Synchronization of the Fractional-order Permanent Magnet Synchronous Motor

Wei Xue
Department of Automation , Tianjin University of Science and Technology,
1038 Dagunanlu Road, Hexi Distric, Tianjin 300222, PR China

Xue Li
Department of Automation , Tianjin University of Science and Technology,
1038 Dagunanlu Road, Hexi Distric, Tianjin 300222, PR China
E-mail:xuewei@tust.edu.cn,lixueTJKJ@163.com
www.tust.edu.cn

Abstract

In this paper, the chaotic synchronization of the fractional-order permanent magnet synchronous motor is investigated. The presented control scheme is simple and flexible, and it is suitable both for design and implement in practice. According to the stability theory of fractional-order linear system, the chaotic synchronization of the drive system and the response system is achieved. Adopting the nonlinear feedback method, a nonlinear feedback controller in theoretical analysis is designed. The obtained numerical simulation results agree with that of theoretical analysis, which can further demonstrate the feasibility and effectiveness of the proposed synchronization scheme.

Keywords:Chaos; Fractional-order; Permanent magnet synchronous motor; Nonlinear feedback synchronization; Numerical simulation.

1. Introduction

Since permanent magnet synchronous motor (PMSM) has excellent properties such as simple structure, high torque-to-weight ratio, low manufacturing cost and high torque-to-inertia ratio, it is used in the industrial applications more widely. Studies have shown that, when the motor works under certain parameters and operating conditions, the chaotic behavior will occur, such as intermittent ripples of torque, the low-performance property speed control, irregular current system noise and so on. The study of the integer-order chaotic model of the PMSM is comprehensive. For PMSM, the fractional-order chaotic model can reflect its chaotic characteristics more accurately. Some investigation of fractional-order PMSM are still in its infancy. Many scholars have researched on fractional-order chaotic systems, such as fractional-order Rössler system,[1] fractional-order Chen system,[2] fractional-order Liu system,[3] fractional-order Lü system,[4] and fractional-order generalized augmented Lü system .[5]

The chaotic behavior of PMSM is harmful in most cases. However, the chaotic behavior of PMSM can also be useful in some applications, such as in industrial mixing process, the chaotic motion itself gives the extension and folding characteristics of good mixing. It is of practical significance to improve the efficiency of industrial mixing. Therefore, it is necessary to study the chaos synchronization of fractional-order PMSM. Recently, many synchronization approaches to the chaotic system have been proposed such as linear feedback synchronization,[6] adaptive synchronization,[7] impulse synchronization,[8-9] projective synchronization,[10] active synchronization,[11] generalized synchronization,[12] sliding mode control,[13] and lag synchronization.[14] Adopting the nonlinear feedback method, a nonlinear feedback controller in theoretical analysis is designed. And the synchronization of the fractional-order PMSM is achieved according to the stability theory of fractional-order linear system. The obtained numerical simulation results agree with the theoretical analysis results, which can further demonstrate the feasibility and effectiveness of the proposed synchronization scheme.

2. Synchronization of fractional-order PMSM

2.1. General nonlinear feedback synchronization scheme

Here, the fractional-order chaotic system is considered as follow:

$$\frac{d^\alpha x}{dt^\alpha} = Ax + f(x) \tag{1}$$

where $\alpha \in (0,1]$, $A \in R^{n\times n}$ is the matrix of system parameters, $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)^{\mathrm{T}} \in R^n$ is the state variable, $f(\boldsymbol{x}) = (f_1(x), f_2(x), \ldots, f_n(x))^{\mathrm{T}} \in \boldsymbol{R}^{n\times 1}$ is continuous nonlinear vector function. The system(1) can be used as a driving system, and the corresponding response system is given by:

$$\frac{d^\alpha y}{dt^\alpha} = By + g(y) + u \tag{2}$$

where $\alpha \in (0,1]$, $B \in R^{n\times n}$ is the matrix of system parameters, $\boldsymbol{y} = (y_1, y_2, \cdots, y_n)^{\mathrm{T}} \in \boldsymbol{R}^n$ is the state variable, $g(\boldsymbol{y}) = (g_1(y), g_2(y), \cdots, g_n(y))^{\mathrm{T}} \in \boldsymbol{R}^{n\times 1}$ is the state variable and $\boldsymbol{u} = (u_1, u_2, \cdots, u_n)^{\mathrm{T}}$ is the controller to be determined in order to realize nonlinear feedback synchronization.

