Adaptive Control of Discrete-Time Systems Using Multiple Fixed and One Adaptive Identification Models

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Abstract

The adaptive control of a linear time-invariant discrete-time system using multiple models is considered in this paper. Based on the prediction errors of a finite number of fixed and one adaptive identification models, a new weighting algorithm is proposed for improving system performance. The principal contributions of the paper are the proofs of global stability and the convergence of the overall system. Computer simulation results are included to complement the theoretical results.

Keywords: discrete-time; multiple model adaptive control; weighting algorithm; stability; convergence.

1. Introduction

The control of dynamical systems with large uncertainties is of great interest. Such problem arises when there are large parameter variations. So, an approach gains development: the use of multiple models to identify the unknown plant. In any time, the closest model will be identified according to the performance index.[1] Then, the corresponding controller is used to control the system.

The earliest research on weighted multiple model adaptive control (WMMAC) appeared in 1970's.[2, 3] In that system, posterior possibility evaluator (PPE) algorithm played the key role. Later, K. S. Narendra put forward the concept of indirect multi-model adaptive control.[4] The basic idea is that multiple fixed models are established off-line to cover uncertainty plant. At the same time, one or two adaptive identification models are present for global stability and the convergence of the overall system.[4] The scheme is designed easily.

Therefore it has been widely used in recent years. This is also one basis of the algorithm in this paper.
In this paper, we made efforts from two aspects to improve the stability of WMMAC. On one side, a new algorithm for calculating controller weights is proposed. On the other side, a finite number of fixed and one adaptive identification models are designed.

It is worth noting that this paper is focused on the linear time-invariant discrete-time system because most practical systems are controlled by computers that are discrete in nature. [3]

2. Description of Weighted Multiple Model Adaptive Control

In order to analyze system clearly, a concise block diagram is shown in Fig.1. The simplified block diagram shows a general discrete-time WMMAC system. The details of every local control strategy and weighting algorithm are omitted.
The meanings of symbols are shown: P is the uncertainty plant; C_i is a local controller, which is corresponded to the fixed model; C_θ is the local controller corresponded to the adaptive identification model; $p_i(k)$ is a weight for its local controller; $p_\theta(k)$ is the weight for the adaptive identification controller. Through the algorithm, local controller weights are known. Then, the weighed sum $u(k)$ is obtained by the local controller weights.

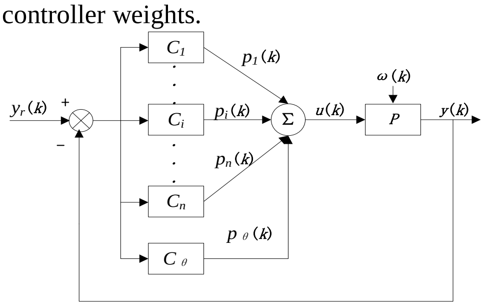

Fig. 1. Concise block diagram of a general WMMAC system

Consider the following discrete-time dynamical plant P, the mathematical model is

$$A(z^{-1})y(k) = z^{-d}B(z^{-1})u(k) + \omega(k) \tag{1}$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{n_a} z^{-n_a} \tag{2}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{n_b} z^{-n_b} \tag{3}$$

$y(k)$ and $u(k)$ are the output and the input. The $\omega(k)$ is a zero-mean white noise with constant variance.
The plant P also can be written as

$$y(k) = \varphi^T(k)\theta + \omega(k) \tag{4}$$

where

$$\varphi(k) = [-y(k-1), \cdots, -y(k-n_a), u(k-d), \cdots, u(k-d-n_b)] \tag{5}$$

$$\theta = [a_1, \cdots, a_{n_a}, b_0, \cdots, b_{n_b}] \tag{6}$$

$\mathrm{M} = \{M_i, M_\theta, i = 1, 2, \cdots, N\}$ is the model set that includes a finite number of fixed and one adaptive identification models in this system. For every fixed model, the corresponded controller is designed by pole placement control. At the same time, the one adaptive identification model is designed by indirect pole assignment self-tuning control. The forgetting factor recursive extended least square is used to estimated parameter [5]. That is

$$\begin{cases} \hat{\theta}(k) = \hat{\theta}(k-1) + K(k)[y(k) - \hat{\varphi}^T(k)\hat{\theta}(k-1)] \\ K(k) = \dfrac{P(k-1)\hat{\varphi}(k)}{\lambda + \hat{\varphi}^T(k)P(k-1)\hat{\varphi}(k)} \\ P(k) = \dfrac{1}{\lambda}[I - K(k)\hat{\varphi}^T(k)]P(k-1) \end{cases} \tag{7}$$

where

$$\begin{cases} \hat{\varphi}(k) = [-y(k-1), \cdots, -y(k-n_a), u(k-d), \cdots, u(k-d-n_b)] \\ \hat{\theta}(k) = [\hat{a}_1, \cdots, \hat{a}_{n_a}, \hat{b}_0, \cdots, \hat{b}_{n_b}]^T \end{cases} \tag{8}$$

For each model, including the adaptive identification model certainly, its output $y_{mi}(k)$ is used to define the output error. That is

$$e_i(k) = y(k) - y_{mi}(k) \tag{9}$$

which is used to calculate the weight for the local controller. As shown in Fig.1, the global control $u(k)$ is obtained by

$$u(k)=\sum_{i=1}^{N+1}p_i(k)u_i(k) \tag{10}$$

In this paper, the algorithm to calculate weight is proposed, that is

$$p_i(0)=l_i(0)=\frac{1}{N+1} \tag{11}$$

$$l_i'(k)=\alpha+\frac{1}{k}\sum_{j=1}^{k}e_i^2(j) \tag{12}$$

where $\alpha>0$, which is a small constant to avoid $l_i'(k)=0$

$$l'_{\min}(k)=\min_i\{l_i'(k)\} \tag{13}$$

$$l_i(k)=\frac{l'_{\min}(k)}{l_i'(k)}l_i(k-1) \tag{14}$$

$$p_i(k)=\frac{l_i(k)}{\sum_{j=1}^{N+1}l_j(k)} \tag{15}$$

This algorithm is very simple in calculation. The results and their proofs regarding its convergence character are presented in Ref. 3 and 6-8.

