAV systems is a major problem to withstand the desired endurance of flight time. A study to manage the power usage is important and a study from fluid mechanic's point of view is much needed. Besides that, flight

movement of quadrotor limits the power of motor. Quadrotor motors cannot operate in 100% power because of another 30% of power is reserved to control for flight direction. Furthermore it is difficult to control the stability and precision of flight movement of quadrotor that powered by fuel or petrol [4][5].

Therefore, the theory is implied to control the quadrotor's direction of flight by increasing or decreasing the length of the support bar. The length of support bar helps in balancing and stabilizing the UAV. The study focus on relation between support bar length and moment of bending of various position of motor. Thus, this study shall be the basis to run the dynamic analysis at the UAV's motor support bar length control systems and also to enhance the UAV's mathematical modelling by using SOLIDWORKS® software, CAD and CAE systems [6].

## 2. Quadrotor Dynamic Model

Quadrotor is an UAV that consist of four motors located at the end of the cross configuration. Each of the motor consists of propeller that generate thrust force for lifting the quadrotor [5]. The front and rear motor rotate counter-clockwise, while the left and the right motor rotate clockwise [7]. The rotations allow to nearly canceling the gyroscopic effects and aerodynamic torques in trimmed flight [8]. The quadrotor is

able to move in yaw, roll, pitch and hover direction with the change of thrust outputs of each propeller. In Figure 1(a), hover movement is obtained when the net thrust of all the rotors is equal to zero. In order to achieve zero net thrust for all rotors, the direction of rotation of rotor 1 and rotor 3 must be in the opposite direction of rotor 2 and rotor 4. Besides that, all the rotors are operated in equal speed. When the speed of motor (1, 2, 3, 4) are increased equally as shown in Figure 1, the quadrotor flies in vertical motion along z-axis according to the thrust output produce due to the propeller rotation. In Figure 1(b), pitch movement is obtained when the torque of motor 3 is increase/decrease and the torque of motor 1 is decrease/increase and keeping constant torque of motor 2 and 4. The quadrotor will pitch forward or backward. In Figure 1(c) roll movement is obtained when the torque of motor 4 is increase/decrease and the torque of motor 2 is decrease/increase and keeping constant torque of motor 1 and 3. The quadrotor will roll right or roll left. In Figure 1(d), yaw movement is obtained when the torque of motor 2 and 4 is increase while decrease the torque of the motor 1 and 3 to move left and vice versa for the right direction. The quadrotor will rotate clockwise or counter-clockwise.

Quadrotor is non-linear system with 6 degree of freedom and contains only 4 motors input [8]. Figure 2 represents the free body diagram and axes of the quadrotor. In Figure 2,  $l$  is the distance of each motor towards the center of the pivot.  $\phi$ ,  $\theta$  and  $\psi$  are representing the Euler angles about the body axes  $x$ ,  $y$ ,  $z$ .  $F_1$ ,  $F_2$ ,  $F_3$  and  $F_4$  are the thrust force produce by the propeller.

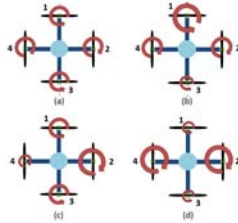


Figure 1. Movement of the quadrotor due to propeller rotation

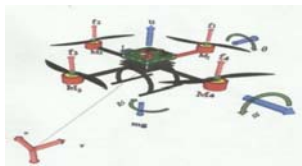


Figure 2. Free body diagram of the quadrotor [4]  
The position and velocity vectors in Earth frame stated as

$$x = [x \ y \ z]^T \quad (1)$$

$$\dot{x} = [\dot{x} \ \dot{y} \ \dot{z}]^T \quad (2)$$

The Pitch, Roll and Yaw angle in body frame stated as

$$\theta = [\theta \ \phi \ \psi]^T \quad (3)$$

$$\dot{\theta} = [\dot{\theta} \ \dot{\phi} \ \dot{\psi}]^T \quad (4)$$

Then,  $R$  indicates the rotational matrix from body to earth frame as

$$R = \begin{pmatrix} c\psi c\theta & -s\psi c\theta + c\psi s\theta s\phi & c\phi c\psi s\theta + s\theta s\psi \\ s\psi c\theta & c\psi c\theta + s\theta s\theta s\phi & c\phi s\psi s\theta - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{pmatrix} \quad (5)$$