Let the error vector be $\boldsymbol{e} = \boldsymbol{y} - \boldsymbol{x} = (e_1, e_2, \cdots, e_n)^{\mathrm{T}}$, and the appropriate controller $\boldsymbol{u}$ should be designed to make $\lim_{t\to\infty} \boldsymbol{e} = 0$.

The chaotic model of PMSM can be expressed as the following form[15]:

$$\begin{cases} \dfrac{d^\alpha x_1}{dt^\alpha} = -x_1 + x_2 x_3 \\ \dfrac{d^\alpha x_2}{dt^\alpha} = -x_2 - x_1 x_3 + \gamma x_3 \\ \dfrac{d^\alpha x_3}{dt^\alpha} = \sigma(x_2 - x_3) \end{cases} \tag{3}$$

where $\alpha \in (0,1]$ is the fractional order, $\sigma > 0$ and $\gamma > 0$ are constants.

In order to observe the nonlinear feedback synchronization of PMSM, the response systems can be defined as follow:

$$\begin{cases} \dfrac{d^\alpha y_1}{dt^\alpha} = -y_1 + y_2 y_3 + u_1 \\ \dfrac{d^\alpha y_2}{dt^\alpha} = -y_2 - y_1 y_3 + \gamma y_3 + u_2 \\ \dfrac{d^\alpha y_3}{dt^\alpha} = \sigma(y_2 - y_3) + u_3 \end{cases} \tag{4}$$

where $u^T = (u_1, u_2, u_3)$ is the nonlinear controller to be designed for PMSM with the same parameters σ and γ in spite of the differences in initial conditions.

2.2. Design of synchronization controller

The drive system(3) can be described as system(1) in the form of:

$$\frac{d^\alpha x}{dt^\alpha} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & \gamma \\ 0 & \sigma & -\sigma \end{pmatrix} x + \begin{pmatrix} +x_2 x_3 \\ -x_1 x_3 \\ 0 \end{pmatrix} \tag{5}$$

Similarly, the response system(4) is written in the form of system(2) as follow:

$$\frac{d^\alpha y}{dt^\alpha} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & \gamma \\ 0 & \sigma & -\sigma \end{pmatrix} y + \begin{pmatrix} +y_2 y_3 \\ -y_1 y_3 \\ 0 \end{pmatrix} + u. \tag{6}$$

Here, the nonlinear feedback method to realize the synchronization of PMSM is adopted. And controller $\boldsymbol{u} = f(\boldsymbol{x}) - g(\boldsymbol{y}) - K\boldsymbol{e}$ can be selected as the following form:

$$u = f(x) - g(y) - Ke = \begin{pmatrix} x_2 x_3 \\ -x_1 x_3 \\ 0 \end{pmatrix} - \begin{pmatrix} y_2 y_3 \\ -y_1 y_3 \\ 0 \end{pmatrix} - Ke \tag{7}$$

Where $\boldsymbol{e} = \boldsymbol{y} - \boldsymbol{x} = (e_1, e_2, e_3)^{\mathrm{T}}$.

From *Eq.*(5), *Eq.*(6), and *Eq.*(7), the error system is obtained, which can be expressed as:

$$\frac{d^\alpha e}{dt^\alpha} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & \gamma \\ 0 & \sigma & -\sigma \end{pmatrix} e - Ke \tag{8}$$

$$K = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \gamma \\ 0 & \sigma & 0 \end{pmatrix} \qquad (9)$$

From *Eq*.(8) and *Eq*.(9), the error system is obtained as follow:

$$\frac{d^{\alpha}e}{dt^{\alpha}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -\sigma \end{pmatrix} e \qquad (10)$$

From *Eq*.(10), the eigenvalues are $\lambda_1 = -1$, $\lambda_2 = -1$, and $\lambda_3 = -\sigma$. Because $\sigma > 0$, all eigenvalues $\lambda_i \ (i = 1,2,3)$ are negative, that is $|\arg(\lambda_i)| > \frac{\alpha\pi}{2}$. According to Ref.[16], it will be asymptotical stability of the origin of the error system, which means that $\lim_{t\to\infty} \boldsymbol{e} = 0$. Therefore, the goal of synchronizing the fractional-order PMSM is achieved.

