Design and Analog Circuit Implementation of a Dynamic Feedback Control System Based on RLC Series Circuit

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Abstract

The purposeas expected performance. The simulation results of the theoretical model and the corresponding results of the real analog circuit implementation are given in the paper to illustrate that the circuit can accomplish the track of this paper is to verify that a dynamic feedback control system can be realized by a simple small analog circuit. A normal RLC series circuit, which is an electrical two-order circuit only consisting of one resistor, one inductor and one capacitor, is taken as the controlled object, and the voltage of the capacitor is taken as the output of the system. The engineering design method of regulator in DC drive control system is applied to design the dynamic feedback compensator, so that the output of the system can track the given input, and the system is stabilized and h ing function of dynamic feedback control system.

Keywords: dynamic output feedback; engineering design method; dynamic compensator; analog circuit implementation

1. Introduction

The servo system is a normal type in electric driving automatic control system. To facilitate engineering applications, scholars have put forward a variety of engineering design methods to simplify the regulator design procedure.[1-5] The purpose of this paper is to design a dynamic feedback compensator with the engineering design method and make the design procedure separate from the complex application background of electric drive system. In this paper, a simple analog circuit will be implemented to achieve the input tracking performance of the servo system.

2. Design of Dynamic Feedback Compensator Based on Engineering Design Method

The structure of RLC series circuit is shown in Fig. 1. From Fig. 1, the state equation of the circuit can be formulated as

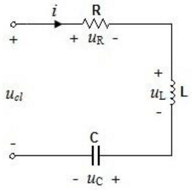

Fig. 1. RLC series circuit

$$u_{cl}=u_{\mathrm{L}}+u_{\mathrm{R}}+u_{\mathrm{C}}=LC\frac{d^2u_{\mathrm{C}}(t)}{dt^2}+RC\frac{du_{\mathrm{C}}(t)}{dt}+u_{\mathrm{C}}(t). \quad (1)$$

Take the RLC series circuit as the controlled object, and let the voltage of the capacitor u_{C} be the output of the system. The transfer function of the RLC series circuit can be presented as

$$G(s)=\frac{U_{\mathrm{C}}(s)}{U_{cl}(s)}=\frac{1}{\mathrm{LC}s^2+\mathrm{RC}s+1}. \quad (2)$$

Where $u_{\mathrm{C}}(0) = 0$, and $i(0) = 0$.

Let L = 10 mH and C = 1 μF. To make the circuit non-oscillatory, the resistance of the resistor R should be $\mathrm{R} > (\mathrm{L/C})^{1/2} = 200\ \Omega$.

Eq. (2) is transformed as

$$G(s)=\frac{1}{\left(\frac{1}{\mathrm{R}}s+1\right)(\mathrm{RLC}s+1)}=\frac{1}{(T_1s+1)(T_2s+1)}. \quad (3)$$

So that the engineering design method can be applied to design the dynamic feedback compensator.

To make Eq. (3) and Eq. (2) equal, the resistance should be $\mathrm{R} = [1/\mathrm{C}/(1\text{-}\mathrm{L})]^{1/2} \approx 1\ \mathrm{k}\Omega > 200\ \Omega$, which satisfies the non-oscillatory condition. Then, $T_1 = 1/\mathrm{R} = 10^{-3}$, and $T_2 = \mathrm{RLC} = 10^{-5}$.

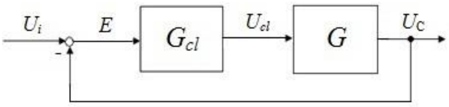

Fig. 2. Block diagram of dynamic feedback control system

The block diagram of the dynamic feedback control system is shown in Fig. 2. The engineering design method, which is simple and can simplify the design procedure, is often used to design the regulator for the DC drive control system. In this paper, the engineering design method is applied to design the dynamic feedback compensator $G_{cl}(s)$, such that the output signal $u_{\mathrm{C}}(t)$ can track the given input signal $u_i(t)$.

To ensure the safety requirements of the elements and the performance of the closed-loop system, such as stability, accuracy and rapidity, the closed-loop system should be designed as a typical I system according to Ref. 1, because the overshoot of the typical I system is smaller than that of the typical II system. Moreover, the typical I system has an acceptable dynamic performance that satisfies the control require of this paper. The open-loop transfer function of the typical I system is

$$G_{\mathrm{I}}(s)=G_{cl}(s)G(s)=\frac{K}{s(Ts+1)}. \quad (4)$$

From Eq. (4) and Eq. (3), the dynamic feedback compensator $G_{cl}(s)$ can be formulated as

$$G_{cl}(s)=\frac{K_{pi}(\tau_1 s+1)}{\tau_1 s}. \quad (5)$$

Where $\tau_1 = T_1 = 10^{-3}$. Thus, the control signal $u_{cl}(t)$ is represented as

$$u_{cl}(t)=K_{pi}e(t)+\frac{K_{pi}}{\tau_1}\int e(t)dt. \quad (6)$$

Substitute Eq. (3) and Eq. (5) into Eq. (4). Then,

$$G_{\mathrm{I}}(s)=G_{cl}(s)G(s)=\frac{K_{pi}/\tau_1}{s(T_2s+1)}=\frac{K}{s(Ts+1)}. \quad (7)$$

Where $K = K_{pi}/\tau_1$, and $T = T_2 = 10^{-5}$.

To guarantee the good tracking performance of the system, let $KT = 0.5$ so that the damping ratio of the system is $\xi = 0.707$. Therefore, $K_{pi} = K\tau_1 = 0.5\tau_1/\mathrm{T} = 50$.

