Research on the Synchronization and Circuit Realization of a Four-Wing Chaotic System

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Abstract

This paper introduces a three-dimensional four-wing chaotic system and the system is analyzed and circuit verified. The paper adopts two synchronous control strategies to control the three - dimensional four-wing chaotic system. One is the feedback synchronization controller based on the observer, and the other is the synchronization controller based on theory of stability. The feasibility of these two methods is proved by analyzing the numerical data and the circuit implementation. On this basis, we conducted signal encryption research. The results verify the feasibility of the synchronous design and encryption design.

Keywords: Chaos, Chaotic synchronization, Secure communication, Circuit realization

1. Introduction

The meaning of chaotic secure communication is that through a series of algebraic operations, the signal that needs to be transmitted can be superimposed on a chaotic signal which is generated by a chaotic system. Because of the pseudo randomness of the chaotic signal, the mixed signal will have no regularity, and be similar to the noise signal. The characteristic guarantees the security of the information that it is difficult for eavesdroppers to restore the original information even if they intercept the mixed signal. After receiving by legitimate people, the useful information can be distinguished from the mixed signal through the synchronization between systems of the sender and the recipient.

2. Analysis of four-wing chaotic systems

The mathematical expression of a 3D four-wing chaotic system is as Formula 1:

$$\begin{cases}\dot{x} = ax + ky - yz \\ \dot{y} = -by - z + xz \\ \dot{z} = -x - cz + xy\end{cases} \quad (1)$$

When the system variables are selected as b=12, c=5, and k=1, we get the bifurcation graphs as shown in Fig.1. The research of encryption and chaotic communication should be based on the chaotic state of the system, so the selected parameters are a=5, b=12, c=5, k=1.

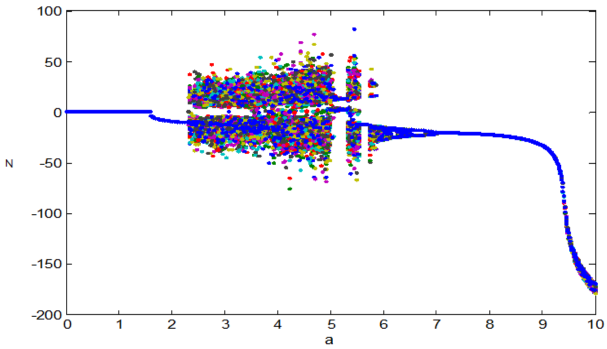


Fig.1. Bifurcation graphs of four-wing chaotic systems

When the parameters are a=5, b=12, c=5, and k=1, the x-y-z plane phase diagram, the x-y plane phase diagram and the y-z plane phase diagram of the system are shown in the following Fig.2.

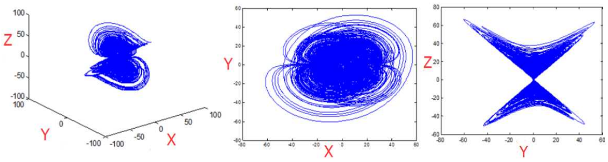


Fig.2. plane phase diagram

There are two kinds of synchronization control of chaotic systems proposed in this paper. The first is feedback synchronization controller based on the observer, and the second is the controller based on stability theory.

3. Design of synchronization system

3.1. The 1st method of synchronization control

When the parameter value is (a, b, c, k) = (5, 12, 5, 1), the observer of drive system is Formula 2.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{z}_1 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 0 \\ 0 & -12 & -1 \\ -1 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + \begin{bmatrix} -y_1 z_1 \\ x_1 z_1 \\ x_1 y_1 \end{bmatrix} + C \quad (2)$$

Set

$$D = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (3)$$

Because A-D matrix has only negative real parts, according to Routh criterion, it is a Hurwitz stable matrix.

According to formula 4, and corresponding to formula 2, the synchronization system can be get as Formula 5.

$$\frac{d_{x_2}}{d_t} = Ax_2 + BF(x_2) + C + s(x_1) - s(x_2) \quad (4)$$

$$\begin{cases} \dot{x}_2 = 10x_1 - 5x_2 + y_2 - y_1 z_1 \\ \dot{y}_2 = -12y_2 - z_2 - x_1 z_1 \\ \dot{z}_2 = -x_2 - 5z_2 + x_1 y_1 \end{cases} \quad (5)$$

The following is the simulation of this design. Set the time of simulation is [0 - 5], the time step is 0.0001, X0 is (3, -6, 7), Y0 is (3, 0, 0). the result of the MATLAB simulation is shown in Fig.3. Through synchronous timing diagram, it can be observed that although the initial value is different, the drive system and the synchronization system have been fully synchronized as the time goes by.

