An efficient structure of organization with complete individual guidance

Mamoru Yoshimura[*]
Graduate School of Engineering, University of Miyazaki
E-mail: hm13056@student.miyazaki-u.ac.jp

Tsutomu Ito[†]
Graduate School of Engineering, Hiroshima University
E-mail: 0va71-2538f211n@ezweb.ne.jp

Makoto Sakamoto*
Department of Computer Science and System Engineering, Miyazaki University
E-mail: sakamoto@cs.miyazaki-u.ac.jp

Takao Ito[†]
Graduate School of Engineering, Hiroshima University
E-mail: itotakao@hiroshima-u.ac.jp

Yuji Shinano[‡]
Zuse Institute Berlin
E-mail: shinano@zib.de

Satoshi Ikeda*
Department of Computer Science and System Engineering, Miyazaki University
E-mail: bisu@cs.miyazaki-u.ac.jp

Abstract

In the previous paper, we reported on the efficient organizational shape under a setting called complete group guidance. In this paper, we investigate the shape of an efficient organization in the case of organizational communication in general, and report the existence of shapes that have not seen so far.

Keywords: Combinatorial Optimization, Efficient structure of organization, Weight policy, Complete guidance

1. Introduction

Recently, a mathematical model that can quantitatively evaluate the structure of the organization was proposed [1, 2, 3]. The basic idea of this model is as follows.

1) Represent a hierarchical organization as a rooted tree with its members as their nodes.
2) The effort of each member of the organization is represented as input / output working in the direction of the leaf node.
3) The output of each member is classified as internal contribution which is the effort to maintain the organization and external contribution which is the effort spent on achieving the purpose of the organization.
4) The evaluation value of the organization structure is the sum of external contributions of each member in the organizational structure.

In the paper [1,2], if the organization has only one evaluation measure, the hierarchical organization structure that maximizes the evaluation value is classified

[*] 1-1 Gakuen Kibanadai, Miyazaki, 889-2192, Japan
[†] 1-4-1 kagamiyama, Higashi-Hiroshima, 739-8527, Japan
[‡] Takustraße 7 14195 Berlin-Dahlem, Germany

into the three types in Fig. 1 according to the capacity value of the member.

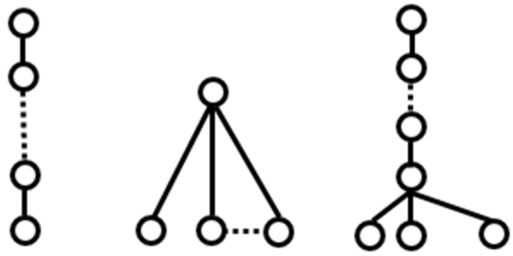

Fig. 1. 3 types of efficient tree

Any of these three have an undifferentiated organizational structure. This result means that the reason why modern organization are divided into multiple divisions is that each division shares a task consisting of different values.

Then in the paper [3], we modified the mathematical model of the paper [1, 2]. Major improvements are follows.

1) Generalization of organization's communication style.
2) Categorize the contribution of each member to internal and external.

The setting of the paper [1, 2] corresponds to the case where the form of communication is a complete group guidance and the external contribution other than the leaf node is set to 0.

In this paper, we report on the most efficient organizational shape of generalized communication style (external contribution is 0).

2. Mathematical Model

Suppose that $G = (V(G), E(G))$ is a graph. Throughout this paper, a graph is a finite undirected simple graph with order $n = |V(G)|(n > 2)$ and size as $m = |E(G)|$. For $u \in V(G)$, by $N(u) = \{v|\{u, v\} \in E(G)\}$, we denote the set of vertices adjacent to u, and call $\deg(u) = |N(u)|$ the degree of $u \in V(G)$.For an arbitrary fixed rooted tree T with its root r and leaves $L(T)$, we denote

$$\deg^*(s) = \begin{cases} \deg(s) & if\ s \in \{r\} \cup L(T_r), \\ \deg(s) - 1 & \text{otherwise.} \end{cases}$$

When $x_0x_1 \dots x_n$ is the path on the tree graph T, we denote

$$T(x_0, x_n) \equiv x_0x_1 \dots x_n.$$

Writing $x \prec y$ for $x \in T_r(r, y)$, we then define a partial ordering on $V(T_r)$, the tree-order associated with T_r.

Suppose that S and $\mathcal{A}(|\mathcal{A}| \geq 1)$ are finite sets. Throughout this paper, S is interpreted as the set of members of a given organization, which consists of $s_0, s_1, \dots, s_n$. And $\mathcal{A}$ is the set of the evaluation measures.

We denote the set of rooted tree graphs with S as vertex set as $\mathcal{T}(S)$, and let each tree graph in $\mathcal{T}(S)$ correspond as an organization tree.

