Analysis and unidirectionally coupled synchronization of a novel chaotic system

Xue Li
Department of Automation , Tianjin University of Science and Technology,
1038 Dagunanlu Road, Hexi Distric, Tianjin 300222, PR China

Wei Xue
Department of Automation , Tianjin University of Science and Technology,
1038 Dagunanlu Road, Hexi Distric, Tianjin 300222, PR China
E-mail:lixueTJKJ@163.com,xuewei@tust.edu.cn
www.tust.edu.cn

Abstract

In this paper, a new three-dimensional autonomous chaotic system is proposed. By means of theoretical analysis and numerical simulation, some basic dynamical properties, such as dissipation, fractal dimension, Lyapunov exponent spectrum and chaotic dynamical behaviors of the new chaotic system are illustrated. The obtained results show clearly that this system is a new chaotic system. Furthermore, based on Lyapunov stability theory of the system, unidirectionally coupled synchronization of the new three-dimensional chaotic system through designing the appropriate coupling coefficient is investigated. Results of numerical simulation validate the accuracy and effectiveness of synchronization scheme of the presented system.

Keywords: Chaos; New chaotic system; Lyapunov exponent; Unidirectionally coupled synchronization.

1. Introduction

In 1963, American mathematical meteorologist Lorenz discovered chaos in a simple system of three autonomous ordinary differential equations, called the Lorenz system,[1] a lot of work have been reported on finding the new chaotic attactors. As the first chaotic model, the Lorenz system has become a paradigm of chaos research. In 1976, Rössler presented a three-dimensional quadratic autonomous chaotic system.[2] Recently, there have been increasing attentions in generating chaotic system since Chen found a new Lorenz-like chaotic system in1999, namely, the Chen system.[3] In 2002, Lü and Chen found a new chaotic system, called the Lü chaotic system,[4] which represents the transition between the Lorenz system and Chen system. In 2003, Liu created a new chaotic system and researched its basic dynamical characteristics, as the name of Liu chaotic system.[5] In 2005, Qi reported a three-dimensional continuous quadratic autonomous chaotic system modified from the Lorenz system,[6] in which each equation contains a single quadratic cross-product term. In this paper, a novel three-dimensional autonomous chaotic system is introduced. Numerical analysis shows that the proposed chaotic attractor is a new attractor, which is not topologically equivalent to the original chaotic system, or the Lorenz system, or the Chen system, or the Lü system, or even the Lorenz system family.

In the past few decades, synchronization of the chaotic systems have attracted considerable interests because of their practical applications in the fields of image encryption, secure communication, electrical engineering, neural network, control processing and so on. Many synchronization approaches have been proposed in order to synchronizing the chaotic systems. For example, linear feedback synchronization,[7] active synchronization,[8] adaptive synchronization,[9] projective synchronization,[10-11] sliding mode synchronization,[12] unidirectionally coupled synchronization,[13] and lag synchronization.[14-15] By means of Lyapunov stability theory of the system, unidirectionally coupled synchronization of the new three-dimensional chaotic system is investigated. Numerical simulation results verify the accuracy and effectiveness of the synchronization scheme presented in this paper.

2. The new chaotic system and its basic dynamical properties

2.1. The new three-dimensional chaotic system

In this paper, a new three-dimensional autonomous system is proposed, which can be expressed as the following form:

$$\begin{cases} \dot{x} = ay - ax - yz \\ \dot{y} = bx + y + xz \\ \dot{z} = -dz + cy^2 \end{cases} \tag{1}$$

When the parameters are fixed as a=33, b= 50, c=3, and d=15, the chaotic behavior of system(1) is obvious. By adopting the time domain method based on predictor-corrector, the phase portrait of system(1) is shown in Fig.1 and the three Lyapunov exponents are 5.767 3, 0.003 4, -7.823 6, respectively.

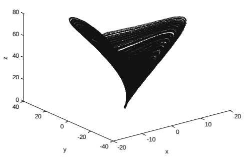


Fig.1 Phase portrait of system(1) with a=33, b= 50, c=3, and d=15.

2.2. Basic dynamical properties of the new chaotic system

(I)Dissipation:

The dissipativity of system(1) is described as:

$$\Delta V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -(a - 1 + d)$$

Therefore, when a, d satisfy $a + d > 1$, system(1) is dissipative, and with an exponential rate:

$$\frac{dV}{dt} = e^{-(a-1+d)t}$$

It means that a volume element V_0 is apparently contracted by the flow into a volume element $V_0 e^{-(a-1+d)t}$ versus time t. That is, each volume element containing the trajectories of this dynamical system shrinks to zero as $t \to \infty$ at an exponential rate. Therefore, all the orbits of the new system(1) are ultimately confined to a specific subset that has zero volume, and the asymptotic motion of system(1) will settle onto an attractor of the system.