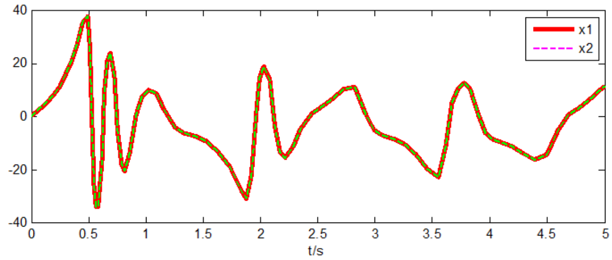


Fig.3. synchronization diagram of The first method

3.2. The 2nd method of synchronization control

The driving system for the four wing chaotic system is shown in Formula 2. Its response system is as Formula 6.

$$\begin{cases} \dot{x}_2 = 5x_2 + y_2 - y_2 z_2 + U_1 \\ \dot{y}_2 = -12y_2 - z_2 + x_2 z_2 + U_2 \\ \dot{z}_2 = -x_2 - 5z_2 + x_2 y_2 + U_3 \end{cases} \quad (6)$$

Ui (i=1, 2, 3) is the controller. The error variables between the response system and the drive system are as Formula 7.

$$e_1 = x_2 - x_1,\ e_2 = y_2 - y_1,\ e_3 = z_2 - z_1 \quad (7)$$

Construct Lyapunov exponent and its derivative can be obtained as Formula 8.

$$\dot{V} = \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 \quad (8)$$

When $\dot{V}$ is always negative, through Lyapunov stability theory, the error system is globally asymptotically stable at the origin. In this case, the system as shown in Formula 2 is synchronized to the system as shown in Formula 6.

Through algebraic calculation, a proper controller can be constructed to synchronize the two systems completely. The controller in the system is selected as Formula 9. It is easy to prove that is always negative.

Through the Formula 9 and the Formula 6, the response system can be obtained as Formula 10.

$$\begin{cases} U_1 = -10e_1 - e_2 e_3 - e_2 \\ U_2 = 0 \\ U_3 = e_1 + e_2 - 2x_1 e_2 \end{cases} \quad (9)$$

$$\begin{cases} \dot{x}_2 = -5x_2 - 2y_2 z_2 + 10x_1 + y_1 + y_2 z_1 + y_1 z_2 + y_1 z_1 \\ \dot{y}_2 = -12y_2 - z_2 + x_2 z_2 \\ \dot{z}_2 = -5z_2 + y_2 - y_1 - x_1 + x_2 y_2 + 2x_1 y_1 - 2x_1 y_2 \end{cases} \quad (10)$$

Matlab simulation results are as Fig.4, and the related parameters are the same as those in the previous.

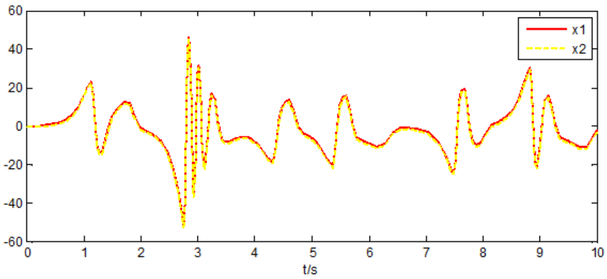


Fig.4. synchronization diagram of The second method

4. Circuit design of four wing chaotic system

The first synchronous control method mentioned above is feedback synchronization controller based on the observer.

