Analog Circuit Implementation and Full State Observation of Chua's Circuit

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Abstract

In this paper, an inductance simulator and a nonlinear resistor, which are constructed by operational amplifiers and resistors, are applied to complete the analog circuit implementation of Chua's circuit as well as improve the accuracy of circuit parameters. For the observation of all variables, the state variable z, which represents the product of the linear resistance and the inductor current in Chua's circuit, should be observed even if it is not an actual measurable physical quantity. It is found that z can be obtained via scaling of the voltage of the resistor in the inductance simulator. The real chaotic curves generated from the analog Chua's circuit are displayed on the oscilloscope clearly and correctly. It demonstrates that the realization of Chua's circuit is correct and available for other research such as chaos synchronization of Chua's circuits.

Keywords: Chua's circuit; inductance simulator; circuit implementation; scaling; full state observation

1. Introduction

Chua's circuit was introduced in 1983 by Leon Ong Chua.[1] It is a three-dimensional autonomous electronic circuit that exhibits classic chaos behavior. Because Chua's circuit has rich chaotic dynamics and a simple circuit structure, it has been one of the best chaotic circuits for experimental observation and study.[2-6]

In Ref. 7 and Ref. 8, hybrid realizations of Chua's circuit, combining the circuit topologies proposed for nonlinear resistor and inductor element, are presented. However, there is no study on how to observe the state variable $z=Ri_L$ which represents the product of the resistance R and the inductor current i_L. In this paper, Chua's circuit will be implemented by an inductance simulator and a piecewise linear resistor consisting of six resistors and two operational amplifiers, as shown in Ref. 8. Moreover, the variable z will be obtained via scaling of the voltage of the resistor in the inductance simulator to show the chaotic attractors of Chua's circuit completely.

2. Chua's Circuit

Chua's circuit consists of two capacitors, one inductor, one linear resistor and one nonlinear resistor named Chua's diode, as shown in Fig. 1. The inductance, resistance, and capacitances respectively are L = 21.83 mH, R = 1.875 kΩ, C1 = 10 nF and C2 = 100 nF.

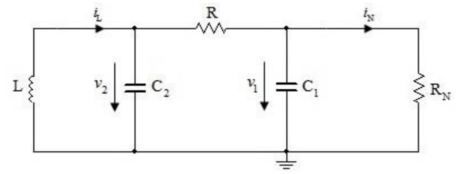

Fig. 1. Chua's circuit

The dynamic equation of Chua's circuit is represented as

$$\begin{aligned}\dot{x} &= \alpha\left[y - x - h(x)\right],\\ \dot{y} &= x - y + z,\\ \dot{z} &= -\beta y.\end{aligned}$$

Where $\alpha = C_2/C_1 = 10$, $\beta = (R^2C_2)/L = 16.1$, and

$$h(x) = m_1 x + \frac{1}{2}(m_0 - m_1)\left[|x+\mathrm{E}| - |x-\mathrm{E}|\right].$$

Where E=1.1, m_0 = -1.4205, and m_1 = -0.7671.

Let the initial values of Chua's circuit be $(x_0, y_0, z_0) = (0.1, 0.2, 0.5)$, then the Lyapunov exponents are $\lambda_1 = 0.3193 > 0$, $\lambda_2 = -0.0001 \approx 0$ and $\lambda_3 = -3.1683 < 0$. It indicates that the system is chaotic. The simulation phase portraits of Chua's circuit are shown in Fig. 3(a1)-(a3).

3. Implementation of Chua's Circuit

3.1. *Inductance simulator*

Because the internal resistance of an ordinary inductor is too large and the inductance is difficult to alter, it is not suitable to put an ordinary inductor in a chaotic circuit which is very sensitive to the parameters. Hence, an inductance simulator should be used to construct Chua's circuit instead of an ordinary inductor.[6-8] The equivalent circuit of the inductance simulator is shown in the left dashed box of Fig. 2, where $R_1 = R_2 = R_3 = 1$ kΩ, $R_4 = 2.183$ kΩ and $C_5 = 10$ nF. The operational amplifiers used in this paper are TL082CP and their supply voltages are ±9V, so that their output saturation voltage is 8.3V. The inductance simulator has no equivalent internal resistance, and its inductance is easy to adjust.