(II)The Lyapunov exponents

As is known, the Lyapunov exponents measure the exponent rates of the divergence and convergence of nearby trajectories in phase space of system. Three Lyapunov exponents of system(1) are calculated as $L_1 = 5.7673$, $L_2 = 0.0034$, and $L_3 = -7.8236$. The Lyapunov exponent spectrum of the new system(1) is showed in Fig.2, when the parameters is taken as a=33, b=50, c=3, and $d \in (0,30)$. Thus, the Lyapunov dimension of the new chaotic system(1) is

$$D_L = 2 + \frac{1}{|L_3|}\sum_{i=1}^{2} L_i = 2 + \frac{L_1 + L_2}{|L_3|} = 2 + \frac{5.7673 + 0.0034}{|-7.8236|} = 2.7376$$

Which means the system(1) is a chaotic system since the Lyapunov dimension of the system(1) is fractional.

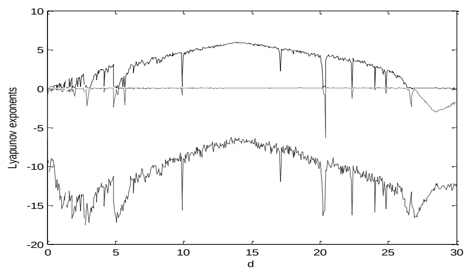


Fig.2 Lyapunov exponent diagrams of system(1) on x -axis versus parameter $d \in (0,30)$

3. Unidirectionally coupled synchronization of the new chaotic system

In order to realize the unidirectionally coupled synchronization of the new chaotic system, the drive system is defined as follow:

$$\begin{cases} \dot{x}_1 = ax_2 - ax_1 - x_2x_3 \\ \dot{x}_2 = bx_1 + x_2 + x_1x_3 \\ \dot{x}_3 = -dx_3 + cx_2x_2 \end{cases} \tag{2}$$

and the response system can be described in the form of:

$$\begin{cases} \dot{y}_1 = ay_2 - ay_1 - y_2y_3 - k(y_1 - x_1) \\ \dot{y}_2 = by_1 + y_2 + y_1y_3 - k(y_2 - x_2) \\ \dot{y}_3 = -dy_3 + cy_2y_2 - k(y_3 - x_3) \end{cases} \tag{3}$$

Where $e_1 = y_1 - x_1$, $e_2 = y_2 - x_2$, and $e_3 = y_3 - x_3$. And k is the coupling coefficient, it can make system(2) and system(3) compose a coupling system.

From *Eq*.(2) and *Eq*.(3), the error system is obtained which can be expressed as:

$$\begin{cases} \dot{e}_1 = ae_2 - ae_1 - y_2e_3 - y_3e_2 + e_2e_3 - ke_1 \\ \dot{e}_2 = be_1 + e_2 + y_1e_3 + y_3e_1 - e_1e_3 - ke_2 \\ \dot{e}_3 = -de_3 + cy_2e_2 + cy_2e_2 - ce_2e_2 - ke_3 \end{cases} \tag{4}$$

Eq.(4) utilizes Laplace transform, the error system is provided which is described by:

$$\begin{cases} sE_1(s) - e_1(0) = aE_2(s) - aE_1(s) - L\{y_2e_3\} \\ \qquad - L\{y_3e_2\} + L\{e_2e_3\} - kE_1(s) \\ sE_2(s) - e_2(0) = bE_1(s) + E_2(s) + L\{y_1e_3\} \\ \qquad + L\{y_3e_1\} - L\{e_1e_3\} - kE_2(s) \\ sE_3(s) - e_3(0) = -dE_3(s) + cL\{y_2e_2\} + cL\{y_2e_2\} \\ \qquad - cL\{e_2e_2\} - kE_3(s) \end{cases} \tag{5}$$