For a given S, we call $(S, \{\phi_i\}_{i \in A})$ an evaluation system if

$$\phi_i: S \to \mathbb{R}^+ \equiv \{x \in \mathbb{R} | x > 0\}\ for\ i \in \mathcal{A}.$$

We call $\phi_i(s)$ *the personal ability of* $s \in S$ *with respect to an evaluation measure* $i \in \mathcal{A}$.

In order for an organization to achieve its purpose to aim at, it is also necessary that appropriate instructions are transmitted to subordinates from superiors. Thus, for a fixed organization tree T_r with $V(T_r) = S$, we considered that the output of s is determined as the interaction of "ability value of subordinate $\phi(s)$" and "accuracy of instruction from superior".

For the subordinate $s \in S$ who received instructions from his superior, it is necessary to transmit appropriate instructions to his own subordinates as superior, while s as subordinate carries out the instructions. For a given organizational structure tree T_r, we assume that the value of the input for subordinate $x \succ s$ with $x \in N(s)$ is obtained by multiplying its weight w_{sx}^i to appropriate instructions to his/her own subordinates as superior.

Weight range is

$$0 \leq w_{sx}^i \leq 1,$$
$$1 \leq \sum_{x \succ s, x \in N(s)} w_{sx}^i \leq \deg^*(s).$$

Let us set

$$\sum_{x \succ s, x \in N(s)} w_{sx}^i = 1.$$

This case corresponds to the organization model which $s \in S$ as superior instruct his/her subordinates individually. The style of communication at this time is called complete individual guidance.

Also,

$$\sum_{x \succ s, x \in N(s)} w_{sx}^i = \deg^*(s) \Leftrightarrow w_{sx}^i = 1.$$

This case corresponds to the organization model which $s \in S$ as superior complete his/her indication to all subordinates with only one instruction.

Throughout this paper, we assume that $\{w_{sx}\}_{x \succ s, x \in N(s)}$ for $s \in S \backslash L(T_r)$ is a sequence depends only on $\deg^*(s)$ and $i \in \mathcal{A}$.the selection that we can do is which weight to assign whom. We call the way of determination of *a weights' policy*. For any weights'

policy, we assume that if $\deg^*(s) \geq \deg^*(s')$ for $s, s' \in S$,

$$w_{sx_1} \geq w_{sx_2} \geq \cdots \geq w_{sx_{\deg^*(s)}}$$

and

$$w_{s'x'_1} \geq w_{s'x'_2} \geq \cdots \geq w_{s'x'_{\deg^*(s')}},$$

then $\{w_{sx}\}_{x>s,x\in N(s)}$ and $\{w_{s'x'}\}_{x'>s,x'\in N(s')}$ satisfy

$$w_{sx_j} \leq w_{s'x'_j}\big(j = 1,2,\dots,\deg^*(s')\big).$$

We will evaluate the rooted tree T_r as organization model by

$$\Phi(T_r) = \sum_{i\in\mathcal{A}} \sum_{l\in L(T_r)} \sum_{s\in T_r(r,l)} w^i_{p(s)s}\phi_i(s).$$

Here, $p(s)$ represents the parent node of $s \in S$, and for convenience, $w^i_{p(r)r} = 1$. We call $\Phi(T_r)$ the ability value of T_r with respect to $(S, \{\phi_i\}_{i\in A})$ for $\{w_{sx}\}_{s,x}$.

3. Results

Let us set $S = \{1,2,3,4,5,6,7\}$, and the interpretation of the members' ability values is shown in Table 1.

Table 1. Interpretation of the members' ability value

ability value	Interpretation.
Higher than 2	High
1 or more and 2 or less	Middle
Low than 1	Low

We considered three different settings of Setting 1 and Setting 2 and Setting 3.

3.1. *Setting 1*

The setting is

$$|\mathcal{A}| = 1,$$
$$w_{sx} = \frac{1}{\deg^*(s)},$$
$$\phi(1) = \phi(2) = \phi(3) = 2.0,$$
$$\phi(4) = \phi(5) = 1.2,$$
$$\phi(6) = \phi(7) = 0.1.$$

This case is a complete individual guidance. A superior instruct his/her subordinates individually. An efficient tree is Fig.2. Evaluation value is 6.42. Fig.2 is efficient shapes not found in Fig. 1.

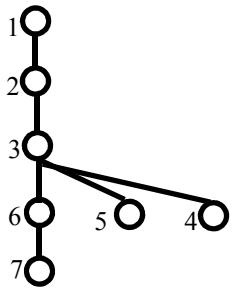

Fig. 2. One of the efficient tree in complete individual guidance in $|\mathcal{A}| = 1$.

3.2. *Setting 2*

The setting is

$$|\mathcal{A}| = 1,$$
$$\{w_{sx}\}_{x>s,x\in N(s)} = \{1.0, 1.0/2.0, \dots, 1.0/\deg^*(s)\},$$
$$\phi(1) = \phi(2) = 2.0,$$
$$\phi(3) = \phi(4) = \phi(5) = 1.2,$$
$$\phi(6) = \phi(7) = 0.1.$$

This case is a general weight policy setting. If $\deg^*(s) = 4$, $\{w_{sx}\}_{x>s,x\in N(s)}$ is $\{1.0, 1.0/2.0, 1.0/3.0, 1.0/4.0\}$. An efficient tree is Fig.3.