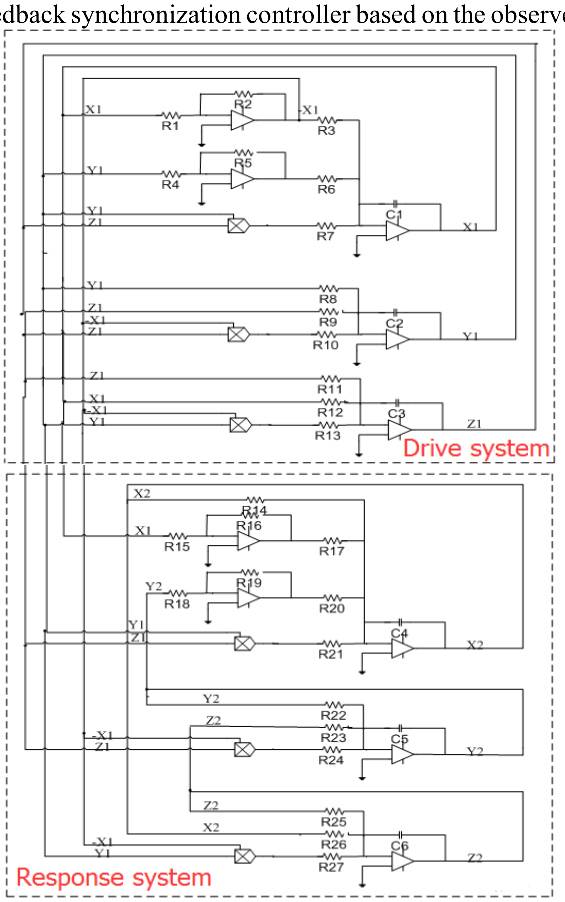


Fig.5. synchronization diagram of The second method

According to the response system shown in Formula 5 and the drive system and the Formula 2, the communication circuit is designed as shown in Fig.5. It is composed of operational amplifiers LF347, analog multipliers AD633, resistors and capacitors to perform additive, subtraction and multiplication operations.

The values of some basic components in this circuit are as follows:

R1=R2=R4=R5=R16=R18=R19=10kOhm,
R6=R9= R12=R20=R23=R26=1000kOhm,
R3=R11=R13=R14= R25=200kOhm,
R8=R22=83.3kOhm,
R7=R10=R17= R21=R24=R27=100kOhm,
C1=C2=C3=C4=C5=C6 =10nF.

The second synchronous control method mentioned above is the controller based on stability theory. According to the response system shown in formula 10 and the drive system and the formula 2, the communication circuit is designed as shown in Fig.6.

The values of some basic components in this circuit are as follows:

R14=R28=200kOhm,
R4=R28=200kOhm,
R18=R19 =R23=R32=100kOhm,
R15=R20=R21=R22=R26=R27 =100kOhm,
R7=R17=R33=R34=50kOhm,
R16=R25 =R29=R30=R31=1000kOhm,
R24=83.3kOhm,
C1=C2 =C3=C4=C5=C6=10nF.

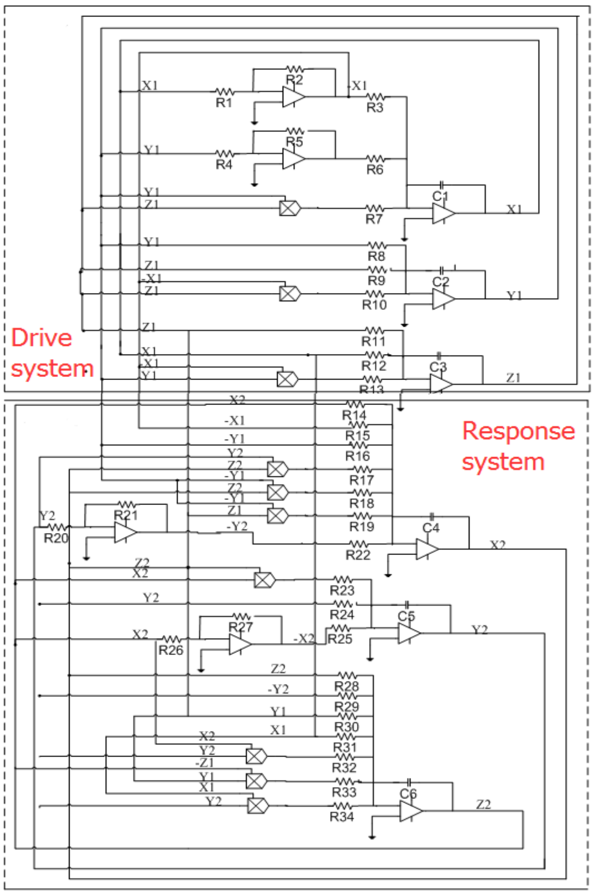
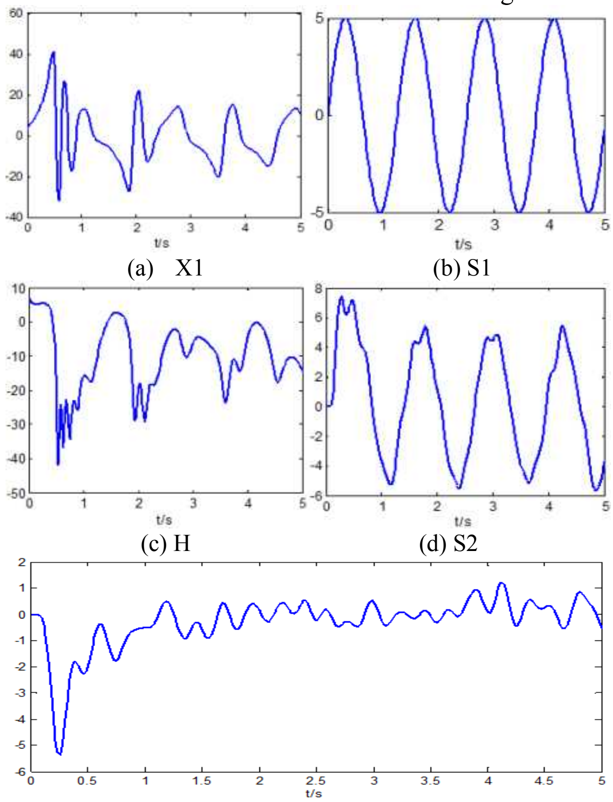


