A Study on Differential Evolution Using BetaCOBL, B^{3}R, and TPBO

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Contents

- Introduction
- Background
- Proposed beta utilization
- Application
- Conclusion

Introduction

- Evolutionary algorithm
  - Mimic biological evolution in nature
  - Not require any prior knowledge about the problem
  - Start with a randomly distributed population and updates the population by reproducing offspring with unique operators
  - Genetic algorithm (GA), evolutionary strategy (ES), evolutionary programming (EP), estimation of distribution algorithm (EDA), differential evolution (DE)

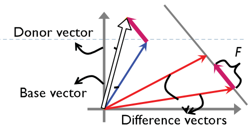
Introduction

- Opposition-based learning (OBL)
  - Estimates and counter estimates are considered simultaneously to accelerate the search or learning process
  - The probability that the opposite point is closer to the solution is higher than probability of a second random guess
- A randomness is added to the algorithm
  - Accelerate the search process
  - Make the algorithm robust
  - Escape the local optima
  - Gaussian, Cauchy, and uniform distributions are most widely used

Introduction

- A beta distribution
  - Advantages
    - Has a bounded input domain
    - Has various shapes depending on the parameter values
- Beta distribution + OBL
  - Control degree of opposition with various shapes
- Beta distribution + Reproduction
  - Offspring reproduction on a bounded search space

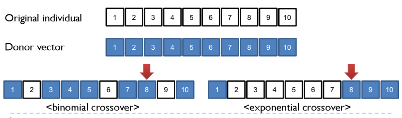
Background



- DE
  - Parent selection
    - Base vector
    - Difference vector
  - Mutation
    - Donor vector: perturb a base vector using a difference between difference vectors
      - Rand/1 $v_{i,j}(t) = x_{r_1,j}(t) + F(x_{r_2,j}(t) - x_{r_3,j}(t))$
      - Rand/2 $v_{i,j}(t) = x_{r_1,j}(t) + F(x_{r_2,j}(t) - x_{r_3,j}(t)) + F(x_{r_4,j}(t) - x_{r_5,j}(t))$
      - Best/1 $v_{i,j}(t) = x_{best,j}(t) + F(x_{r_1,j}(t) - x_{r_2,j}(t))$
      - Best/2 $v_{i,j}(t) = x_{best,j}(t) + F(x_{r_1,j}(t) - x_{r_2,j}(t)) + F(x_{r_3,j}(t) - x_{r_4,j}(t))$
      - Current-to-best/1 $v_{i,j}(t) = x_{i,j}(t) + K(x_{best,j}(t) - x_{i,j}(t)) + F(x_{r_1,j}(t) - x_{r_2,j}(t))$
      - Current-to-rand/1 $u_{i,j}(t) = x_{i,j}(t) + K(x_{r_1,j}(t) - x_{i,j}(t)) + F(x_{r_2,j}(t) - x_{r_3,j})(t)$

Background

- DE-cont'd
  - Crossover
    - Trial vector: mix an original with its donor vector
      - Binomial $u_{i,j}(t) = \begin{cases} v_{i,j}(t), & \text{if } rand(0,1) \le CR \text{ or } j = j_{rand} \\ x_{i,j}(t), & \text{otherwise.} \end{cases}$
      - Exponential $u_{i,j}(t) = \begin{cases} v_{i,j}(t), & \text{if } j = \langle n \rangle_D, \langle n+1 \rangle_D, \ldots, \langle n+L-1 \rangle_D \\ x_{i,j}(t), & \text{otherwise.} \end{cases}$



Background

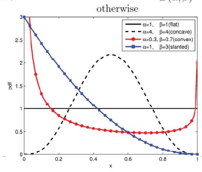
- DE-cont'd
  - Entire procedure

```
Algorithm 2 DE
Initialize population
while !(Termination condition meets) do
    for all individuals in population do
        Select parents
        Generate a donor vector
        Mix the donor with the original individual to get a trial vector
        Evaluate the trial vector
        if The trial is better then
            Replace the original with the trial
        end if
    end for
end while
```

Background

- Beta distribution
  - Parameterized by two parameters α, β
  - Probability density function

$$f(x;\alpha,\beta) = \begin{cases} \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases} \qquad B(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1} dx$$