Assume that the currents through the resistors R_1, R_2, R_3, R_4 and the capacitor C_5 respectively are i_1, i_2, i_3, i_4 and i_5, whose reference directions are all from top to bottom. When the operational amplifiers U1A and U1B are being operated in linear mode, it is obtained that

$$i_2 = i_3, i_4 = i_5, i_1\mathrm{R}_1 = -i_2\mathrm{R}_2, i_3\mathrm{R}_3 = -i_5\left(1/\mathrm{j}\omega\mathrm{C}_5\right).$$

By solving the equations above, the relationship between the currents i_1 and i_4 is expressed as

$$i_4 = \left(\mathrm{j}\omega\mathrm{R}_1\mathrm{R}_3\mathrm{C}_5/\mathrm{R}_2\right)i_1.$$

According to the definition of two-terminal network impedance, the equivalent impedance of the inductance simulator is

$$\mathrm{Z} = i_4\mathrm{R}_4/i_1 = \mathrm{j}\omega\mathrm{R}_1\mathrm{R}_3\mathrm{R}_4\mathrm{C}_5/\mathrm{R}_2 \to \mathrm{L} = 21.83\mathrm{mH}.$$

As a result, the desired inductance is obtainable by adjusting the resistance of the resistor R_4.

3.2. *Chua's diode*

Chua's diode consists of two operational amplifiers U2A, U2B and six resistors, as shown in the right dotted box of Fig. 2. The resistances are $R_6 = R_7 = 22$ kΩ, $R_8 = 3.3$ kΩ, $R_9 = R_{10} = 220$ Ω and $R_{11} = 2.2$ kΩ. The design procedure of Chua's diode's equivalent circuit can be found in detail in Ref. 6.

3.3. *Realization of Chua's circuit*

Chua's circuit is implemented by replacing the ordinary inductor and Chua's diode shown in Fig. 1 with the inductance simulator and the equivalent circuit of Chua's diode. The whole analog Chua's circuit is presented in Fig. 2.

4. Phase Portraits of Chua's Circuit

4.1. *Observation of state variable z*

For the observation of all variables of Chua's circuit, the state variable $z = Ri_L$ should be observed even if it is not an actual measurable physical quantity. The observation circuit of *z* consists of four operational amplifiers U3A, U3B, U4A, U4B and some related resistors, as shown in

Fig. 2. The resistances are $R_{12} = R_{13} = R_{14} = R_{15} = R_{16} = R_{18} = 10$ kΩ, and the resistance of the resistor R_{17} is variable.

The relationship between the voltage u_{R1} and the state variable z can be described as

$$i_L = -i_1 \rightarrow z = Ri_L = -\frac{R}{R_1}u_{R1} = -\frac{R_{17}}{R_{16}}u_{R1}.$$

Thus, the design procedure of the observation circuit can be divided into three parts. Firstly, U3A and U3B are two input voltage followers used to get the potentials of both ends of R_1. Secondly, U4A and the resistors R_{12}-R_{15} constitute the differential input proportion operational circuit used to work out the voltage u_{R1}. Finally, the state variable z can be obtained from the output of the inverse proportion operational circuit consisting of U4B and the resistors R_{16} and R_{17}. Furthermore, the operational amplifiers U3A and U3B can also insulate the main Chua's circuit from impact of the observation circuit.

It is found that some peak (valley) values of z are larger than the output saturation voltage of the operational amplifier by numerical simulation, such that the cut-off (saturation) distortion will occur when observing the experiment phase portraits of x-z and y-z on the oscilloscope. Hence, scaling of z should be performed. Adjust the resistance of the resistor R_{17} to $R_{17} = 14.77$ kΩ, then the distortions will no longer occur. Consequently, the experiment value of z is about 0.8 times its simulation value. For a convenient comparison, the simulation value of z is adjusted to 0.8 times its original value in Fig. 3(a2)-(a3).

