Trajectory Tracking Control for Omnidirectional Mobile Robots with Input Constraints

Wenhao Zheng
The Seventh Research Division and the Department of Systems and Control, Beihang University (BUAA)
Beijing, 100191, China

Yingmin Jia
The Seventh Research Division and the Department of Systems and Control, Beihang University (BUAA)
Beijing, 100191, China
E-mail: zhengwenhao@163.com, ymjia@buaa.edu.cn
www.buaa.edu.cn

Abstract

In this paper, a trajectory tracking controller based on kinematics for omnidirectional mobile robots with input constraints is presented. The tracking error model with the control law is proved to be global asymptotic stability by Lyapunov stability theory. The input limits can be described as an octahedron in three-dimensional space, so that a spatial vector analysis method is proposed to design time-varying feedback parameters to limit robot inputs. Simulation results show the feasibility and effectiveness of the control strategy.

Keywords: omnidirectional mobile robot, tracking control, input constraints, time-varying feedback control.

1. Introduction

Omnidirectional mobile robots, with the ability of three degree-of-freedom motion in the plane, have been widely applied in different fields of the society, which has brought to the forefront in recent years. Many methods about tracking control have been proposed, such as sliding mode control[1], model predictive control[2], fuzzy control[3] and their combinations. In the practical systems, the velocities of driving motors is limited, which means that the inputs of the mobile robot are subject to constraints. The control law will be affected by input constraints. Some results about tracking control with input constraints can be found. In Ref.4, the control signal to a given reference system was modified to make the error dynamics robust to the saturation constraints. In Ref.5, the diamond-shaped input constraints was considered which made the controller more effective.

This paper mainly focuses on the tracking control of omnidirectional mobile robots with input constraints. The control law is proposed with time-varying feedback parameters to satisfy the input constraints. Then, a spatial vector analysis method is used to design parameters which is devoted to find a suitable robot inputs in the restricted area. And the tracking error system can be global asymptotic stability with the controller. Comparing with existing results, this paper primarily contributes to the novel solution of input constraints of omnidirectional mobile robots.

The structure of the rest paper is organized as follows. The section 2 introduces the tracking error system. In the section 3, the tracking controller is designed with input constraints using the spatial vector analysis method. Simulation results are presented to show the validity of the control law in the section 4. In the end, the section 5 summarizes the whole paper and draws the conclusion.

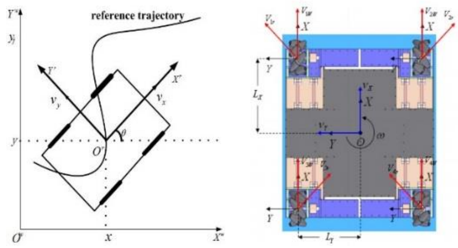
2. Problem Statement

2.1. *Kinematic model of omnidirectional mobile robot*

As shown in Fig.1.(a), the four wheeled omnidirectional mobile robot is considered in this paper. The kinematic of omnidirectional mobile robots is

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ \omega \end{pmatrix} \tag{1}$$

Where (x, y) are the robot position in Cartesian coordinates and θ is the robot orientation. The control inputs of the robot are (v_x, v_y, ω) : (v_x, v_y) here denote robot's longitudinal velocity and transverse velocity, and ω represents the rotate speed of the robot. As a matter of fact, the kinematic of all kinds of omnidirectional robots including three wheeled robots or others with different omnidirectional wheels can be expressed as equation (1).



(a) (b)

Fig. 1. The schematic of trajectory tracking (a) and Four-wheel structure of the robot (b).

In the Fig.1.(b), the framework of omnidirectional mobile robot has been proposed. The transformation relation between the speeds $(v_{1w}, v_{2w}, v_{3w}, v_{4w})$ of the four driving wheels and the omnidirectional robot's speed (v_x, v_y, ω) is descried as the equation (2), where $d = L_X + L_Y$ and L_X is the X-axis distance from each wheel to the center of gravity (the similar definition for L_Y).

$$\begin{pmatrix} v_{1w} \\ v_{2w} \\ v_{3w} \\ v_{4w} \end{pmatrix} = \begin{pmatrix} 1 & -1 & -d \\ 1 & 1 & d \\ 1 & 1 & -d \\ 1 & -1 & d \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ \omega \end{pmatrix} \tag{2}$$

2.2. *Input constraints*

In the practical systems, the velocities of motors are limited. Assume that four driving wheels have the same mechanical characteristics, therefore, it is obvious that the same input constraint $|v_{iw}| \le V$ $(i = 1, 2, 3, 4)$ should be satisfied, and V is the maximum wheel velocity. Using the constraint and the equation (2), we can deduce the equation (3) as follows:

$$\left|\frac{v_x}{V}\right| + \left|\frac{v_y}{V}\right| + \left|\frac{\omega}{V/d}\right| \le 1 \tag{3}$$

which can be described in three-dimensional space as shown in Fig.2: the restricted zone of robot inputs is an octahedron with the geometric representation.