Background

- Beta distribution-cont'd
  - Mean

$$mean = \frac{\alpha}{\alpha+\beta}$$

  - Mode

$$mode = \frac{\alpha-1}{\alpha+\beta-2}$$

  - Variance

$$variance = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

Background

- OBL
  - Opposite number

$$\breve{x} = a+b-x$$

  - Opposite point

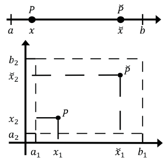
$$P(x_1, x_2, \ldots, x_D)$$

$$\breve{P}(\breve{x}_1, \breve{x}_2, \ldots, \breve{x}_D)$$

$$\breve{x}_j = a_j + b_j - x_j$$

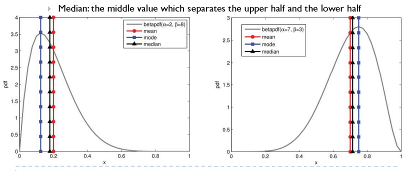
  - OBL

The learning continues with x if $g(f(x)) \geq g(f(\breve{x}))$; otherwise, it continues with $\breve{x}$.



Proposed beta distribution utilization

- Beta distribution using peak and spread
  - Difference between mean, mode, and median
    - Mean: the sum of all values divided by the number of values (cardinality)
    - Mode: the most frequent value
    - Median: the middle value which separates the upper half and the lower half



Proposed beta distribution utilization

- Beta distribution using peak and spread-cont'd
  - Parameter calculation from peak and spread
    - New parameters: *peak* and *spread*

when $mode < 0.5$	when $mode \geq 0.5$
$\alpha = spread \cdot peak$	$\alpha = spread$
$\beta = spread$	$\beta = spread \cdot peak$

given $mode$ and $spread$

$$peak = \frac{(spread-2)mode+1}{spread(1-mode)} \qquad peak = \frac{2-spread}{spread} + \frac{spread-1}{spread \cdot mode}$$

Proposed beta distribution utilization

Theorem 1. *If* $\boldsymbol{spread < 1}$, *then* $\boldsymbol{\alpha < 1}$ *and* $\boldsymbol{\beta < 1}$

If $\boldsymbol{spread > 1}$, *then* $\boldsymbol{\alpha > 1}$ *and* $\boldsymbol{\beta > 1}$

When $mode < 0.5$, the value of $peak$ for given $spread$ can be rearranged as follows:

$$peak = \frac{1-2mode}{(1 \quad mode)spread} + \frac{mode}{1 \quad mode} \tag{4.4}$$

Because $mode < 0.5$, the value of $peak$ depending on $spread$ is monotonically decreasing when $spread \in [0\ \infty)$, so $peak > 0$ when $spread \in [0\ \infty)$.
If $peak$ is subtracted from $1/spread$, the result can be expressed as follows:

$$\frac{1}{spread} - peak = \frac{mode(spread-1)}{(mode-1)spread} \tag{4.5}$$

In Eqn. 4.5, $mode/(mode-1)$ is always negative, whereas $(spread-1)/spread$ is negative when $spread \in [0\ 1]$ and positive when $spread \in [1\ \infty)$. Hence, $peak$ is always smaller than $1/spread$ when $spread \in [0\ 1]$ and greater than $1/spread$ when $spread \in [1\ \infty)$. Therefore, the parameter $\alpha(= spread \cdot peak)$ is always smaller than 1 when $spread \in [0\ 1]$ and greater than 1 when $spread \in [1\ \infty)$. Furthermore, the parameter $\beta(= spread)$ follows the same pattern.

Proposed beta distribution utilization

Similarly, when $mode \geq 0.5$, the value of $peak$ for given $spread$ can be rearranged as follows:

$$peak = \frac{1-mode}{mode} + \frac{2mode-1}{mode \cdot spread} \tag{4.6}$$

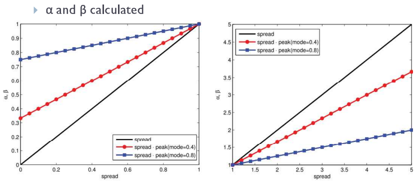
Because $mode \geq 0.5$, the value of $peak$ depending on $spread$ is monotonically decreasing when $spread \in [0\ \infty)$, so $peak > 0$ when $spread \in [0\ \infty)$.
If $peak$ is subtracted from $1/spread$, the result can be expressed as follows:

$$\frac{1}{spread} - peak = \frac{(mode-1)(spread-1)}{mode \cdot spread} \tag{4.7}$$