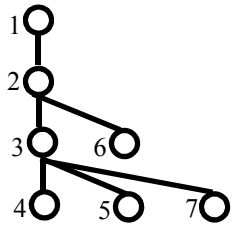

Fig. 3. One of the efficient tree in general communication style in $|\mathcal{A}| = 1$.

Evaluation value is 9.36, the weight is

$$w_{12} = 1.0, w_{23} = 1.0, w_{26} = 0.5,$$
$$w_{34} = 1.0, w_{35} = 0.5, w_{37} = 0.33.$$

Fig.3. is efficient shapes not found in Fig. 1.

3.3. *Setting 3*

The setting is

$$\mathcal{A} = \{a, b, c\},$$
$$\{w_{sx}\}_{x>s,x\in N(s)} = \{1.0, 1.0/2.0, \dots, 1.0/deg^*(s)\},$$
$$\phi_a(1) = 2.0, \phi_b(1) = 2.0, \phi_c(1) = 2.0,$$
$$\phi_a(2) = 2.0, \phi_b(2) = 0.2, \phi_c(2) = 1.2,$$
$$\phi_a(3) = 1.2, \phi_b(3) = 2.0, \phi_c(3) = 0.2,$$
$$\phi_a(4) = 1.2, \phi_b(4) = 2.0, \phi_c(4) = 0.2,$$
$$\phi_a(5) = 1.2, \phi_b(5) = 2.0, \phi_c(5) = 0.2,$$
$$\phi_a(6) = 0.2, \phi_b(6) = 0.2, \phi_c(6) = 2.0,$$

$$\phi_a(7) = 0.2, \phi_b(7) = 0.2, \phi_c(7) = 2.0.$$

This case is a general weight policy setting in the same Setting 2. An efficient tree is Fig. 4.

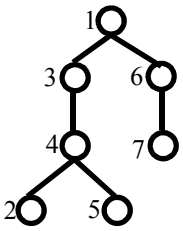

Fig. 4. One of the efficient tree in general communication style in $|\mathcal{A}| = 3$.

Evaluation value is 27.28, the weight is

$$w_{13} = 1.0, w_{216} = 0.5, w_{34} = 1.0,$$
$$w_{42} = 0.5, w_{45} = 1.0, w_{67} = 1.0.$$

Fig. 4. has a differentiated organizational structure which is two divisions.

4. Conclusion

Based on section 3, we found that the general shape of the efficient organization tree when the evaluation measure is only one is Fig. 5.

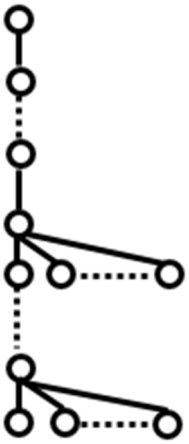

Figure. 5. General shape of an efficient tree.

This means that it is generally shown that there is an efficient organization tree different from the efficient organization tree shown in the paper [1, 2].

Furthermore, from this result, it is inferred that the general efficient tree shape in the evaluation system with multiple evaluation criteria will be as shown in Fig. 6.

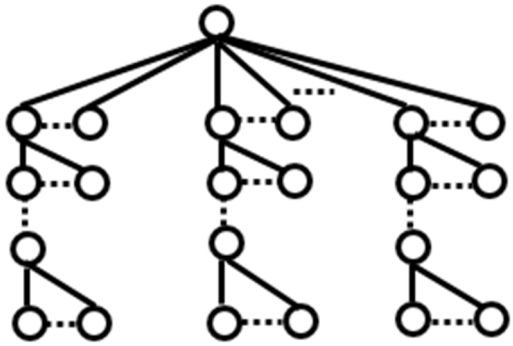

Figure. 6. General form of an efficient tree in the evaluation system with multiple evaluation criteria.

References

1. S.Ikeda, T.Ito and M.Sakamoto,"Discovering the efficient organization structure: horizontal versus vertical", *Artificial Life and Robotics*, vol.15, 4, pp.478-481, 2011.
2. S.Ikeda, "An Efficient Structure of Organization", *International Interdisciplinary Workshop on Robotics, Ecosystem, and Management*, pp.72-78, 2010.
3. S.Ikeda, T.Ito, M.Sakamoto, T.Ito,"An Evaluation Model of Internal and External Contributions in Hierarchical Organizations", I*nternational Journal of Mathematical Model and Methods in Applied Sciences*, Vol.9, pp.636-645,2015.
4. Reinhard Diestel, *"Graph Theory(Third Edition)"*, Springer, 2000.
5. Alvesson Mats, "A Flat Pyramid: ASymbolic Processing of Organizational Structure ", *International Studies of Management and Organization*, 19-4,5-23,1989/1990.