From Fig. 2 and Eq. (7), the closed-loop transfer function is formulated as

$$\varphi(s)=\frac{G_{\mathrm{I}}(s)}{1+G_{\mathrm{I}}(s)}=\frac{K}{Ts^2+s+K}. \quad (8)$$

It is easy to prove that all of the eigenvalues of Eq. (8) have the negative real parts, so the closed-loop system is stabilized.

When the given input signal is a step signal whose amplitude is a, i.e., $u_i(t)=a\cdot 1(t)$ and $U_i(s)=a/s$, the steady-state error of the system (8) is

$$e_{ss}=\lim_{s\to 0}sE(s)=0. \quad (9)$$

When the given input signal is a ramp signal whose slope is *b*, i.e., $u_i(t) = b \cdot t \cdot 1(t)$ and $U_i(s) = b/s^2$, the steady-state error of the system (8) is

$$e_{ss} = \lim_{s \to 0} sE(s) = b/K. \tag{10}$$

According to the output saturation voltage of the operational amplifier, the parameters *a* and *b* should not be too large to avoid distortion of the output signal. Because the parameter $K = K_{pi}/\tau_1 = 5\times10^4$, which is large enough, the steady-state error of the system (8) under the ramp signal input is also equal to 0, i.e., $e_{ss} = 0$.

When the given input signal is a sine signal whose amplitude is *A*, i.e., $u_i(t) = A \cdot \sin(\omega t)$ and $U_i(s) = A\omega/(s^2 + \omega^2)$, the steady-state error of the system (8) is

$$e_{ss} = \lim_{s \to 0} sE(s) = 0. \tag{11}$$

Similarly, the amplitude of the input sine signal should also be kept in a proper range.

3. Numerical Simulation

In this paper, there are three different given input signals. The first one is a square wave, the second one is a triangular wave, and the third one is a sine wave, as shown in Fig. 3(a). The corresponding output response waves are shown in Fig. 3(b).

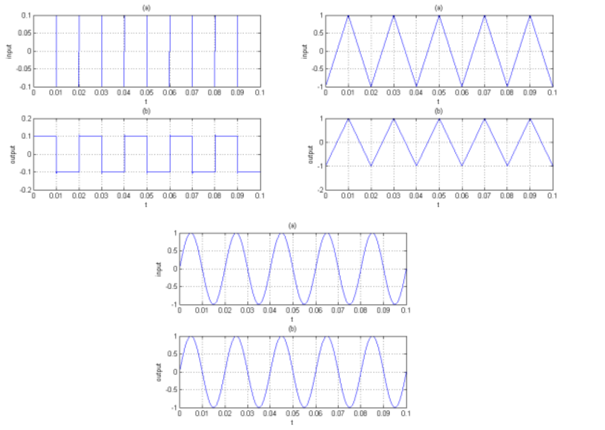

Fig. 3. Waves of the input signal and the output signal: (a) input; (b) output

The numerical simulation waves illustrate that the output signals can track the different given input signals well. It demonstrates that the dynamic feedback compensator designed by the engineering design method is feasible and effective.

4. Circuit Implementation

The analog circuit implementation of the servo system (8) is shown in Fig. 4. The type and parameters of the elements have been labelled in the figure. The real output response waves under the same input signals of the numerical simulation are shown in Fig. 5.

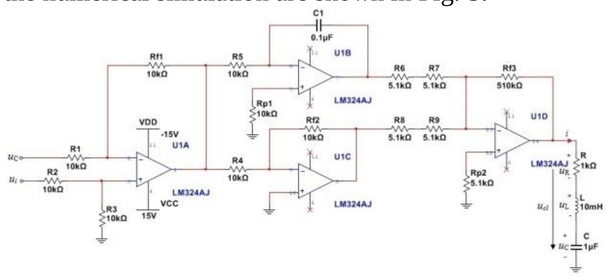

Fig. 4. Analog circuit implementation of the servo system (8)

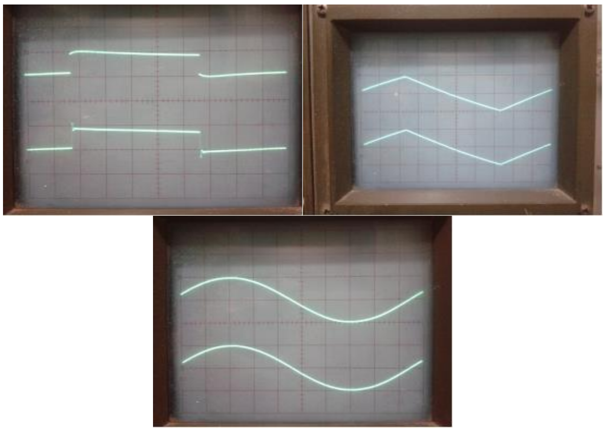

Fig. 5. Real waves of the input signal and the output signal: (up) input; (down) output

By comparing Fig. 5 with Fig. 3, it can be verified that here is a good qualitative agreement between the numerical simulation and the experimental realization. It shows that the real analog circuit accomplishes the purpose of tracking the given input signals.

Furthermore, the amplitude of the given input signal is limited to the output saturation voltage of the operational amplifier. Hence, it should not be too large, otherwise, the distortion of the output signal would occur, and the elements would even be destroyed.

5. Conclusions

In this paper, a servo system is implemented by a simple analog circuit only consisting of nineteen elements. The numerical simulation results, as well as the experiment realization results, prove that the output voltage of the capacitor in the RLC series circuit tracks the different given input signals well. The proposed circuit can be used as an example for the courses of control theory and application, such as the course of automatic control theory, the course of motion control system and so on.

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