In Eqn. 4.7, $(mode-1)/mode$ is always negative, whereas $(spread-1)/spread$ is negative when $spread \in [0\ 1]$ and positive when $spread \in [1\ \infty)$. Hence, $peak$ is always smaller than $1/spread$ when $spread \in [0\ 1]$ and greater than $1/spread$ when $spread \in [1\ \infty)$. Therefore, the parameter $\beta(= spread \cdot peak)$is always smaller than 1 when $spread \in [0\ 1]$ and greater than 1 when $spread \in [1\ \infty)$. Furthermore, the parameter $\alpha(= spread)$ follows the same pattern.

Proposed beta distribution utilization

- Beta distribution using peak and spread
  - α and β calculated



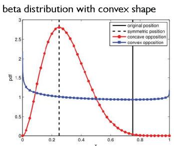
spread<1 *spread*>1

Proposed beta distribution utilization

- Existing OBLs
  - Stochastic calculation using a **uniform** distribution
  - **All of the dimensions are changed** to the opposite values together
  - **The best individuals** among the original individuals and their opposites are passed on to the next generation
- BetaCOBL: OBL using a beta distribution with changes in partial dimensions and selection scheme
  - Stochastic calculation using a **beta** distribution
  - **A subset of dimensions are changed** to the opposite values together
  - **Selection scheme switching**

Proposed beta distribution utilization

- BetaCOBL-cont'd
  - Definition of **opposition degree**
    - **Strong** degree: **concave opposition**
      - Using a beta distribution with concave shape
    - **Weak** degree: **convex opposition**
      - Using a beta distribution with convex shape



Proposed beta distribution utilization

- BetaCOBL-cont'd
  - Concave opposition

$$\breve{x}_{i,j}(t) = a_j + (b_j - a_j) \cdot Beta(\alpha_j, \beta_j)$$

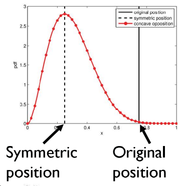
$$\alpha_j = \begin{cases} spread_j \cdot peak_j, & \text{if } mode_j < 0.5 \\ spread_j, & \text{otherwise} \end{cases}$$

$$\beta_j = \begin{cases} spread_j, & \text{if } mode_j < 0.5 \\ spread_j \cdot peak_j, & \text{otherwise} \end{cases}$$

$$mode_j = \frac{(a_j + b_j - x_{i,j}(t)) - a_j}{b_j - a_j}$$

$$spread_j = \left(\frac{1}{\sqrt{normDiv}}\right)^{\max(N(1,0.5),0)}$$

$$peak_j = \begin{cases} \frac{(spread_j - 2)mode_j + 1}{spread_j(1 - mode_j)}, & \text{if } mode_j < 0.5 \\ \frac{2 - spread_j}{spread_j} + \frac{spread_j - 1}{spread_j \cdot mode_j}, & \text{otherwise} \end{cases}$$



Proposed beta distribution utilization

- BetaCOBL-cont'd
  - Concave opposition

$$\breve{x}_{i,j}(t) = a_j + (b_j - a_j) \cdot Beta(\alpha_j, \beta_j)$$

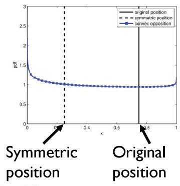
$$\alpha_j = \begin{cases} spread_j \cdot peak_j, & \text{if } mode_j < 0.5 \\ spread_j, & \text{otherwise} \end{cases}$$

$$\beta_j = \begin{cases} spread_j, & \text{if } mode_j < 0.5 \\ spread_j \cdot peak_j, & \text{otherwise} \end{cases}$$

$$mode_j = \frac{x_{i,j}(t) - a_j}{b_j - a_j}$$

$$spread_j = 0.1\sqrt{normDiv} + 0.9$$

$$peak_j = \begin{cases} \frac{(spread_j - 2)mode_j + 1}{spread_j(1 - mode_j)}, & \text{if } mode_j < 0.5 \\ \frac{2 - spread_j}{spread_j} + \frac{spread_j - 1}{spread_j \cdot mode_j}, & \text{otherwise} \end{cases}$$