3. Numerical simulation

From system(3), the phenomenon of chaos can be observed when α =0.95, therefore we will concentrate on the 2.85-order system in the following part. By adopting the time domain method based on predictor-corrector to conduct the numerical simulation, the chaotic attractors of system(3) is obtained. When α=0.95,σ=11, and γ=90, and phase portraits are shown in Fig.1:

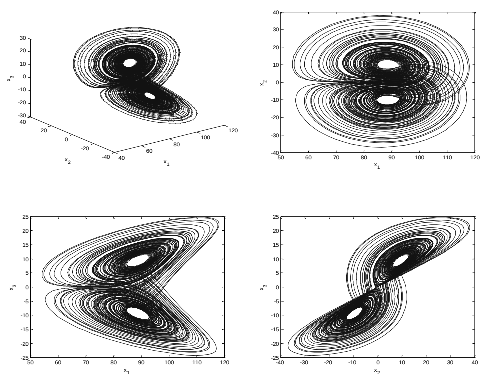

Fig.1. Phase portraits of system(3) with α =0.95,σ=11, and γ=90: (a) $x_1 - x_2 - x_3$;(b) $x_1 - x_2$ (c) $x_1 - x_3$;(d) $x_2 - x_3$.

In this section, we conduct the numerical simulation of the synchronization via Matlab, taking the 2.85-order PMSM as an example. The system parameters are chosen to be α = 0.95, σ =11, and γ=90, and in time steps of 0.01. The initial states of the drive system and response system are assumed as $x_1(0)$ = 0.8, $x_2(0)$ = 5, $x_3(0)$ = 1.3, $y_1(0)$ = 3, $y_2(0)$ = 6, and $y_3(0)$ = 1, respectively. It can be observed that the drive system and the response system are in asymptotic synchronization in Fig.2. In addition, the curves of synchronization error is obtained as shown in Fig.3. It's clearly showed the synchronization error vector converge to zero as time *goes* to infinity. The obtained numerical simulation results agree with theoretical analysis results.

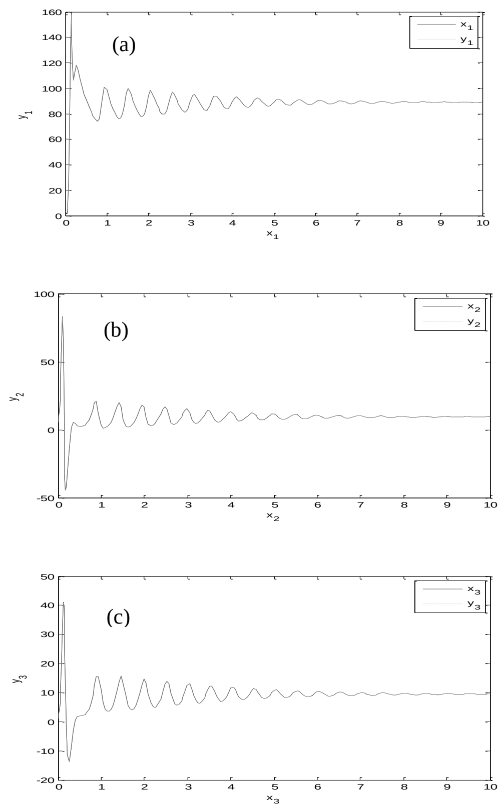

Fig.2. Variation of state variables with time: (a) (x_1, y_1); (b) (x_2, y_2); (c) (x_3, y_3).