4.2. *Comparison of simulation and experiment phase portraits*

The experiment phase portraits of Chua's circuit are shown in Fig. 3(b1)-(b3). The scales of x, y and z are 2 V/Div, 0.5 V/Div and 2 V/Div, respectively. By comparing figures (b1)-(b3) with (a1)-(a3), it can be confirmed that here is a good qualitative agreement between the numerical simulation and the experimental realization. It demonstrates that the realization of Chua's circuit is correct and available for other research such as chaos synchronization of Chua's circuits.

5. Conclusions

The hybrid analog circuit implementation of Chua's circuit, which exploits the circuit topologies for the inductor element and nonlinear resistor, is proposed in this paper. The observation circuit for the directly nonmeasurable variable z is also designed for comprehensive investigation of the chaotic attractors of Chua's circuit. It is necessary to observe all chaotic state variables, especially for full state chaos synchronization. For example, in synchronization physical test of two identical three-dimensional chaotic circuits, the assumption that all three states would synchronize as long as two of them synchronized may be incorrect, so that all of the states generated from the circuits should be investigated to verify the synchronization phenomena. Therefore, the study in this paper has some engineering significance.

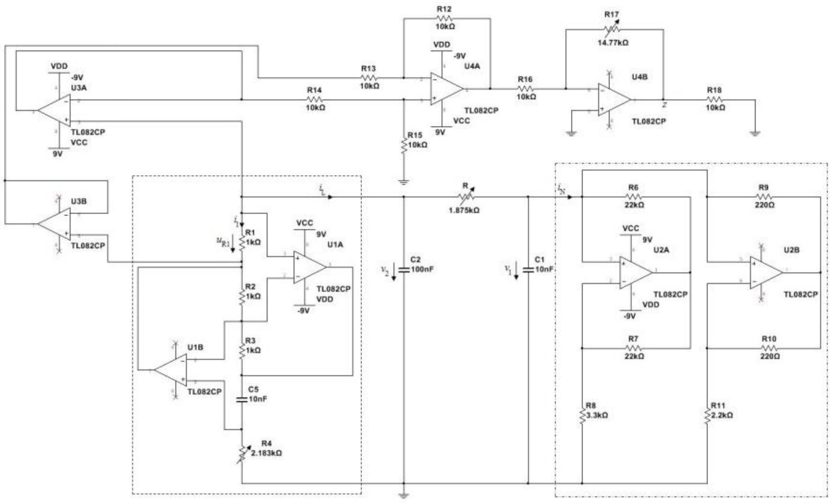

Fig. 2. Analog Chua's circuit

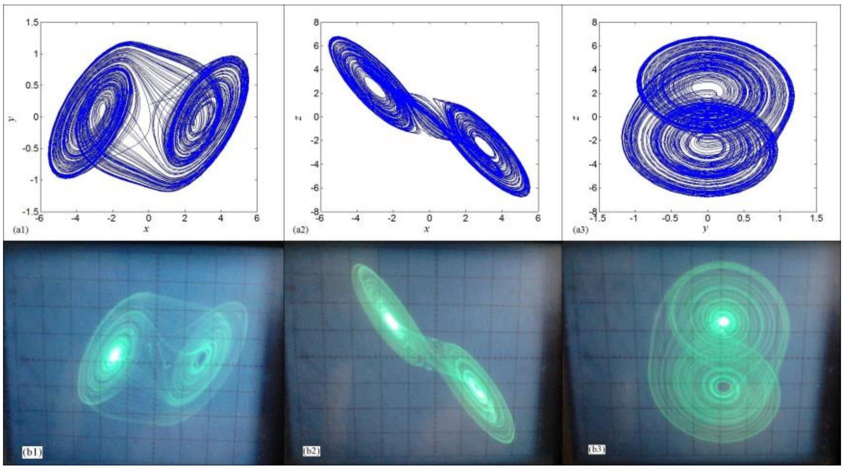

Fig. 3. Comparison of phase portraits: Simulation-Experiment: (a1)-(b1) *x*-*y*; (a2)-(b2) *x*-*z*; (a3)-(b3) *y*-*z*

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