Proposed beta distribution utilization

- BetaCOBL-cont'd
  - Normalized diversity

$$normDiv = \frac{1}{NP}\sum_{i=1}^{NP} CD(x_i, POP)$$

$$CD(x_i, POP) = \min_{c \in POP, x_i \neq c} d(c, x_i)$$

$$d(c, x_i) = \sqrt{\frac{1}{D}\sum_{j=1}^{D}\left(\frac{x_{i,j} - c_j}{b_j - a_j}\right)^2}$$

Proposed beta distribution utilization

- BetaCOBL-cont'd
  - Partial dimensional change
    - Opposite point calculation as DE mutation

$$\breve{x}_{i,j}(t) = a_j + b_j - x_{i,j}(t) = a_j + F(b_j - x_{i,j}(t)) \quad F = 1$$

    - **Mix original individual and opposite point** using crossover
      - Effect of crossover rate
        - Low crossover rate is beneficial for separable functions
        - High crossover rate is beneficial for non-separable functions
      - Generate **two opposite point** calculations with both **low and high crossover rates**
  - Selection switching
    - If **diversity is high**, **(μ+λ) selection** in ES
      - The best individuals from the original population and its opposite are picked up
    - If **diversity is low**, **(μ,λ) selection** in ES
      - The worst half of the original individuals are replaced by their opposites one-to-one

Proposed beta distribution utilization

- BetaCOBL-cont'd
  - Entire procedure

Algorithm 3 BetaCOBL

```
Calculate Diversity
if Diversity is high then
    for all individuals in population do
        Calculate a full dimensional convex or concave opposite point
        Mix the full dimensional opposite with the original individual to get partial dimensional opposite point candidates
        Evaluate the opposite point candidates
    end for
    Choose the best individuals from the original and opposite populations
else
    Sort individuals according to fitness value
    for the worst half individuals in population do
        Calculate a full dimensional convex or concave opposite point
        Mix the full dimensional opposite with the original individual to get partial dimensional opposite point candidates
        Evaluate the opposite point candidates
        Replace the original with the better candidate
    end for
end if
```

Proposed beta distribution utilization

- BetaCODE: DE embedding BetaCOBL
  - Entire procedure

Algorithm 4 BetaCODE

```
Initialize population
BetaCOBL
while !(Termination condition meets) do
    if rand(0,1) < JR then
        BetaCOBL
    else
        for all individuals in population do
            Select parents
            Generate a donor vector
            Mix the donor with the original individual to get a trial vector
            Evaluate the trial vector
            if The trial is better then
                Replace the original with the trial
            end if
        end for
    end if
end while
```

Proposed beta distribution utilization

- Existing bare bones reproductions
  - Using **Gaussian, Cauchy, and polynomial** distributions
  - Using a beta distribution with **mean and standard deviation**
- B^3R: beta distribution-based bare bones reproduction
  - Using a beta distribution
    - suffer from out-of-range phenomenon
  - Using a beta distribution with *mode* and *spread*

Proposed beta distribution utilization

- B^3R
  - Candidate generation using a **beta distribution with *mode* and *spread***

$$v_{i,j}(t) = a_j + (b_j - a_j) \cdot Beta(\alpha_j, \beta_j)$$

$$(\alpha_j, \beta_j) = \begin{cases} (spread_j \cdot peak_j, spread_j), & \text{if } mode_j < 0.5 \\ (spread_j, spread_j \cdot peak_j), & \text{if } mode_j \geq 0.5 \end{cases}$$

$$peak_j = \begin{cases} \frac{(spread_j - 2)mode_j + 1}{spread_j(1 - mode_j)}, & \text{if } 0 \leq mode_j < 0.5 \\ \frac{2 - spread_j}{spread_j} + \frac{spread_j - 1}{spread_j \cdot mode_j}, & \text{if } 0.5 \leq mode_j \leq 1 \end{cases}$$

$$mode_j = \frac{x_{r_1,j}(t) - a_j}{b_j - a_j} \qquad dist_j = \frac{|x_{r_2,j}(t) - x_{r_3,j}(t)|}{b_j - a_j}$$

$$spread_j = \left(\frac{1}{dist_j}\right)^{\max(2+N(0,0.5),0)}$$

Proposed beta distribution utilization

- B^3R-cont'd
  - Crossover
    - Exponential crossover $CR_i(t) = Beta(5, 2.71)$
    - Self-adaptive scheme in crossover rate
- TPBO: two-phase B^3R optimization
  - Exploration-oriented strategy
    - B^3R
  - Exploitation-oriented strategy
    - DE/Current-to-best/2 with self-adaptation *F*

$$v_i(t) = x_i(t) + rand() \cdot (x_{best}(t) - x_i(t)) + F_i(t)(x_{r_1}(t) - x_{r_2}(t)) + F_i(t)(x_{r_3}(t) - x_{r_4}(t))$$

$$F_i(t+1) = \begin{cases} N(0.5, 0.1), & \text{if } f(x_i(t)) < f(v_i(t)) \text{ and } rand() < 0.1 \\ F_i(t), & \text{otherwise} \end{cases}$$