In equation (5),  $c$  indicates  $\cos(\ )$  and  $s$  indicates  $\sin(\ )$ . The thrust force and control torque act on the body and produced by the propeller rotation. The vector of thrust moves from Body frame to Earth frame. Applying the Newton-Euler method for rigid body as:

$$m\ddot{x} = FR + [0 \ 0 \ -mg]^T - F_a \quad (6)$$

$$r\dot{w} = w \times rw + \tau \quad (7)$$

$\ddot{x}$  represents the linear acceleration vector,  $F$  is the thrust that produced by the motors,  $F_a$  indicates the frictional force,  $\tau$  is the control torque produced by motors,  $m$  is the mass of the body. Thus, from the equation (6) and (7), quadrotor dynamic equation is derived as:

$$\begin{aligned} m\ddot{x} &= u_1(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) - k_1\dot{x} \\ m\ddot{y} &= u_1(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) - k_2\dot{y} \\ m\ddot{z} &= u_1(\cos\theta\cos\phi) - mg - k_3\dot{z} \\ I_y\ddot{\phi} &= (I_x - I_z)\phi\psi + u_3 - k_5\dot{\phi} \\ I_x\ddot{\psi} &= (I_x - I_y)\theta\phi + u_4 - k_6\dot{\psi} \\ I_x\ddot{\theta} &= (I_x - I_z)\dot{\theta}\psi + u_2 - k_4\dot{\theta} \end{aligned} \quad (8)$$

Where  $I$  is the moment of inertia. The thrust induced variation the speed of the motors is the input vector and stated as:

$$U = [u_1 \ u_2 \ u_3 \ u_4]^T \quad (9)$$

Thus, the input vectors are defined as:

$$\begin{aligned} u_1 &= b(w_1^2 + w_2^2 + w_3^2 + w_4^2) \\ u_2 &= bl(w_3^2 - w_1^2) \\ u_3 &= bl(w_4^2 - w_2^2) \\ u_4 &= d(w_2^2 + w_4^2 - w_1^2 - w_3^2) \end{aligned} \quad (10)$$

Where,  $w$  is the motor speed,  $b$  and  $d$  are thrust coefficient and drag coefficient. The lift force to hover the quadrotor is defined as  $u_1$ ,  $u_2$ ,  $u_3$ , and  $u_4$  are the input torques that locate the quadrotor to the pitch, roll and yaw attitude. In equation (10), the increasing and decreasing of motor speed produce the input torque that oriented toward pitch, roll or yaw attitude.

### 3. Varying Arm Length

Figure 3(a) indicates the pitch movement that is produced by the decreasing the arm length  $l1$  and increasing the arm length  $l3$  and keeping both arm length  $l2$  and  $l4$  length unchanged. Figure 3(b) shows the pitch movement produced when the arm length  $l1$  is increased and the arm length  $l3$  is decreased and kept constant length in both arm length  $l2$  and  $l4$ . Figure 3(c) illustrates the roll movement that is obtained when the arm length  $l2$  is decreased and the arm length  $l4$  is increased while keeping the length of both  $l1$  and  $l3$  fixed. Figure 3(d) shows the roll movement that is obtained when the arm length  $l2$  is increased and the arm length  $l4$  is increased while keeping the length of both  $l1$  and  $l3$  fixed. In Figure 3(e), the yaw movement is accomplished by increasing the arm length  $l2$ ,  $l4$  and decreasing the arm length  $l1$ ,  $l3$ . Thus, the quadrotor tilted in counter-clockwise direction. Figure 3(f) indicates the yaw movement in clockwise direction produced by decreasing the arm length  $l1$ ,  $l3$  and increasing the arm length  $l2$ ,  $l4$ .

Equation (10) that stated earlier explains the input torque produced by the increasing and decreasing the motor speed. According to torque law:

$$\tau = f_i l \quad (11)$$

$$f_i = b w_i^2 \quad (12)$$

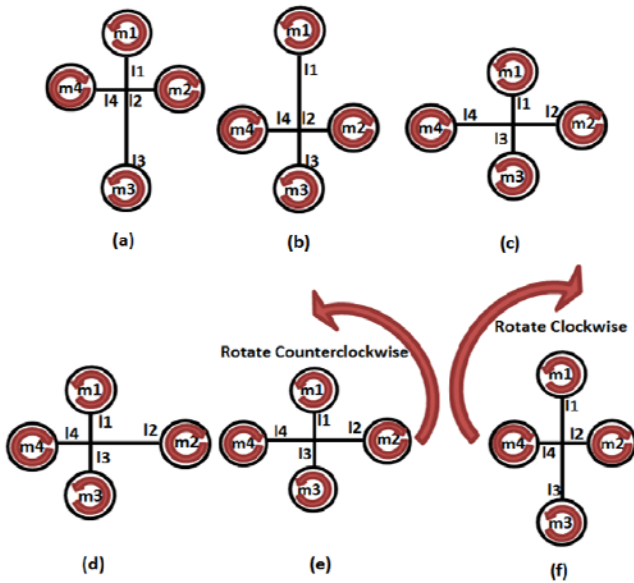


Figure 3. Pitch, roll, yaw moment respect to arm length

The input torque is derived as:

$$\begin{aligned} u1 &= b(w1^2 + w2^2 + w3^2 + w4^2) \\ u2 &= b w_i^2 (l3 - l1) \\ u3 &= b w_i^2 (l4 - l2) \\ u4 &= d w_i^2 (l2 + l4 - l3 - l1) \end{aligned} \quad (13)$$