From *Eq*.(5) , the error system is obtained as follow:

$$\begin{cases} E_1(s) = \dfrac{aE_2(s)}{s+k+a} - \dfrac{e_1(0)}{s+k+a} - \dfrac{L\{y_2e_3\}}{s+k+a} \\ \qquad - \dfrac{L\{y_3e_2\}}{s+k+a} + \dfrac{L\{e_2e_3\}}{s+k+a} \\ E_2(s) = \dfrac{bE_1(s)}{s+k-1} + \dfrac{e_2(0)}{s+k-1} + \dfrac{L\{y_1e_3\}}{s+k-1} \\ \qquad + \dfrac{L\{y_3e_1\}}{s+k-1} - \dfrac{L\{e_1e_3\}}{s+k-1} \\ E_3(s) = -\dfrac{e_3(0)}{s+k+d} + \dfrac{cL\{y_2e_2\}}{s+k+d} + \dfrac{cL\{y_2e_2\}}{s+k+d} \\ \qquad - \dfrac{cL\{e_2e_2\}}{s+k+d} \end{cases} \tag{6}$$

Based on the final value theorem of Laplace transform, the error system is described in the form of:

$$\begin{aligned} \lim_{t\to\infty} e_1(t) &= \lim_{s\to 0} E_1(s) = \frac{a}{s+k+a} \lim_{t\to\infty} e_2(t) - \lim_{s\to 0} \frac{sL\{y_2e_3\}}{s+k+a} \\ &\quad - \lim_{s\to 0} \frac{sL\{y_3e_2\}}{s+k+a} + \lim_{s\to 0} \frac{sL\{e_2e_3\}}{s+k+a} \\ &= 0 \end{aligned} \tag{7}$$

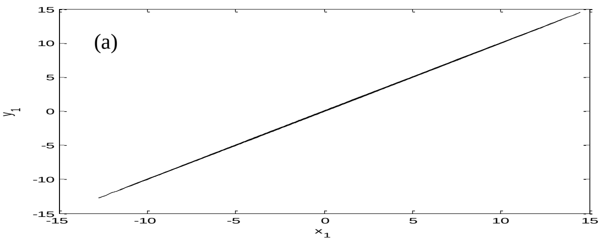
$$\begin{aligned} \lim_{t\to\infty} e_2(t) &= \lim_{s\to 0} E_2(s) = \frac{b}{s+k-1} \lim_{t\to\infty} e_1(t) + \lim_{s\to 0} \frac{sL\{y_1e_3\}}{s+k-1} \\ &\quad + \lim_{s\to 0} \frac{sL\{y_3e_1\}}{s+k-1} - \lim_{s\to 0} \frac{sL\{e_1e_3\}}{s+k-1} \\ &= 0 \end{aligned} \tag{8}$$

$$\begin{aligned} \lim_{t\to\infty} e_3(t) &= \lim_{s\to 0} E_3(s) = c \lim_{s\to 0} \frac{sL\{y_2e_2\}}{s+k+d} + c \lim_{s\to 0} \frac{sL\{y_2e_2\}}{s+k+d} \\ &\quad - c \lim_{s\to 0} \frac{sL\{e_2e_2\}}{s+k+d} \\ &= 0 \end{aligned} \tag{9}$$

From *Eq*.(7), *Eq*.(8), and *Eq*.(9), $\lim_{t\to\infty} e_i(t) = 0$ $(i = 1,2,3)$ can be obtained. Theoretical analysis demonstrate system(2) can synchronize system(3) completely.

4.Numerical simulation

In following, unidirectionally coupled synchronization of the new three-dimensional chaotic system through designing the appropriate coupling coefficient is illustrated. The system parameters are chosen as *a*=33, *b*=50, *c*=3, and *d*=15. The initial values of the drive system(2) and the response system(3) are taken as $x_1(0) = -3$, $x_2(0) = 0$, $x_3(0) = 2$, $y_1(0) = -3$, $y_2(0) = -0.6$, and $y_3(0) = -1.6$, respectively. When the coupling coefficient is taken as $k = 200$, the simulation results of state parameters are displayed in Fig.3. Furthermore, the curves of synchronization error is described in Fig.4. It's clearly showed that corresponding numerical simulations agree with the analytical results.