Proposed beta distribution utilization

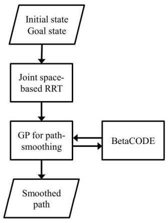
- TPBO-cont'd
  - Entire procedure

Algorithm 5 TPBO

```
Initialize population
while !(Termination condition meets) do
    for all individuals in population do
        if First phase then
            B3R
        else
            if rand() < MR then
                B3R
            else
                DE/Current-to-best/2 reproduction
            end if
        end if
        Evaluate the offspring
        if The offspring is better then
            Replace the original with the offspring
        end if
    end for
end while
```

Application

- Path smoothing algorithms using Gaussian process (GP)
  - Consider the scattering in the path generated by RRT as noise
  - The scattering can be removed by GP regression
  - Hyperparameters of GP is optimized by BetaCODE



Application

- Path smoothing algorithms using GP
  - Simulation setup
    - GP
      - Diagonal squared exponential covariance function
      - Gaussian likelihood function
    - Performance metric
      - Fitness function
        - Sum of the negative log marginal likelihood
      - The number of successful runs
        - The smoothed path using the GP is regarded as successful if the initial and goal positions of the smoothed path are not far away from those of the original path
    - Compared algorithm
      - BetaCODE
      - DE/Best/1/Bin (DEB)
      - Conjugate gradient method (CG)

Application

- Path smoothing algorithms using GP-cont'd
  - Result
    - Fitness function

BetaCODE		DEB		CG	
Mean	SD	Mean	SD	Mean	SD
-1305.83	8.03	-1301.78	11.56	749.69	1469.29

i	Algorithm	Average ranking	$z = \lvert R_0 - R_i \rvert / SE$	Unadjusted p-value	Holm APV
2	CG	3.0000	26.500	0	**0**
1	DEB	1.6760	5.5656	2.6124E-08	**2.6124E-08**
0	BetaCODE	**1.3240**	.	.	.

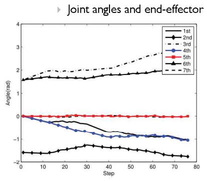
Application

- Path smoothing algorithms using GP-cont'd
  - Result-cont'd
    - The number of successful runs

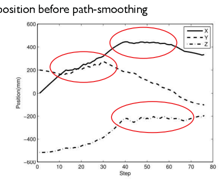
BetaCODE	DEB	CG
25	25	2

Application

- Path smoothing algorithms using GP-cont'd
  - Result-cont'd
    - Joint angles and end-effector position before path-smoothing



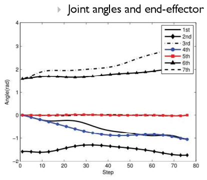
Joint angles



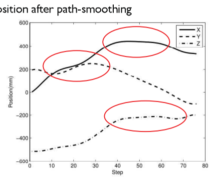
End-effector position

Application

- Path smoothing algorithms using GP-cont'd
  - Result-cont'd
    - Joint angles and end-effector position after path-smoothing



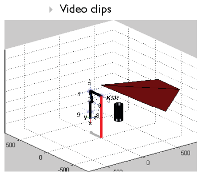
Joint angles



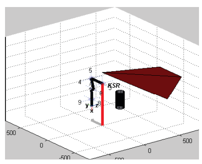
End-effector position

Application

- Path smoothing algorithms using GP-cont'd
  - Result-cont'd
    - Video clips



Before path smoothing



After path smoothing

Application

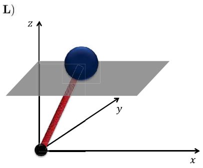
- Bipedal walking of a humanoid robot
  - Vertical center of mass (COM) motion
    - Single 3-D linear inverted pendulum model (3DLIPM)