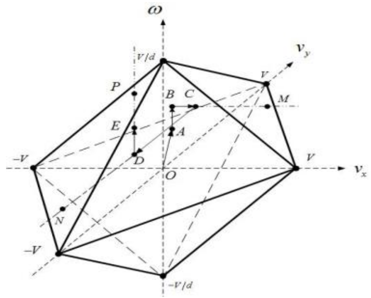


Fig. 2. Input constraints area in three-dimensional space

2.3. *Error model of trajectory tracking*

Assume that the reference trajectory for the omnidirectional mobile robot satisfies the kinematic

$$\begin{pmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{pmatrix} = \begin{pmatrix} \cos\theta_r & -\sin\theta_r & 0 \\ \sin\theta_r & \cos\theta_r & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_{xr} \\ v_{yr} \\ \omega_r \end{pmatrix} \tag{4}$$

and the input constraints

$$\left|\frac{v_x}{V}\right| + \left|\frac{v_y}{V}\right| + \left|\frac{\omega}{V/d}\right| \le 1 - \varepsilon \tag{5}$$

In the equation (4) and (5), $(x_r, y_r, \omega_r, v_{xr}, v_{yr}, \omega_r)$ are the desired values for $(x, y, \omega, v_x, v_y, \omega)$. Besides, ε is a constant with the condition $0 < \varepsilon < 1$ which guarantees the robot has the ability to track the reference trajectory successfully.

We can define the tracking errors as follows:

$$\begin{pmatrix} x_e \\ y_e \\ \theta_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{pmatrix} \tag{6}$$

Combining the equation (1) and (4) with differentiating both side of (6), error model can be expressed as

$$\begin{aligned} \dot{x}_e &= v_{xr}\cos\theta_e - v_{yr}\sin\theta_e + \omega y_e - v_x \\ \dot{y}_e &= v_{xr}\sin\theta_e + v_{yr}\cos\theta_e - \omega x_e - v_y \\ \dot{\theta}_e &= \omega_r - \omega \end{aligned} \tag{7}$$

The destination of control law design is to find the appropriate inputs (v_x, v_y, ω) which are subject to the input constraints (3) to meet the desired outcome:

$$\lim_{t\to 0} x_e \to 0, \lim_{t\to 0} y_e \to 0, \lim_{t\to 0} \theta_e \to 0 \tag{8}$$

3. Controller Design

3.1. *Control law design without input constraints*

Inspired by the tracking error model and tracking control methods for differential-drive mobile robots, we come up with the control law

$$\begin{aligned} v_x &= v_{xr}\cos\theta_e + k_x x_e \\ v_y &= v_{yr}\cos\theta_e + k_y y_e \\ \omega &= \omega_r + k_t(v_{xr} y_e - v_{yr} x_e)\sin\theta_e / \theta_e + k_\theta \theta_e \end{aligned} \tag{9}$$

Where k_x, k_y, k_θ are positive time-varying parameters and k_t is positive constants. In order to ensure the continuity of the robot inputs, we define $\sin\theta_e/\theta_e = 1$ when $\theta_e = 0$. And it's noticeable that $\sin\theta_e/\theta_e$ is bounded. The tracking errors x_e, y_e and θ_e will converge to zero under the control law (9), which can be proven as follows:

Let Lyapunov function be

$$V = \frac{1}{2}\left(x_e^2 + y_e^2 + \frac{\theta_e^2}{k_t} \right) \tag{10}$$

Considering the tracking error system (7) and the controller (9), the derivative of this Lyapunov function can be expressed as $\dot{V} = -k_x x_e^2 - k_y y_e^2 - \left(k_\theta / k_t\right)\theta_e^2$. When the conditions $k_x, k_y, k_\theta, k_t > 0$ are satisfied, we can infer $\dot{V} \le 0$ and $V(t) \le V(0)$. Furthermore, it's convenient to get the conclusion that $x_e, y_e, \theta_e \to 0$ as $t \to \infty$ with Barbalat's lemma.