Fig.6. synchronization diagram of The second method

5. Testing and conclusion

We set the original sine signal as S1, the decrypted signal as S2, the mixed signal as H, the signal produced by the drive system as X1, and the signal error as deviation. The initial value X0 is (3, -6, 7), Y0 is (3, 0, 0), and the simulation time is (0s - 5s).

The simulation result of feedback synchronization controller based on the observer is shown as Fig.7.



(a) X1 (b) S1

(c) H (d) S2

(e) Deviation

Fig.7. simulation result of The first method

The simulation result of the controller based on stability theory is shown as Fig.8.

We set the original sine signal as S1, the decrypted signal as S2, the mixed signal as H, the signal produced by the drive system as X1, and the signal error as E. The initial value X0 is (3, -6, 7), Y0 is (3, 0, 0), and the simulation time is (0s - 10s).

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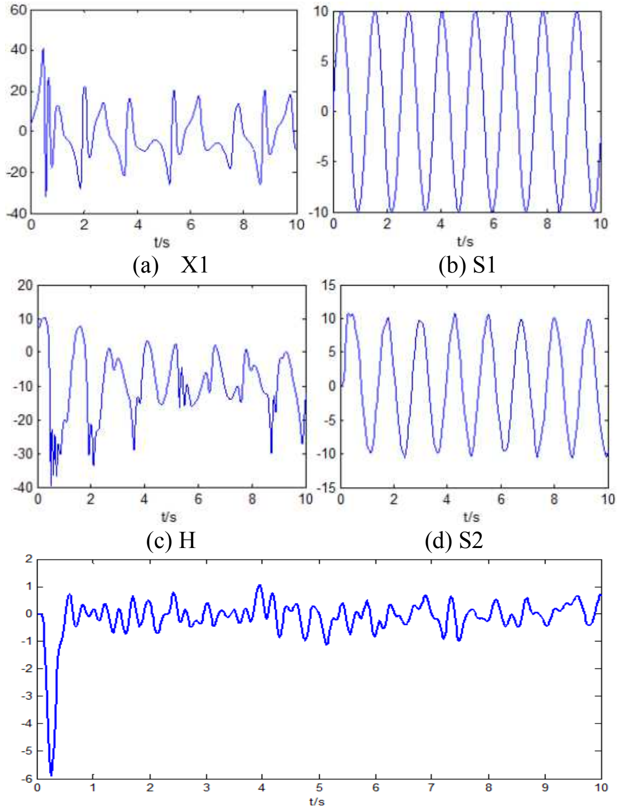

(a) X1 (b) S1

(c) H (d) S2

(e) Deviation

Fig.8. simulation result of The second method

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