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Therefore, the pitch, yaw, roll movement of the quadrotor can be controlled by increasing or decreasing arm length to generate the certain input torque. The motor speed is kept at constant where  $w_i = w1 = w2 = w3 = w4$ .

#### 4. Mathematical Analysis of Quadrotor Arm Length

The analysis is conducted by referring KDE data performance. KDEdirect.com provides data performance of specific motor version for its suitable propeller size and voltage. Motor version that suited well with 18.5" diameter and 6.3 mm pitch propeller is KDE5215XF-435(435KV). The data consists of amperage, power input, thrust output, rpm and efficiency for different throttle range between 25% to 100%. The data is obtained from the experiments conducted by the KDE Company.

Table 1. KDE data performance

MOTOR VERSION	VOLTAGE [V]	PROPELLER SIZE	THROTTLE RANGE	AMPERAGE [A] (LOWER IS BETTER)	POWER INPUT [W] (HIGHER IS BETTER)	THRUST OUTPUT [g] (HIGHER IS BETTER)	RPM (HIGHER IS BETTER)	EFFICIENCY [g/W] (HIGHER IS BETTER)	[lb-HP]			
KDE5215XF-435(435KV) KDEXP-EAS7SHVC S.R.ENABLED	14.8V(4S) 16.8V MAX	18.5"×6.3 KDE-CF18S-DP DUAL BLADE	25.00%	2.1	31	0.04	430	4.22	0.95	1980	13.87	22.8
			37.50%	4.6	68	0.09	860	8.43	3.9	2820	12.65	20.79
			50.00%	8.3	122	0.16	1390	13.63	3.06	3600	11.39	18.73
			62.50%	13.6	201	0.27	2010	19.71	4.43	4320	10	16.44
			75.00%	20.6	304	0.41	2710	26.58	5.97	4980	8.91	14.66
			87.50%	29.7	439	0.59	3510	34.42	7.74	5640	8	13.14
100.00%	39.3	581	0.78	4420	43.35	9.74	6240	7.61	12.51			

#### 5. Determining Moment of Bending for Every Motor Speed (RPM)

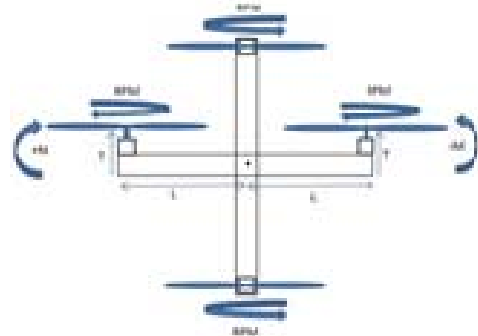


Figure 4. Free body diagram respect to moment thrust and length

Figure 4 shows the free body diagram of the quadrotor. In order to produce hover movement, the equivalent net torque of the quadrotor is equal to zero and all motor spinning in equal speeds. Moment of bending acting on quadrotor arm can be calculated using the formula as shown in equation (14).

$$(14)$$

$$M_i = T_i \times L$$

$M$  is the moment of force,  $T$  is the force or thrust output produce by the propeller rotation and  $L$  is the quadrotor arm length, and  $i=1, 2, 3, 4, 5, 6, 7$ . In the first step of the analysis, there are 2 variables, a fixed variable and a manipulated variable. The fixed variable is the length of the quadrotor arm,  $L$ . The manipulated variable is the thrust output produced from variation motor speeds. The length of the quadrotor arm is 400 mm suitable for 18.5" propeller. Quadrotor hover in vertical direction, when the all four-motor spinning in equal speed, produces the same amount of thrust. Higher speed produces more thrust so that the quadrotor flies higher and vice versa for lower speed. Therefore, the moment of bending can be calculated for every speed of motor provided from KDE data performance.