3.2. *Feedback parameters design with input constraints*

Define the space vectors

$$\begin{aligned} \overrightarrow{OE} &= \begin{pmatrix} v_x & v_y & \omega_r \end{pmatrix}^T \\ \overrightarrow{OA} &= \begin{pmatrix} v_{xr}\cos\theta_e & v_{yr}\cos\theta_e & \omega_r \end{pmatrix}^T \\ \overrightarrow{AB} &= \begin{pmatrix} 0 & 0 & k_t(v_{xr} y_e - v_{yr} x_e)\sin\theta_e / \theta_e \end{pmatrix}^T \\ \overrightarrow{BC} &= \begin{pmatrix} k_x x_e & 0 & 0 \end{pmatrix}^T, \overrightarrow{CD} = \begin{pmatrix} 0 & k_y y_e & 0 \end{pmatrix}^T \\ \overrightarrow{DE} &= \begin{pmatrix} 0 & 0 & k_\theta \theta_e \end{pmatrix}^T \end{aligned} \tag{11}$$

The controller (7) can be expressed as a space vector $\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}$. Fig.2 shows the relationships of the vectors. According to the equation (5),

$$\left|\frac{v_{xr}\cos\theta_e}{V}\right| + \left|\frac{v_{yr}\cos\theta_e}{V}\right| + \left|\frac{\omega_r}{V/d}\right| \le 1 - \varepsilon \tag{12}$$

$\overrightarrow{OA}$ is in the input area.

If the condition

$$k_t = \frac{\mu\varepsilon}{2d\sqrt{V(0)+\alpha}} \quad (\alpha > 0, 0 < \mu < 1) \tag{13}$$

holds, $\overrightarrow{OB}$ is in the octahedron. We can prove this result as follows:

$$\begin{aligned} &\left|\frac{v_{xr}\cos\theta_e}{V}\right| + \left|\frac{v_{yr}\cos\theta_e}{V}\right| + \left|\frac{\omega_r + k_t(v_{xr} y_e - v_{yr} x_e)\sin\theta_e/\theta_e}{V/d}\right| \\ &\le \left|\frac{v_{xr}\cos\theta_e}{V}\right| + \left|\frac{v_{yr}\cos\theta_e}{V}\right| + \left|\frac{\omega_r}{V/d}\right| + \\ &k_t\left|\frac{(v_{xr} y_e - v_{yr} x_e)\sin\theta_e/\theta_e}{V/d}\right| \\ &\le 1 - \varepsilon + dk_t(|x_e| + |y_e|) \le 1 - \varepsilon + dk_t\sqrt{4V(t)} \\ &\le 1 - \varepsilon + 2dk_t\sqrt{V(0)} < 1 - \varepsilon + \mu\varepsilon < 1 \end{aligned}$$

We can obtain the value of k_t from equation (13)

$$k_t = \frac{-\theta_e^2(0)d + \sqrt{\theta_e^4(0)d^2 + 2[x_e^2(0) + y_e^2(0) + 2\alpha]\mu^2\varepsilon^2}}{2[x_e^2(0) + y_e^2(0) + 2\alpha]d}$$

Define a function $\eta(x)$ which has three features: $\eta(x) \in [0,1]$ *for* $x \in [0,\infty)$, $\eta(x)$ is non-decreasing and $\eta(0) = 0, \eta(\infty) = 1$. In order to design the time-varying feedback parameters k_x, k_y, k_θ, we come up with a geometric method. Sign function is defined as $sign(x)$.

As shown in Fig.2, we can make a line through B and in parallel with v_x, which goes through the constrained boundary with a point of intersection M. Take $\overrightarrow{BC} = \eta(|x_e|)\overrightarrow{BM}$, then, we can get the expression of k_x. And construct another line going through C and in parallel with v_y, there will be another intersection N in the constrained boundary, then make $\overrightarrow{CD} = \eta_2(|y_e|)\overrightarrow{CN}$. It is easy to get the expression of k_y. We can use the same thought to define P and $\overrightarrow{DE} = \eta_3(|\theta_e|)\overrightarrow{DP}$.