$$\mathbf{T}_{gr} + \mathbf{r}_{COM} \times \mathbf{F}_{gr} = \frac{d}{dt}(\mathbf{r}_{COM} \times \mathbf{L})$$

$$\begin{bmatrix} \ddot{y} - \frac{g}{Z_c} y \\ \ddot{x} - \frac{g}{Z_c} x \end{bmatrix} = \begin{bmatrix} -\frac{T_x}{mZ_c} \\ \frac{T_y}{mZ_c} \end{bmatrix}$$

$$\mathbf{T}_{gr} - \mathbf{r}_{ZMP} \times \mathbf{F}_{gr} = [0\ 0\ M_z]^T$$

$$\begin{bmatrix} \ddot{y} - \frac{g}{Z_c} y \\ \ddot{x} - \frac{g}{Z_c} x \end{bmatrix} = -\frac{g}{Z_c} \begin{bmatrix} y_{ZMP} \\ x_{ZMP} \end{bmatrix}$$

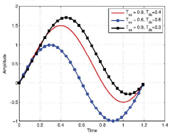


Application

- Bipedal walking of a humanoid robot-cont'd
  - Amplified sinusoidal function

$$f(t) = k_s \sin\left(\omega\left(t - \frac{\pi(\gamma-1)}{2\omega(\gamma+1)}\right)\right) + \sin\left(\frac{\pi(\gamma-1)}{2(\gamma+1)}\right)$$

$$\omega = \frac{2\pi}{T^{ss}_{l/r} + T^{ds}_{l/r}}, \ \gamma = \frac{T^{ss}_{l/r}}{T^{ds}_{l/r}}$$



  - Feedback from the GRF

$$feed = k_f|F_l + F_r - mg|$$

  - Proposed vertical COM trajectory

$$z = Z_c + z_{sf}$$

$$z_{sf} = (k_s + feed)\sin\left(\omega\left(t - \frac{\pi(\gamma-1)}{2\omega(\gamma+1)}\right)\right) + \sin\left(\frac{\pi(\gamma-1)}{2(\gamma+1)}\right)$$

Application

- Bipedal walking of a humanoid robot-cont'd
  - Simulation setup
    - Performance metric
      - Fitness function

$$fitness(k_s, k_f) = W_z|F_l + F_r - mg| + W_x|x_c - x_{ZMP}| + W_y|y_c - y_{ZMP}| + P$$

    - Compared algorithm
      - TPBO (k_s and k_f)
      - SDE (k_s and k_f)
      - TPBO(k_s only)

Application

- Bipedal walking of a humanoid robot-cont'd
  - Result
    - Fitness function

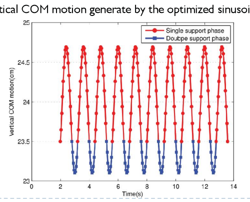
TPBO(k_s, k_f)		SDE(k_s, k_f)		TPBO(k_s only)	
Mean	SD	Mean	SD	Mean	SD
25693.82	1.69	25694.69	2.16	25695.25	0.04

No movement
25806.11

i	Algorithm	Average ranking	$z = \|R_0 - R_i\|/SE$	Unadjusted p-value	Holm APV
2	TPBO(k_s only)	2.6000	4.3134	1.6080E-05	**3.2160E-05**
1	SDE(k_s, k_f)	2.0200	2.2627	2.3652E-02	**2.3652E-02**
0	TPBO(k_s, k_f)	**1.3800**	.	.	.

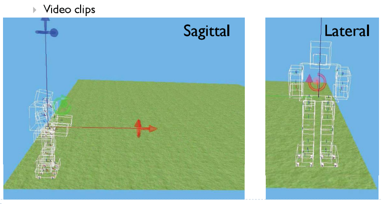
Application

- Bipedal walking of a humanoid robot-cont'd
  - Result-cont'd
    - Vertical COM motion generate by the optimized sinusoidal function



Application

- Bipedal walking of a humanoid robot-cont'd
  - Result-cont'd
    - Video clips



Conclusion

- New parameter definition for a beta distribution
  - *peak* and *spread*
  - Shape variation without limitation
- BetaCODE
  - Control degree of opposition using a beta distribution
  - Selection switching based on population diversity
  - Partial dimensional change
- TPBO
  - Bounded reproduction strategy using beta distribution
  - Hybridization with DE/Current-to-best/2

Thank you for your attention