### 6. Determining The Change in Quadrotor Arm Length

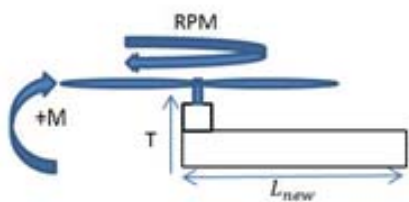


Figure 5: Free body diagram for 1 arm

Finding arm length needed to produce a required moment from previous data using thrust output produce from 3 motor speeds of 3600 RPM, 4320 RPM and 4980 RPM. The constant variable now is change to thrust output produced from a fixed motor speed and the manipulated variable is the moment of bending,  $M_i$ . Their relationship is shown in the equation (15).

$$L_{inew} = M_i / T \tag{15}$$

Where  $i=1,2,3,4,5,6,7$  and  $M$  is the moment of force,  $T$  is the thrust output.

### 7. Assembly Design of The Quadrotor Arm

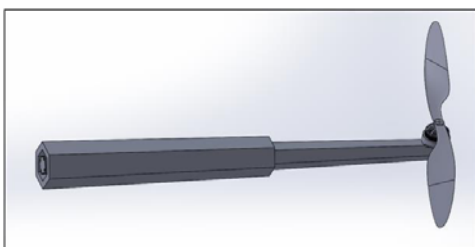


Figure 6. Assembly design of the quadrotor arm

The design requires a stepper motor, a fixed arm, a moving arm, 4 x 18.5" propellers and a brushless DC motor.

### 8. Results

Figure 7 shows the relation of moment of bending against the thrust output. In the diagram, we can see that the moment of bending is increased directly that is proportional to the thrust output. Thrust output produced by the rotation of the propeller is also related to the speed of motor. The increased in motor speed results in higher thrust generated. The minimum value of moment of bending is 1.69 N.m acting on 0.4 m quadrotor arm length, with the thrust output of 4.22 N. The moment of bending increases until it reaches to the maximum value of 17.34 N.m with the thrust output of 43.35 N.

Table 2. Data moment of bending due to various motor speed

MOTOR VERSION	PROPELLER SIZE	THROTTLE RANGE	RPM [rev/min] (HIGHER IS BETTER)	THRUST OUTPUT		ARM LENGTH (m)	MOMENT OF BENDING (N.m)
				[l]	[N] [lb]		
KDE5215XF-435(435kv) KDEXF-UAS7SHVC S.RENABLED	18.5"×4.3 KDE CF18S DP DUAL-BLADE	25.00%	1980	430	4.22	0.95	1.69
		37.50%	2820	860	8.43	1.9	3.37
		50.00%	3600	1390	13.63	3.06	5.54
		62.50%	4320	2010	19.71	4.43	7.88
		75.00%	4980	2710	26.58	5.97	10.63
		87.50%	5640	3510	34.42	7.74	13.77
		100.00%	6240	4420	43.35	9.74	17.34

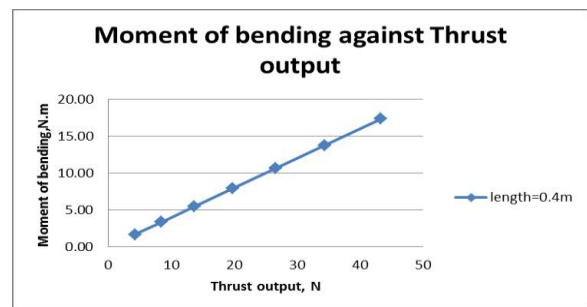


Figure 7. Graph moment of bending against thrust output

### 9. Result of Arm Length for Required Moment of Bending

Figure 8 shows the arm length against moment of bending for 3 different motor speeds 3600 RPM, 4320 RPM and 4980 RPM. In the graph, we can see that the arm length is proportional to the moment of bending. Additionally, the slope showing in the graphs differs from each other. The slope of the motor speed of 3600 RPM is highest compared to 2 other motor speeds, 4320 RPM and 4980 RPM. Motor speeds of 3600 RPM, 4320 RPM and 4980 RPM generate thrust output of 13.63 N, 19.71 N and 26.58 N respectively.

**Table 3:** Data of arm length for required moment of bending (3600 RPM)

MOTOR VERSION	VOLTAGE [V]	PROPELLER SIZE	THRUST[N]	RPM [rev/min]	MOMENT [N.m]	Lnew [m]	
KDE5215XF-435(435Kv)	14.8V(4S)	18.5"× 6.3	13.63	3600	1.69	0.12	
			13.63	3600	3.37	0.25	
			13.63	3600	5.45	0.40	
KDEXF-UAS75HVC	16.8V(MAX)	KDE-CF185-DP	13.63	3600	7.88	0.58	
S.RE.NABLE			DUAL-BLADE	13.63	3600	10.63	0.78
				13.63	3600	13.77	1.01
			13.63	3600	17.34	1.27	