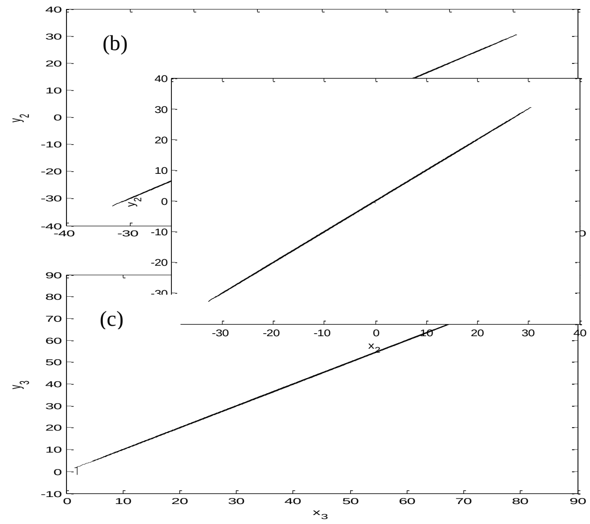


Fig.3 Curves of state variable (a) (x_1, y_1) ; (b) (x_2, y_2) ; (c) (x_3, y_3) .

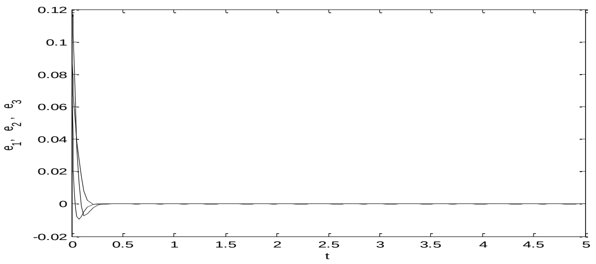


Fig.4 Synchronization errors versus time t

5. Conclusion

In this paper, a new three-dimensional autonomous chaotic system is proposed and investigated. By utilizing theoretical analysis and numerical simulation, some basic dynamical properties, such as dissipation, fractal dimension, Lyapunov exponent spectrum and chaotic dynamical behaviors of the new chaotic system are introduced. In addition, based on Lyapunov stability theory, unidirectionally coupled synchronization of the new chaotic system is illustrated. Numerical simulation are performed to validate the effectiveness of the presented synchronization scheme. The synchronization scheme proposed in our work can provide technical basis and support for the further study of the secure communication and automatic control.

6. Acknowledgement

This work is supported by the Young Scientists Fund of the National Natural Science Foundation of China (Grant No.11202148).

7. References

1. Lorenz E N. Deterministic nonperiodic Flow. *Journal of the Atmosphere Science.* 1963, 20(2): 130-141.
2. Rössler O E. An equation for continuous chaos. *Physics Letters A.* 1976, 57(3): 397-398.
3. Chen G R, Ueta T. Yet another chaotic attractor. *International Journal of Bifurcation and Chaos.* 1999, 9(7): 1465-1466.
4. Lü J H, Chen G R. A new chaotic attractor coined. *International Journal of Bifurcation and Chaos*. 2002, 12(3): 659-661.
5. Liu W B, Chen G R. A new chaotic system and its generation. *International Journal of Bifurcation and Chaos.* 2003, 13(1): 261-267.
6. Qi G Y, Du S Z, Chen G R, *et al*. On a 4-dimensional chaotic system. *Chaos, Solitons and Fractals.* 2005, 23: 1671-82.
7. Yassen M T. Controlling chaos and synchronization for new chaotic system using linear feedback control. *Chaos, Solitons and Fractals.* 2005, 26: 913-920.
8. Li G H. An active control synchronization for two modified chua circuits. *Chinese Physics B.* 2005, 14(3): 472-475.
9. Kim J H, Park C W, Kim E, et al. Adaptive synchronization of T-S fuzzy chaotic systems with unknown parameters. *Chaos, Solitons and Fractals.* 2005, 24: 1335-1361.
10. Meng J, Wang X Y. Generalized projective synchronization of a class of delayed neural networks. *Physics Letters A.* 2008, 22(3): 181-190.
11. Xu D L, Li Z. Controlled projective synchronization in nonpartially-linear chaotic systems. *Physics Letters A.* 2002, 292(6): 1395-1402.
12. Yau H T, Yan J J. Desgn of sliding mode controller for Lorenz chaotic system with nonlinear input. *Chaos, Solitons and Fractals.* 2004, 19(4): 891-898.
13. Yang X S. On the existence of generalized synchronization in unidirectionally coupled systems. *Computers and Mathematics with Applications*. 2001, 122(1): 71-79.
14. Shahverdiev E M, Sivaprakasam S, Shore K A. Lag synchronization in time-delayed systems. *Physics Letters A.* 2002, 292(96): 320-324.
15. Rosenblum M G, Pikovsky A S, Kurths J. From phase to lag synchronization in coupled chaotic oscillators. *Physics Letters A.* 1997, 78(22): 4193-4196.