3. Simulation Studies

In the previous section, the algorithm for calculating controller weights is presented. And a finite number of fixed and one adaptive identification models are discussed. In this section, the results of computer simulations will be presented. Using a single example, the effectiveness of the adaptive control of discrete-time systems with multiple fixed and one adaptive identification models, is discussed in detail.

Consider an uncertain discrete-time plant

$$(1+a_1z^{-1}+a_2z^{-2})y(k)=z^{-1}(b_0+b_1z^{-1})u(k)+\omega(k) \tag{16}$$

where $\omega(k)$ is a zero-mean white noise with constant variance. This discrete-time model can be obtained by the simple continuous-time in (17) with sample time $t_s=0.5s$ and the zero order hold.

$$G(s)=\frac{k}{s^2-3s+2} \tag{17}$$

For simplicity, four fixed local models are used as the uncertainty of k, that is, $k_1=0.7$, $k_2=0.8$, $k_3=1.2$, $k_4=1.3$. The four corresponded controllers are designed by pole placement control. At the same time, the one adaptive identification model is designed by indirect pole assignment self-tuning control. The forgetting factor recursive extended least square is used to estimated parameter.

The controller 1 is

$$\begin{aligned}&(1-4.3670z^{-1}+4.4817z^{-2})y(k)\\&=q^{-1}(0.1473z^{-1}+0.2428z^{-2})u(k)+\omega(k)\end{aligned} \tag{18}$$

The controller 2 is

$$\begin{aligned}&(1-4.3670z^{-1}+4.4817z^{-2})y(k)\\&=q^{-1}(0.1683z^{-1}+0.2775z^{-2})u(k)+\omega(k)\end{aligned} \tag{19}$$

The controller 3 is

$$\begin{aligned}&(1-4.3670z^{-1}+4.4817z^{-2})y(k)\\&=q^{-1}(0.2525z^{-1}+0.4163z^{-2})u(k)+\omega(k)\end{aligned} \tag{20}$$

The controller 4 is

$$\begin{aligned}&(1-4.3670z^{-1}+4.4817z^{-2})y(k)\\&=q^{-1}(0.2735z^{-1}+0.4510z^{-2})u(k)+\omega(k)\end{aligned} \tag{21}$$

The controller 5 is designed according to the adaptive identification model by indirect pole assignment self-tuning control. The forgetting factor recursive extended least square is used to estimated parameter. The initial value of θ is $0.001*ones(na+nb+1,1)$; the initial value of P is $10^6*eye(na+nb+1)$. The forgetting factor λ is 1. The calculation in detail is shown in formula (7) and (8).

The expected closed-loop characteristic polynomial is corresponds to the characteristic polynomial of the following continuous-time second-order system

$$\frac{{\omega_n}^2}{s^2+2\xi\omega_n s+{\omega_n}^2} \tag{22}$$

where ξ is 0.7, ω_n is 1.

The true model of plant is not included in the four fixed model, which k is 1. The simulation results are shown in Fig.2 to Fig.4.

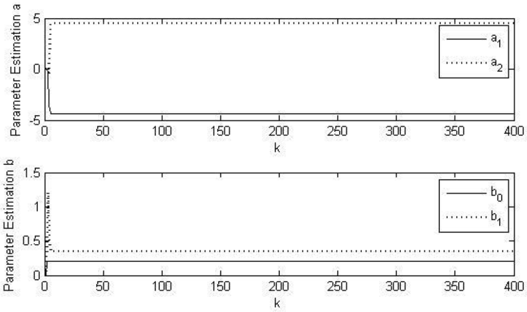

Fig. 2. Parameter estimation of the adaptive identification model

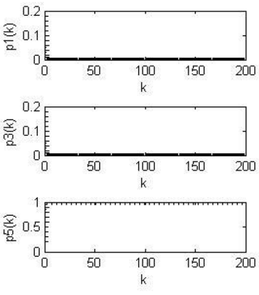

Fig. 3. Controller weight signals

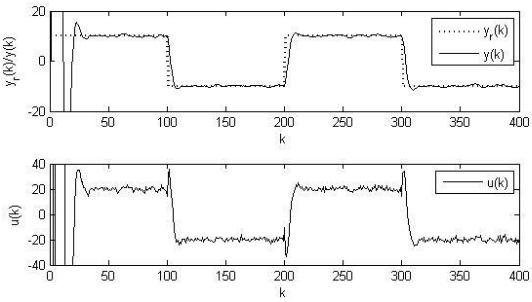

Fig. 4. Output, reference and control signals

As shown in Fig. 3, if the true model of plant is not included in the four fixed models, controller 5 designed according to the adaptive identification model is chosen rapidly in a good performance.

4. Conclusions

In this paper, a finite number of fixed and one adaptive identification models are designed. At the same time, the new algorithm for calculating controller weights is proposed. The weighted value can converge quickly. The controller designed according to the adaptive identification model performs well when the true one is not included in the fixed models, which improves the stability of WMMAC.

Acknowledgements

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