**Table 4:** Data of arm length for required moment of bending (4320 RPM)

MOTOR VERSION	VOLTAGE [V]	PROPELLER SIZE	THRUST[N]	RPM [rev/min]	MOMENT [N.m]	Lnew [m]	
KDE5215XF-435(435Kv)	14.8V(4S)	18.5"× 6.3	19.71	4320	1.69	0.09	
			19.71	4320	3.37	0.17	
			19.71	4320	5.45	0.28	
KDEXF-UAS75HVC	16.8V(MAX)	KDE-CF185-DP	19.71	4320	7.88	0.40	
S.RE.NABLE			DUAL-BLADE	19.71	4320	10.63	0.54
				19.71	4320	13.77	0.70
			19.71	4320	17.34	0.88	

**Table 5:** Data of arm length for required moment of bending (4980 RPM)

MOTOR VERSION	VOLTAGE [V]	PROPELLER SIZE	THRUST[N]	RPM [rev/min]	MOMENT [N.m]	Lnew [m]	
KDE5215XF-435(435Kv)	14.8V(4S)	18.5"× 6.3	26.58	4980	1.69	0.06	
			26.58	4980	3.37	0.13	
			26.58	4980	5.45	0.21	
KDEXF-UAS75HVC	16.8V(MAX)	KDE-CF185-DP	26.58	4980	7.88	0.30	
S.RE.NABLE			DUAL-BLADE	26.58	4980	10.63	0.40
				26.58	4980	13.77	0.52
			26.58	4980	17.34	0.65	

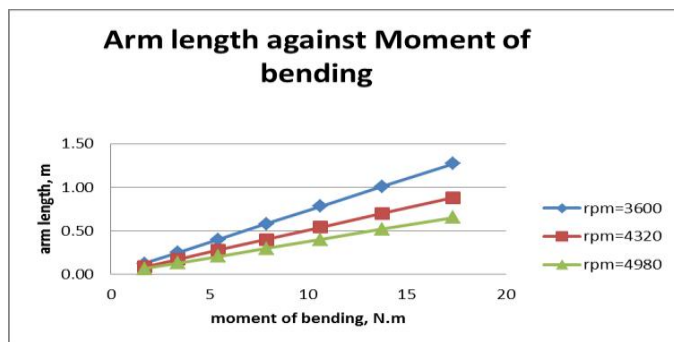


Figure 8. Graph arm length against moment of bending

In order to generate moment of bending of 1.69 N.m to 17.34 N.m with motor speed of 3600 RPM, we need to increase the arm length from 0.12 m to 1.7 m. For 4320 RPM motor speed, the arm length needs to be increased from 0.09 m to 0.88 m. For motor speed of 4980 RPM, the arm lengths needs to be increased from 0.06 m to 0.65 m in order to generate moment of bending of 1.69 N.m to 17.34 N.m. It

shows that lower thrust output or lower motor speed requires longer arms to produce sufficient moment.

## 10. Result of Solidworks Motion Simulation

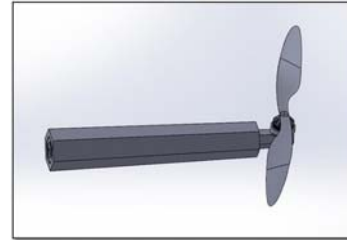


Figure 9: Arm moves inside the fixed arm

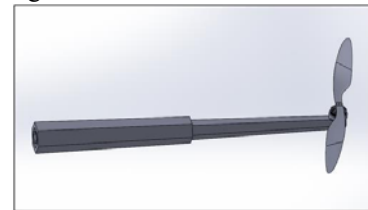


Figure 10: Arm move outside the fixed arm

Figure 9 shows the moving arm moving inside the fixed arm. This occurs when the stepper motor rotates counter-clockwise thus the moving arm moves in negative x-direction. Figure 10 shows that the moving arm is move outside the fixed arm. This motion occurred when the stepper motor rotates clockwise and achieved positive linear motion (forward motion).

## 11. Conclusions

Based on the mathematical analysis conducted, the varying in arm lengths affects the moment of bending of the quadrotor. Increasing the arm length changes the moment of bending of the quadrotor to increase or decrease the arm length will decrease the moment of bending. The speed of motor is kept at constant so that the thrust generated by the propeller rotation is at constant. The analysis also proves that the change in mechanism of the existing UAV's quadrotor that is the quadrotor flight direction can be obtain by increasing or decreasing motors speed. The existing methods have been extended to varying the speed of the motor, the quadrotor flight direction can be controlled by varying the length of the quadrotor's arms.

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