In the end, we can get the controller

$$
\begin{aligned}
v_x &= v_{xr}\cos\theta_e + \eta_1(|x_e|)[sign(x_e)(V - |v_{yr}\cos\theta_e| - \\
&\quad d\,|\omega_r + k_t(v_{xr}y_e - v_{yr}x_e)\sin\theta_e/\theta_e|) - v_{xr}\cos\theta_e] \\
v_y &= v_{yr}\cos\theta_e + \eta_2(|y_e|)[sign(y_e)(V - |v_x| - \\
&\quad d\,|\omega_r + k_t(v_{xr}y_e - v_{yr}x_e)\sin\theta_e/\theta_e|) - v_{yr}\cos\theta_e] \\
\omega &= \omega_r + k_t(v_{xr}y_e - v_{yr}x_e)\sin\theta_e/\theta_e + \\
&\quad \eta_2(|\theta_e|)[sign(\theta_e)(V - |v_x| - |v_y|)/d \\
&\quad -\omega_r - k_t(v_{xr}y_e - v_{yr}x_e)\sin\theta_e/\theta_e]
\end{aligned}
\tag{15}
$$

And the time-varying feedback parameters can be expressed as $k_x = \eta_1(|x_e|)[sign(x_e)(V - |v_{yr}\cos\theta_e| - d\,|\omega_r + k_t (v_{xr}y_e - v_{yr}x_e)\sin\theta_e/\theta_e|) - v_{xr}\cos\theta_e]/x_e$, $k_y = \eta_2(|y_e|)[sign(y_e)(V - |v_x| - d\,|\omega_r + k_t(v_{xr}y_e - v_{yr}x_e)\sin\theta_e/\theta_e|) - v_{yr}\cos\theta_e]/y_e$ and $k_\theta = \eta_2(|\theta_e|)[sign(\theta_e)(V - |v_x| - |v_y|)/d - \omega_r - k_t(v_{xr}y_e - v_{yr}x_e)\sin\theta_e/\theta_e]/\theta_e$.

4. Simulation Results

Set the reference trajectory as $x_r = 0.75\sin(2\pi t/45)$, $y_r = \sin(4\pi t/45)$ and $\theta_r = \pi\cos(\pi t/16)/4$, and the initial robot pose is $(x(0), y(0), \theta(0)) = (-0.5m, 0.2m, 0)$. Let $V = 2m/s, d = 0.7m, \varepsilon = 0.01, \alpha = 1, \mu = 0.99$. Thus $k_t = 1.6203\text{e-}04$. The simulation results are presented in Fig.3 and Fig.4.

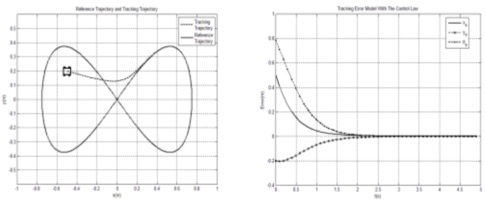


Fig. 4. Tracking trajectory and tracking errors.

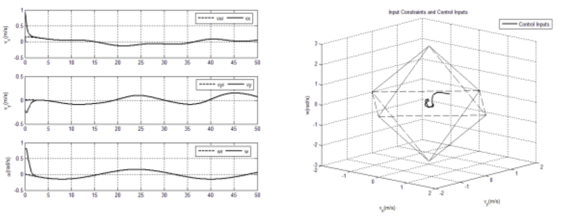


Fig. 4. Tracking velocity and control inputs.

Fig.3 and Fig.4 show that the tracking errors will converge to zero with the control law (15), and the control inputs are always in the area of input constraints.

5. Conclusion

In this paper, the tracking control problem of omnidirectioanl mobile robot with input constraints has been solved. A controller with time-varying feedback parameters is proposed using the spatial vector analysis method. The simulation results show that the control law can guarantee great tracking performance and the inputs will be located in the restricted area.

Acknowledgements

This work was supported by the National Basic Research Program of China (973 Program: 2012CB821200, 2012CB821201) and the NSFC (61134005, 60921001, 61327807).

References

1. Li, Yunqi, et al. "Sliding mode trajectory tracking control based on dynamic model for four-wheel drive omni-directional mobile robots." *Science Mosaic* (2014).
2. Zeng, Zhi Wen, et al. "Trajectory tracking based on model predictive control for omni-directional mobile robot." *Control Engineering of China* (2011).
3. Hashemi, Ehsan, M. G. Jadidi, and N. G. Jadidi. "Model-based PI–fuzzy control of four-wheeled omni-directional mobile robots." *Robot. Auton. Syst.* 59.11(2011):930–942.
4. A. Leonessa, W. M. Haddad, T. Hayakawa, and Y. Morel, "Adaptive control for nonlinear uncertain systems with actuator amplitude and rate saturation constraints." *Int. J. Adapt. Control Signal Process.* 23.1(2009):73-96.
5. Chen, Xiaohan, Y. Jia, and F. Matsuno. "Tracking control for differential-drive mobile robots with diamond-shaped input constraints." *IEEE Trans. Control Syst. Technol.* 22.5(2014):1999-2006.