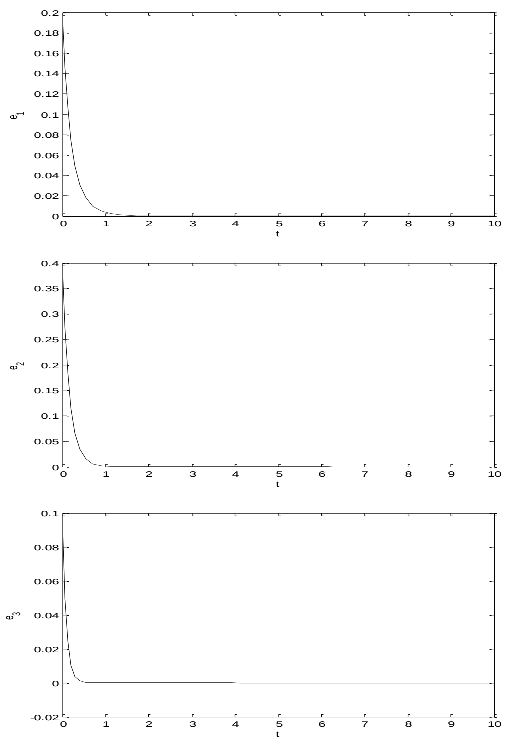

Fig.3 Synchronization errors versus time t (a) $e_1 - t$;

(b) $e_2 - t$;(c) $e_3 - t$.

4. Conclusion

In this paper, a general nonlinear feedback chaotic synchronization scheme for fractional-order PMSM is introduced. By analyzing phase portraits of fractional-order PMSM, we have found there are abundant dynamical behaviors in the fractional-order PMSM. Moreover, the synchronization problem of the fractional-order PMSM has been investigated. Based on the nonlinear feedback method, the synchronization controller to achieve synchronization between the drive system and the response system is designed. The obtained numerical simulation results agree with theoretical analysis, which can further demonstrate the feasibility and effectiveness of the proposed synchronization scheme.

5. Acknowledgement

This work is supported by the Young Scientists Fund of the National Natural Science Foundation of China (Grant No.11202148).

6. References

1. Li C G, Chen G R. Chaos and hyperchaos in the fractional order Rössler equations. *Physics Letters A*. 2004, 341: 55-61.
2. Li C P, Peng G J. Chaos in Chen’s system with a fractional order. *Chaos, Solitons and Fractals*. 2004, 22: 443-450.
3. Daftardar-Gejji V, Bhalekar S. Chaos in fractional ordered Liu system. *Computers and Mathematics with Applications*. 2010, 59(3): 1117-1127.
4. Lu J G. Chaotic dynamics of the fractional order Lü system and its synchronization. *Physics Letters A*. 2006, 354(4): 305-311.
5. Xue W, Xu J K, Cang S J, et al. Synchronization of the fractional-order generalized augmented Lü system and its circuit implementation. *Chinese Physics B*. 2014, 23(6): 060501.
6. Liu F, Ren Y, Shan X M, et al. A linear feedback synchronization theorem for a class of chaotic system. *Chaos, Solitons and Fractals*. 2002, 13(4): 723-730.
7. Wang Y W, Guan Z H, Wen X J. Adaptive synchronization for Chen chaotic system with fully unknown parameters. *Chaos, Solutions and Fractals*. 2004, 19(4): 899-903.
8. Ren Q S, Zhao J Y. Impulsive synchronization of coupled chaotic systems via adaptive-feedback approach. *Physics Letters A*. 2006, 356(3): 226-230.
9. Mohammad H, Mahsa D. Impulsive synchronization of Chen’s hyperchaotic systems. *Physics Letters A*. 2006, 355(4, 5): 342-347.
10. Meng J, Wang X Y. Generalized projective synchronization of a class of delayed neural networks. *Physics Letters A*. 2008, 22(3): 181-190.
11. Li G H. An active synchronization for two modified chua circuits. *Chinese Physics B*. 2005, 14(3): 472-475.
12. Shih Y L, Zheng M G. Generalized synchronization of chaotic systems with different order by fuzzy logic constant controller. *Expert Systems with Applications*. 2011, 38(3): 2302-2310.
13. Yau H T, Yan J J. Desgn of sliding mode controller for Lorenz chaotic system with nonlinear input. *Chaos, Solitons and Fractals*. 2004, 19(4): 891-898.
14. Shahverdiev E M, Sivaprakasam S, Shore K A. Lag synchronization in time-delayed systems. *Physics Letters A*. 2002, 292(6): 320-324.
15. Xue W , Li Y L , Cang S J , Jia H Y , Wang Z H. Chaotic behavior and circuit implementation of a fractional-order permanent magnet synchronous motor model. *Journal of the Franklin Institute*. 2015, 352: 2887-2898.
16. Matignon D. Stability results for fractional differential equations with application to control processing. *In Computational Engineering in Systems and Application multiconference, Lille*. 1996, 2: 963-968.